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Novel results on observer-based control of one-sided Lipschitz systems under input saturation

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Abstract: This paper addresses the design of an observer-based control system for the one-sided Lipschitz (OSL) nonlinear systems under input saturation. A nonlinear matrix inequality-based control law for the nonlinear systems under input saturation and unavailable states is derived to ensure convergence of the state vector to the origin. A decoupling approach is provided for attaining simple design constraints for computing the controller and observer gains through a cone complementary linearization algorithm. In contrast to the conventional decoupling methods, the proposed approach considers OSL nonlinearity and saturation function to demonstrate both the necessity and sufficiency of the decoupled design constraints for the nonlinear matrix inequality-based main condition. To the best of our knowledge, observer-based stabilization of OSL systems under input saturation has been addressed for the first time. Novel results for the observer-based control of input-constrained Lipschitz nonlinear systems are provided as specific scenarios of the proposed results. Simulation results of the proposed control scheme to a flexible-joint robot and a complex nonlinear circuit are presented.

Keywords: Observer-based control; input saturation; one-sided Lipschitz nonlinearity; decoupling technique; cone complementary linearization

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1. Introduction

Almost all physical systems are subject to the input saturation nonlinearity, which degrades the closed-loop performance, causes lag in the output, and induces instability in the closed-loop response owing to the so-called windup phenomenon [1]-[7]. Careful attentions are required for the control of sensitive, expensive, and critical systems for attaining safety from accidents, for preservation from hazards, and to avoid financial losses. To attain the desired control performance under input saturation, many control methods such as anti-windup compensators [7]-[10] and feedback controllers with windup protection ([4]-[6], [10]-[14]), have been exercised in the existing literature.

Stabilization of linear or nonlinear systems under input saturation by taking feedback of the state vector and by employing the windup protection controllers have been studied in the literature. A state feedback control approach for the stabilization of linear systems under nested input saturation by employing a local generalized sector condition has been provided in [6]. Two interesting robust adaptive computationally complex tracking control approaches, based on back-stepping, are recently developed ([4], [11]). An adaptive control law by relaxing the nonsingular input coefficient matrix was also studied for the nonlinear systems, as seen in the work [12], for adaptive neural controller synthesis. In [13], local robust synchronization and anti-synchronization methodologies for the Lipschitz nonlinear systems with saturating actuators that require feedback of the full state vector have been worked out. In [14], a simple state feedback stabilization methodology for the discrete-time linear systems under input delay and saturation is proposed by application of a parametric Lyapunov function. Local state feedback stabilization approaches, providing an estimate of the region of stability, for linear systems under input and state delays have been explored in [15] and [16] using a

Lyapunov-Krasovskii functional. Regional state feedback stabilization schemes for linear systems under quantization effects and bilinear terms have been focused in [17] and [18], respectively.

A main drawback of the stabilization approaches mentioned above is the requirement of the exact states for feedback, which cannot be measurable in practical scenarios due to the restricted number of sensors. When these states are not available, observer-based feedback controllers are employed to simultaneously estimate and control the states of linear or nonlinear systems ([19]-[25]). To compute the gains of the controller and observer, the main observer-based controller design conditions can be decoupled into convex constraints that can be easily solved using numerical algorithms. The goal of a decoupling technique is to develop the less conservative design conditions that provide at least sufficient and, preferably, both sufficient and necessary conditions for the main constraints. However, derivation of an observer-based feedback control law for nonlinear systems facing the input constraint is a non-trivial research problem owing to the apparent complexities like saturation nonlinearity, state nonlinearity, unavailability of exact states, simultaneous computation of the observer and controller gains, and the formulation of convex design constraints.

Several nonlinear and adaptive nonlinear control approaches have been developed for the control of nonlinear systems under input saturation. The work of [26] presented a feedback linearization control scheme with dynamic compensation to control the linearizable nonlinear systems. In the approach [11], an adaptive control scheme to deal with the input-constrained nonlinear systems has been developed. These global approaches lack in the attainment of a performance objective, because it is easy to attain the specific performance goals for a local region via a regional control approach.

Recently, a dynamic nonlinear (computationally complex) compensation approach, utilizing the system states for feedback, is provided for global or local stability in [10]. In practical scenarios, these methods have limited applicability due to consideration of restrictive classes of nonlinear systems and owing to the computational complexity for their implementation.

Control of linear and nonlinear systems has several industrial applications to deal with the saturation effects [27]. As far as biomedical application is concerned, a recent study demonstrates that the controller design for a prosthetic hand for anthropomorphic coordination should be considered by accounting the saturation nonlinearity bound [28]. Flexible engineering structures should be attained by accounting the actuator saturation for achieving an intelligent control scheme [29]. Boundary adaptive controls for a complicated scenario of a flexible riser using the adaptive approach are considered in [30, 31], which works have their roots in the practical study [32]. Control of robotic manipulators and riser vessel by modeling the external perturbations and disturbances have been performed in the exceptional works [33, 34]. Actuator constraints are critical issues to develop control strategies for sensitive applications [35, 36] in rigid space crafts and surface vessels. Control of an induction motor under nonlinearities, developed in [37] to deal with stochastic parameters, suggests the incorporation of an actuator limits for an efficient control dilemma. Observers and state estimators are more likely to be involved for uncertain systems when states of a plant are not available as seen in [38, 39] owing to practical limits of measurement systems. All these recent control schemes and applications recommended the consideration of the input constraint and observers for attaining a high quality and risk-free operation of industrial systems.

It has been observed in several recent studies that one-sided Lipschitz (OSL) systems represent a generalized and less conservative class of nonlinear systems (please see [40]-[44] and the references therein). For the classical Lipschitz functions, the inner product between (i) the difference of a nonlinearity for two points and (ii) the difference in same points can be characterized by a two-sided inequality, leading to both lower and upper bounds. However, in one-sided Lipschitz function case, the above-said inner product is bounded by an upper bound only, leading to a one-sided inequality. This one-sided Lipschitz condition is therefore more general than the classical Lipschitz property. This generalized property leads to a less conservative controller or observer design along with inherent difficulties in dealing with the corresponding generalized models. For instance, we often need to incorporate the quadratic inner-boundedness condition in a design paradigm for attaining feasible results. Although this property introduces conservatism in the design method in terms of reduced applicability domain; however, the results are still general than the classical techniques for Lipschitz models.

Several observer-based control approaches are developed for the OSL nonlinear plants. For instance, the work of [25] highlights the observer-based controller scheme for the OSL nonlinear systems. Further, an observer-based robust stabilization of uncertain nonlinear systems in the presence of parametric uncertainties has been focused in [40] by considering complex matrix inequality procedures. Moreover, the idea of a robust control system design for the general class has been extended for discrete-time systems to cope with digital technologies [41]. The ideas have been extended for the control of multiple nonlinear agents [42, 43] and for iterative learning control [44] to deal with the complex environment and to attain intelligent control applications. However, all of these studies in [40-44] do not incorporate the input saturation, which can lead to performance degradation or even instability of the closed-loop response for a practical system. The effect of input saturation for the OSL nonlinear systems has been rarely addressed in the literature. The recent approach of [46] considers anti-windup design for controlling the OSL nonlinear systems; however, the application of the method is limited due to the requirement of the full state vector of the plant. Therefore, further work is needed to device observer-based control strategies, not requiring the exact state of the system, for the stabilization of input-saturated OSL nonlinear systems.

This paper addresses an observer-based controller design, based on a Luenberger-type nonlinear observer and feedback of the estimated state vector, for the OSL nonlinear systems under input saturation constraint. A nonlinear matrix inequality-based condition for finding the controller and observer gains, simultaneously, is derived by application of inequality tools, saturation sector condition, OSL condition, and quadratic inner-boundedness approach. Further, a decoupling method is provided to decouple the main nonlinear matrix inequality-based design condition into simple and numerically tractable design constraints for straightforward computation of the observer-based controller gains. It is demonstrated that the proposed decoupled constraints are necessary and sufficient for obtaining a solution for the main design approach, which reflects the less conservativeness of the proposed approach.

Compared with the existing decoupling methodologies, the proposed decoupling approach can be applied to a wide range of nonlinear systems under input saturation and state nonlinearity. This paper considers two types of nonlinearities, namely, the nonlinearity in the system dynamics and the actuator saturation nonlinearity, to design an observer-based control strategy in contrast to the conventional techniques [40-44]. In contrast to [46], the requirement of full state vector for the generalized nonlinear systems has been overcome in our study. It is notable that a novel observer-based control scheme by considering the input saturation for the Lipschitz nonlinear systems is derived as a specific case of the proposed method. Numerical simulation results for the control of a robotic arm and a complex circuit are pursued to demonstrate the application of the proposed control methodology. The main contributions of the present study are as follows:

i) *To the best of authors' knowledge,* an observer-based stabilization method, not requiring the full state information, for the input-saturated one-sided Lipschitz nonlinear systems has been provided for the first time. Further, a methodology for the straightforward computation of the observer-based control parameters is attained by using the cone complementary linearization approach and knowledge of sign of OSL and quadratic inner-boundedness condition parameters.

ii) A guaranteed region of stability for the control of input-saturated nonlinear systems in terms of states and state estimation error by utilizing the local sector condition and Lyapunov redesign is ensured.

iii) Simple design conditions are also formulated by a novel decoupling scheme to ensure the necessity and sufficiency of the decoupled conditions for the primary design approach.

Standard notation is used throughout this paper. Given a matrix Z, $Z_{(m)}$ represents the *m*th row of Z. Euclidian norm of a vector x is represented by $\|x\|$ and $diag\{s_1, s_2, ..., s_m\}$ stands for a diagonal matrix with entry s_i at *i*th-diagonal element. For an input signal $u \in R^m$, the input saturation nonlinearity is defined as

$\psi_{(i)}(u_{(i)}) = \text{sgn}(u_{(i)})\min\left(\bar{u}_{(i)}, \left|u_{(i)}\right|\right)$, where $\bar{u}_{(i)} > 0$ refers to the ith bound on the saturation function. The notation $\langle a,b \rangle$ has been employed to denote the dot product of vectors a and b.

2. System description

Consider a nonlinear system, given by

$$\begin{aligned} \dot{x} &= Ax(t) + f(t,x) + B\psi(u), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \Re^n$, $y \in \Re^p$ and $u \in \Re^m$ are the state, output and input vectors, respectively, $f(t,x) \in \Re^n$ represents the nonlinearity associated with the state vector, and A, B and C are matrices representing the linear components of the system. The function $\psi(u)$ is used to represent the input saturation nonlinearity.

Assumption 1: The function $f(t,x)$ with OSL constant ρ satisfies $f(t,0) = 0$ and

$$\left\langle f(t,x) - f(t,\bar{x}), x - \bar{x} \right\rangle \le \rho \left\| x - \bar{x} \right\|^2. \tag{2}$$

Assumption 2: For $f(t,x)$, the following condition for $\alpha, \beta \in \Re$ and $\forall x, \bar{x} \in \Re^n$ is valid:

$$\begin{aligned} &\left(f(t,x) - f(t,\bar{x}) \right)^T \left(f(t,x) - f(t,\bar{x}) \right) \\ &\qquad \le \beta \left\| x - \bar{x} \right\|^2 + \alpha \left\langle x - \bar{x}, f(t,x) - f(t,\bar{x}) \right\rangle. \end{aligned} \tag{3}$$

It is worth mentioning that the conditions in (2)-(3) are a generalization of the conventional Lipschitz condition. It is also notable that the OSL constant ρ has smaller or at most equal value to the Lipschitz constant, which fact can be used for a less conservative control system design. Moreover, ρ can have any positive, zero or even negative value, while the Lipschitz constant is always positive. The proposed controller takes the form as

$$u(t) = F\hat{x}(t), \tag{4}$$

where $F \in \Re^{m\times n}$ and $\hat{x}(t) \in \Re^{n}$ represent the controller gain matrix and the estimated state vector. Using (1) and (4) and, further, substituting $\varphi(u) = u - \psi(u)$, we obtain

$$\begin{aligned} \frac{dx}{dt} &= Ax(t) + f(t,x) + BF\hat{x}(t) - B\varphi(u), \\ y(t) &= Cx(t). \end{aligned} \tag{5}$$

For state vector estimation, we consider the following observer:

$$\begin{aligned} \frac{d\hat{x}}{dt} &= A\hat{x}(t) + f(t,\hat{x}) + B\psi(u) + L\left(y(t) - \hat{y}(t)\right), \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \tag{6}$$

where $L \in \Re^{n\times p}$ is the observer gain. The proposed observer in (6) provides an estimated state $\hat{x}(t)$, which will be employed through the feedback controller (4). Let us define the state estimation error as $e = x - \hat{x}$. Using (5), (6), $\varphi(u) = u - \psi(u)$, and $e = x - \hat{x}$, an augmented system is obtained as

$$\dot{z}(t) = (\tilde{A} + \tilde{G})z(t) + g(t,x,\hat{x}) + \tilde{B}\varphi(u), \tag{7}$$

$$z(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T, \tag{8}$$

$$g(t,x,\hat{x}) = \begin{bmatrix} f^T(t,x) & \left(f(t,x) - f(t,\hat{x})\right)^T \end{bmatrix}^T, \tag{9}$$

$$\tilde{A} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \; \tilde{G} = \begin{bmatrix} BF & -BF \\ 0 & -LC \end{bmatrix}, \; \tilde{B} = \begin{bmatrix} -B \\ 0 \end{bmatrix}. \tag{10}$$

For a diagonal matrix *W*, the local sector condition

$$\varphi^T(u)W\left(w - \varphi(u)\right) \geq 0, \; W > 0 \tag{11}$$

remains valid, if the region

$$\Upsilon(\bar{u}) = \left\{ w \in \Re^m;\ -\bar{u} \le u - w \le \bar{u} \right\} \tag{12}$$

exists for an auxiliary defined vector $w \in \Re^m$, where $\bar{u}$ is a vector containing the saturation bounds (see [6] and [7]). It can be observed that the overall closed-loop system dynamics has been transformed into (7), which contains the dead-zone nonlinearity $\varphi(u)$, rather than the original input constraint $\psi(u)$. This variation can be interesting due to two reasons: First, the resultant matrix $\tilde{A}+\tilde{G}$ has term $A+BF$, which can be used to ensure stability of the closed-loop system. Second, it is more convenient to apply the sector condition (11), which can be more complicated if the original saturation function is employed for the design in the present scenario.

The present study explores simultaneous controller (4) and observer (6) design for the stabilization of the nonlinear system (1) under the input saturation constraint by considering the conditions in Assumptions 1-2 and by incorporating the saturation properties of (11)-(12).

3. Main results

In this section, we develop conditions for determining the observer and controller gains L and F for stabilization of (1) under input saturation. A nonlinear matrix inequality-based design condition for observer (6) and controller (4) is derived herein.

Theorem 1: *Consider the input-constrained nonlinear system (1) satisfying Assumptions 1-2. There exist a controller (4) and an observer (6) ensuring the asymptotic convergence of the augmented system's state vector $z(t)$ to the origin for initial condition validating $z^T(0)Pz(0) \le 1$, if for matrices $P \in \Re^{2n\times 2n}$, $F \in \Re^{m\times n}$,*

$L \in \Re^{n\times p}$ and $J \in \Re^{m\times n}$, a diagonal matrix $W \in \Re^{n\times n}$, and scalars $\varepsilon_1, \varepsilon_2$, ε_3 and ε_4, the matrix inequalities

$$P = P^T > 0, W > 0, \ \varepsilon_1 > 0, \ \varepsilon_2 > 0, \ \varepsilon_3 > 0, \ \varepsilon_4 > 0, \tag{13}$$

$$\begin{bmatrix} P & \eta^T F_{(i)}^T - \eta^T J_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \ge 0, \ \forall i = 1, \dots, m, \tag{14}$$

$$\begin{bmatrix} X_1 + \lambda_1 & P + \lambda_2 & P\tilde{B} + \eta^T J^T W \\ * & X_2 & 0 \\ * & * & -2W \end{bmatrix} < 0, \tag{15}$$

are satisfied, where

$$\begin{aligned} &\eta = \begin{bmatrix} I & -I \end{bmatrix}, \\ &X_1 = \tilde{A}^T P + P\tilde{A} + \tilde{G}^T P + P\tilde{G}, \\ &X_2 = -diag\{\varepsilon_2 I, \varepsilon_4 I\}, \\ &\lambda_1 = diag\{\rho\varepsilon_1 I, \rho\varepsilon_3 I\} + diag\{\beta\varepsilon_2 I, \beta\varepsilon_4 I\}, \\ &\lambda_2 = -\frac{1}{2} diag\{\varepsilon_1 I, \varepsilon_3 I\} + \frac{1}{2} diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}. \end{aligned} \tag{16}$$

Proof: Consider a Lyapunov function as

$$V(t,z) = z^T(t) P z(t). \tag{17}$$

The time-derivative of (17) along (7) becomes

$$\begin{aligned} \dot{V}(t,z) = {} & z^T \left(\tilde{A}^T P + P\tilde{A} + \tilde{G}^T P + P\tilde{G} \right) z + z^T P g(t,x,\hat{x}) \\ & + g^T(t,x,\hat{x}) P z + z^T P \tilde{B} \varphi(u) + \varphi^T(u) \tilde{B}^T P z. \end{aligned} \tag{18}$$

From Assumption 1, under $f(t,0) = 0$, we have

$$x^T f(t,x) \le \rho x^T x.$$

By taking $\bar{x} = \hat{x}$ in (2), we obtain

$$(x - \hat{x})^T \left(f(t,x) - f(t,\hat{x}) \right) \le \rho (x - \hat{x})^T (x - \hat{x}).$$

Multiplying the above conditions with positive scalars ε_1 and ε_3, combining them, applying $e = x - \hat{x}$, and using (8)-(9), we have

$$z^T(t)\,diag\{\rho\varepsilon_1 I, \rho\varepsilon_3 I\}\,z(t) - z^T(t)\,diag\{\varepsilon_1 I, \varepsilon_3 I\}\,g(t,x,\hat{x}) \geq 0. \tag{19}$$

Taking $\bar{x} = 0$ in Assumption 2 and multiplying the resultant with scalar $\varepsilon_2 > 0$ lead to

$$\varepsilon_2 f^T(t,x)\,f(t,x) \leq \varepsilon_2 \beta x^T x + \varepsilon_2 \alpha x^T f(t,x).$$

For the case when $\bar{x} = \hat{x}$ (under $\varepsilon_4 > 0$), the inequality (3) implies

$$\begin{aligned}&\varepsilon_4\left(f(t,x) - f(t,\hat{x})\right)^T\left(f(t,x) - f(t,\hat{x})\right)\\ &\quad\leq \varepsilon_4\beta(x-\hat{x})^T(x-\hat{x}) + \varepsilon_4\alpha(x-\hat{x})^T\left(f(t,x) - f(t,\hat{x})\right).\end{aligned}$$

Combining the above two inequalities along with the formulations $e = x - \hat{x}$ and (8)-(9), it produces

$$\begin{aligned}&z^T(t)\,diag\{\beta\varepsilon_2 I, \beta\varepsilon_4 I\}\,z(t) - g^T(t,x,\hat{x})\,diag\{\varepsilon_2 I, \varepsilon_4 I\}\,g(t,x,\hat{x})\\ &\qquad + z^T(t)\,diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}\,g(t,x,\hat{x}) \geq 0.\end{aligned} \tag{20}$$

Combining the conditions (18)-(20) entails

$$\begin{aligned}\dot{V}(t,z) \leq\; & z^T\left(\tilde{A}^T P + P\tilde{A} + \tilde{G}^T P + P\tilde{G}\right)z\\ & + z^T\left(P - \frac{1}{2}diag\{\varepsilon_1 I, \varepsilon_3 I\} + \frac{1}{2}diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}\right)g(t,x,\hat{x})\\ & + g^T(t,x,\hat{x})\left(P - \frac{1}{2}diag\{\varepsilon_1 I, \varepsilon_3 I\} + \frac{1}{2}diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}\right)z\\ & + z^T P\tilde{B}\varphi(u) + \varphi^T(u)\tilde{B}^T P z - g^T(t,x,\hat{x})\left(diag\{\varepsilon_2 I, \varepsilon_4 I\}\right)g(t,x,\hat{x})\\ & + z^T\left(diag\{\rho\varepsilon_1 I, \rho\varepsilon_3 I\} + diag\{\beta\varepsilon_2 I, \beta\varepsilon_4 I\}\right)z.\end{aligned} \tag{21}$$

Using (8), (16), and $e = x - \hat{x}$ and, further, setting $w = J\hat{x}$, the constraints in (11)-(12) are rewritten as

$$\varphi^T(u)W\left[J\eta z - \varphi(u)\right] \geq 0, \tag{22}$$

$$\Upsilon(\bar{u}) = \left\{w \in \Re^m;\; -\bar{u} \leq (F\eta - J\eta)z \leq \bar{u}\right\}. \tag{23}$$

Incorporating (22) into (21), it produces

$$\begin{aligned}\dot{V}(t,z) \le\ & z^T\left(\tilde{A}^T P + P\tilde{A} + \tilde{G}^T P + P\tilde{G}\right)z \\ & + z^T\left(P - \frac{1}{2}diag\{\varepsilon_1 I, \varepsilon_3 I\} + \frac{1}{2}diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}\right)g(t,x,\hat{x}) \\ & + g^T(t,x,\hat{x})\left(P - \frac{1}{2}diag\{\varepsilon_1 I, \varepsilon_3 I\} + \frac{1}{2}diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\}\right)z \\ & + z^T P\tilde{B}\varphi(u) + \varphi^T(u)\tilde{B}^T Pz - g^T(t,x,\hat{x})\left(diag\{\varepsilon_2 I, \varepsilon_4 I\}\right)g(t,x,\hat{x}) \\ & + z^T\left(diag\{\rho\varepsilon_1 I, \rho\varepsilon_3 I\} + diag\{\beta\varepsilon_2 I, \beta\varepsilon_4 I\}\right)z \\ & + \varphi^T(u)W\left[J\eta z - \varphi(u)\right] + \left[J\eta z - \varphi(u)\right]^T W\varphi(u).\end{aligned} \tag{24}$$

The inequality (24) further implies that $\dot{V}(t,z) \le Z^T \Psi Z$, by employing the S-procedure, where

$$\Psi = \begin{bmatrix} \begin{pmatrix} X_1 + diag\{\rho\varepsilon_1 I, \rho\varepsilon_3 I\} \\ + diag\{\beta\varepsilon_2 I, \beta\varepsilon_4 I\} \end{pmatrix} & \begin{pmatrix} P - \frac{1}{2}diag\{\varepsilon_1 I, \varepsilon_3 I\} \\ + \frac{1}{2}diag\{\alpha\varepsilon_2 I, \alpha\varepsilon_4 I\} \end{pmatrix} & P\tilde{B} + \eta^T J^T W \\ * & X_2 & 0 \\ * & * & -2W \end{bmatrix}, \tag{25}$$

$$Z = \begin{bmatrix} z^T & g^T(t,x,\hat{x}) & \varphi^T(u) \end{bmatrix}^T. \tag{26}$$

Note that $\Psi < 0$ ensures $\dot{V}(t,z) < 0$, which further renders (15). As $\dot{V}(t,z) < 0$, the augmented state $z(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$ will converge to the origin. It leads to the stability of the system's state $x(t)$ and state estimation error $e(t)$. However, this stability will be guaranteed for a local region of initial conditions in the neighborhood of the origin due to the presence of the input saturation nonlinearity $\psi(u)$. To investigate this region, note that $\dot{V}(t,z) < 0$ implies $V(t,z) \le V(0,z)$ (for time $t \ge 0$). By using (17), we obtain $z^T(t)Pz(t) \le z^T(0)Pz(0)$, which by application of the given initial conditions $z^T(0)Pz(0) \le 1$ produces $z^T(t)Pz(t) \le 1$. We require to include the region $z^T(t)Pz(t) \le 1$

into $\Upsilon(\bar{u})$ for validating the region $\Upsilon(\bar{u})$, required to satisfy the sector condition (22) for saturation function. By including the ellipsoidal region $z^T(t)Pz(t) \le 1$ into the region $\Upsilon(\bar{u})$ given in (23) and further employing $w = J\hat{x}$ and $u(t) = F\hat{x}(t)$, we obtain

$$P - \bar{u}_{(i)}^{-2}\left(\eta^T F_{(i)}^T - \eta^T J_{(i)}^T\right)\left(F_{(i)}\eta - J_{(i)}\eta\right) \ge 0 . \tag{27}$$

It further produces (14) by application of the Schur complement, which completes the proof of Theorem 1. □

Remark 1: Several studies concerning control or observer-based control of the OSL nonlinear systems are available in the literature [25], [40]-[44]. These approaches ignore the presence of saturation nonlinearity at the control input due to bounded-input restriction of physical systems. In contrast to these traditional methods [25] and [40]-[44], the proposed methodology in Theorem 1 develops a control approach for the OSL nonlinear plants by regarding the input saturation constraint. This input saturation cannot be ignored in physical systems due to bounded-input limitation of actuators and can cause undesirable performance degradation. It should be noted that observer-based stabilization of the more generic OSL nonlinear plants under input saturation constraint has been lacking in the previous studies.

Remark 2: Recently, a new robust control approach for the OSL systems along with applications by considering the actuator saturation is developed in [46]. Nevertheless, this conventional control scheme cannot be practically used, for instance, when states of a system are not available for feedback. The anti-windup compensation scheme, applied in the study, relies on the feedback of states of nonlinear systems for the protection of the closed-loop response against saturation. Contrastingly, the present study employs an estimated state vector, rather than the actual states, to stabilize the nonlinear systems. In addition, if an observer is added to the approach of [46], the resultant scheme becomes quite complex. This complexity issue can cause extra computations and additional hardware for implementation of the schema over digital and analog technologies, respectively. It is worth mentioning that the proposed approach uses output feedback (rather than the state feedback) for controlling the OSL plants. To the best of our knowledge, such a control scheme for stabilization of the generalized form of nonlinear systems under input saturation and using output feedback has been revealed for the first time.

Remark 3: The conventional methods like ([4], [6], [11]-[18]) investigated the control schemes for linear or nonlinear systems under input saturation by using state feedback to ensure convergence of states of systems. These conventional methods cannot be implemented, if all states of a plant are not available for feedback. However, measurement of all states of a plant requires extra sensors and extra hardware for amplification and calibration, which limits the applicability of the scheme in practical scenarios. The present work employs estimated states rather than the actual states for the control purpose, which is more practical approach.

Remark 4: Analysis of the region of stability in the classical control methods [4], [6], [11]-[18] is a relatively less complicated task due to simplification in the control structure by avoiding the state estimates. In the present case, a region of stability is needed for the augmented state $z(t)=\begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$. Consequently, an estimate of the region for stability is obtained for the augmented vector $z(t)$, containing both the state and the estimation error. Obtainment of a region of stability for the proposed observer-based stabilization scenario is a non-trivial research problem in contrast to the conventional methods due to consideration of both actual and estimated states for determining the region of stability. In addition, the proposed approach provides a new direction for controlling systems under input saturation and employs a novel treatment of the local sector condition (11)-(12) for the observer-based control.

Corollary 1: *Consider the input-constrained Lipschitz nonlinear system (1) satisfying Assumption 2 with* $\alpha = 0$ *and* $\beta = \Lambda^2$. *There exist a controller (4) and an observer (6) ensuring the asymptotic convergence of the augmented system's state vector* $z(t)$ *to the origin for all initial condition* $z^T(0)Pz(0) \le 1$, *if for matrices* $P \in \Re^{2n\times 2n}$, $F \in \Re^{m\times n}$, $L \in \Re^{n\times p}$, *and* $J \in \Re^{m\times n}$, *a diagonal matrix* $W \in \Re^{n\times n}$, *and scalars* ε_2 *and* ε_4, *the matrix inequality (14) and the following constraints are satisfied:*

$$P = P^T > 0, W > 0,\ \varepsilon_2 > 0,\ \varepsilon_4 > 0, \tag{28}$$

$$\begin{bmatrix} X_1 + \lambda_3 & P & P\tilde{B} + \eta^T J^T W \\ * & X_2 & 0 \\ * & * & -2W \end{bmatrix} < 0, \tag{29}$$

$$\lambda_3 = diag\left\{\varepsilon_2 \Lambda^2 I, \varepsilon_4 \Lambda^2 I\right\}. \tag{30}$$

Remark 5: The proposed observer-based control methodology for the OSL nonlinear systems in Theorem 1 has been reduced to provide a specific (but novel) result for the Lipschitz nonlinear systems in Corollary 1. The condition in Assumption 2 for $\alpha = 0$ and $\beta = \Lambda^2$ reduces to the conventional Lipschitz condition. By applying these substitutions along with $\varepsilon_1 = 0$ and $\varepsilon_3 = 0$, the approach of Theorem 1 produces the result in Corollary 1. This novel result highlights the flexibility of the proposed controller design in Theorem 1 for application to a less conservative and more general class of nonlinear systems under input saturation. It is notable that the observer-based control of Lipschitz nonlinear systems for dealing with the input saturation has been remained rare in the existing works, which further validates the novelty of the proposed results in Theorem 1.

It is hard to compute the controller and observer gain matrices using Theorem 1 (or Corollary 1). Therefore, we provide a necessary and sufficient condition for the existence of the solution to Theorem 1 by application of a decoupling approach.

Theorem 2: *A solution to the conditions in Theorem 1 exists, if and only if there exist matrices* $P_1 \in \Re^{n\times n}$, $P_2 \in \Re^{n\times n}$, $Z \in \Re^{m\times n}$, $S \in \Re^{m\times n}$, *and* $M \in \Re^{n\times p}$, *a diagonal matrix* $U \in \Re^{n\times n}$, *and scalars* $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2$, $\tilde{\varepsilon}_3$ *and* $\tilde{\varepsilon}_4$ *such that the matrix inequalities*

$$P_1 = P_1^T > 0,\ P_2 = P_2^T > 0,\ U > 0,\ \tilde{\varepsilon}_1 > 0, \tilde{\varepsilon}_2 > 0, \tilde{\varepsilon}_3 > 0, \tilde{\varepsilon}_4 > 0, \tag{31}$$

$$\begin{bmatrix} P_1 & Z_{(i)}^T - S_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0,\ \forall i = 1,\ldots,m, \tag{32}$$

$$\cdot\begin{bmatrix} P_1 P_2 P_1 & -Z_{(i)}^T + S_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0,\ \forall i = 1,\ldots,m, \tag{33}$$

$$\begin{bmatrix} P_1 A^T + AP_1 + BZ + Z^T B^T + P_1 \rho \tilde{\varepsilon}_1 P_1 + P_1 \beta \tilde{\varepsilon}_2 P_1 & I - \dfrac{P_1 \tilde{\varepsilon}_1 I}{2} + \dfrac{P_1 \alpha \tilde{\varepsilon}_2 I}{2} & -BU + S^T \\ * & -\tilde{\varepsilon}_2 I & 0 \\ * & * & -2U \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} A^T P_2 + P_2 A + MC + C^T M^T + \beta \tilde{\varepsilon}_4 I + \rho \tilde{\varepsilon}_3 I & P_2 - \dfrac{\tilde{\varepsilon}_3 I}{2} + \dfrac{\alpha \tilde{\varepsilon}_4 I}{2} \\ * & -\tilde{\varepsilon}_4 I \end{bmatrix} < 0, \tag{35}$$

are satisfied. Various gains for the proposed observer-based control approach can be computed via $F = ZP_1^{-1}$, $J = SP_1^{-1}$ *and* $L = P_2^{-1} M$.

Proof: *Sufficiency:* To prove sufficiency, we combine (32)-(33) and (34)-(35) to obtain (14) and (15), respectively. Applying the congruence transformation to (34) through $diag\{\bar{P}_1, I, W\}$, choosing $\bar{P}_1 = P_1^{-1}$ and $W = U^{-1}$, and using $F\bar{P}_1^{-1} = Z$ and $J\bar{P}_1^{-1} = S$, we obtain

$$\Pi_{11} = \begin{bmatrix} A^T\bar{P}_1 + \bar{P}_1 A + \bar{P}_1 BF + F^T B^T \bar{P}_1 + \rho\tilde{\varepsilon}_1 I + \beta\tilde{\varepsilon}_2 I & \bar{P}_1 - \frac{\tilde{\varepsilon}_1 I}{2} + \frac{\alpha\tilde{\varepsilon}_2 I}{2} & -\bar{P}_1 B + J^T W \\ * & -\tilde{\varepsilon}_2 I & 0 \\ * & * & -2W \end{bmatrix} < 0. \quad (3$$

Using $P_2 L = M$ in (35), we have

$$\Pi_{22} = \begin{bmatrix} A^T P_2 + P_2 A + P_2 LC + C^T L^T P_2 + \beta\tilde{\varepsilon}_4 I + \rho\tilde{\varepsilon}_3 I & P_2 - \frac{\tilde{\varepsilon}_3 I}{2} + \frac{\alpha\tilde{\varepsilon}_4 I}{2} \\ * & -\tilde{\varepsilon}_4 I \end{bmatrix} < 0. \quad (37)$$

By application of the congruence transformation through $diag\{\bar{P}_1, I\}$ and using $F = Z\bar{P}_1$ and $J = S\bar{P}_1$, the sets of inequalities in (32)-(33) are rewritten as

$$\begin{bmatrix} \bar{P}_1 & F_{(i)}^T - J_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0, \ \forall i = 1, \ldots, m, \quad (38)$$

$$\begin{bmatrix} P_2 & -F_{(i)}^T + J_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0, \ \forall i = 1, \ldots, m. \quad (39)$$

Applying the Schur complement to (39) implies

$$P_2 - \bar{u}_{(i)}^{-2}\left(F_{(i)}^T - J_{(i)}^T\right)\left(F_{(i)} - J_{(i)}\right) \geq 0, \ \forall i = 1, \ldots, m. \quad (40)$$

$\Pi_{11} < 0$, $\Pi_{22} < 0$, (38) and (40) imply that there exists a sufficiently large positive scalar λ such that the inequalities

$$\begin{bmatrix} \Pi_{11} & \Pi_{12}^T \\ \Pi_{12} & \lambda\Pi_{22} \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} \bar{P}_1 & F_{(i)}^T - J_{(i)}^T & 0 \\ * & \bar{u}_{(i)}^2 & -F_{(i)} + J_{(i)} \\ * & * & \lambda\left(P_2 - \bar{u}_{(i)}^{-2}\left(F_{(i)}^T - J_{(i)}^T\right)\left(F_{(i)} - J_{(i)}\right)\right) + \Theta \end{bmatrix} \geq 0 \tag{42}$$

are satisfied, for all $i = 1, \dots, m$ and for any $\Theta \geq 0$, where

$$\Pi_{12} = \begin{bmatrix} 0 & 0 & -J^T W & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{43}$$

Under $\Pi_{11} < 0$ and $\Pi_{22} < 0$, a positive scalar λ can always be taken arbitrarily large to ensure (41). Similarly, the condition (42) can be ensured for a large positive number λ. Selecting

$$\Theta = \lambda \bar{u}_{(i)}^{-2}\left(F_{(i)}^T - J_{(i)}^T\right)\left(F_{(i)} - J_{(i)}\right) \tag{44}$$

and, further, exchanging the rows and columns of (41) and (42), we obtain (14) and (15). The substitutions $\tilde{\varepsilon}_1 = \varepsilon_1$, $\tilde{\varepsilon}_2 = \varepsilon_2$ $\tilde{\varepsilon}_3 = \varepsilon_3/\lambda$, $\tilde{\varepsilon}_4 = \varepsilon_4/\lambda$, and $P = diag(\bar{P}_1, \lambda P_2)$ have been employed for the derivation.

Necessity: To demonstrate the necessity, we consider the constraints (14) and (15) to deduce (32)-(35). Suppose there exists a matrix P satisfying (14)-(15), partitioning P as

$$P = \begin{bmatrix} \tilde{P}_1 & \otimes \\ * & \otimes \end{bmatrix}, \tag{45}$$

where $\otimes$ represents an entry which will not be employed in the sequel. Substituting (45) into (14)-(15), it yields

$$\begin{bmatrix} \tilde{P}_1 & \otimes & F_{(i)}^T - J_{(i)}^T \\ * & \otimes & \otimes \\ * & * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0, \ \forall i = 1, \dots, m, \tag{46}$$

$$\begin{bmatrix} \begin{pmatrix} A^T\tilde{P}_1+\tilde{P}_1A+\tilde{P}_1BF+F^TB^T\tilde{P}_1 \\ +\rho\varepsilon_1 I+\beta\varepsilon_2 I \end{pmatrix} & \otimes & \tilde{P}_1-\frac{\varepsilon_1 I}{2}+\frac{\alpha\varepsilon_2 I}{2} & \otimes & -\tilde{P}_1B+J^TW & \otimes \\ * & \otimes & \otimes & \otimes & \otimes & \otimes \\ * & * & -\varepsilon_2 I & \otimes & 0 & \otimes \\ * & * & * & \otimes & \otimes & \otimes \\ * & * & * & * & -2W & \otimes \\ * & * & * & * & * & \otimes \end{bmatrix}<0. \quad (47)$$

Pre- and post-multiplication of

$$\begin{bmatrix} \tilde{P}_1^{-1} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

and its transpose to (46) along with substitutions $P_1=\tilde{P}_1^{-1}$, $F=ZP_1^{-1}$, and $J=SP_1^{-1}$ verifies the constraint (32). Pre- and post-multiplication of

$$\begin{bmatrix} \tilde{P}_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W^{-1} & 0 \end{bmatrix}$$

and its transpose to inequality (47) and using transformations $P_1=\tilde{P}_1^{-1}$, $F=ZP_1^{-1}$, $J=SP_1^{-1}$, $U=W^{-1}$ and $\varepsilon_i=\tilde{\varepsilon}_i$ (for $i=1,2$), we obtain (34). Hence, the conditions (32) and (34) are necessary for the constraints in Theorem 1. Now, applying the partition

$$P=\begin{bmatrix} \otimes & \otimes \\ * & \tilde{P}_2 \end{bmatrix} \quad (48)$$

to inequality (14), it produces

$$\begin{bmatrix} \otimes & \otimes & \otimes \\ * & \tilde{P}_2 & F_{(i)}^T-J_{(i)}^T \\ * & * & \bar{u}_{(i)}^2 \end{bmatrix}\geq 0,\ \forall i=1,\ldots,m. \quad (49)$$

Substituting (48) into (15) implies

$$\begin{bmatrix} \otimes & \otimes & \otimes & \otimes & \otimes & \otimes \\ * & \begin{pmatrix} A^T\tilde{P}_2+\tilde{P}_2A+\tilde{P}_2LC+C^TL^T\tilde{P}_2 \\ +\beta\varepsilon_4 I+\rho\varepsilon_3 I \end{pmatrix} & \otimes & \tilde{P}_2-\frac{\varepsilon_3 I}{2}+\frac{\alpha\varepsilon_4 I}{2} & \otimes & \otimes \\ * & * & \otimes & \otimes & \otimes & \otimes \\ * & * & * & -\varepsilon_4 I & \otimes & \otimes \\ * & * & * & * & \otimes & \otimes \\ * & * & * & * & * & \otimes \end{bmatrix} < 0. \quad (50)$$

Similarly, pre- and post-multiplication of

$$\begin{bmatrix} 0 & \tilde{P}_1^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}$$

and its transpose to (49) and applying the transformations $P_1=\tilde{P}_1^{-1}$, $P_2=\tilde{P}_2$, $F=ZP_1^{-1}$, and $J=SP_1^{-1}$ imply (33). In the same line, pre- and post-multiplication of

$$\begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and its transpose to (50) and further applying transformations $P_2=\tilde{P}_2$, $M=\tilde{P}_2L$ and $\varepsilon_i=\tilde{\varepsilon}_i$ (for $i=3,4$), it results into (35). Hence, (33) and (35) are necessary conditions for (14) and (15). This completes the proof. □

Remark 6: By application of a rigorous decoupling approach, an equivalent form of Theorem 1 has been developed in the proposed scheme of Theorem 2. Mainly, the bulky conditions of (14)-(15) in Theorem 1 are degenerated into relatively simple conditions (32)-(35) in Theorem 2 without loss of generality. Compared to Theorem 1, this new approach of Theorem 2 can be quite easy to resolve for computation of the proposed observer (6) and controller (4) gains through matrix inequality procedures. It is also noted that the technique in Theorem 2 is less conservative because it provides both necessity and sufficiency for solving the main design constraints in Theorem 1.

Remark 7: The decoupling approach used in the present work comparably differs from [19]-[22], [23] and [25]. The presented decoupling approach in Theorem 2 is a sophisticated extension of [19]-[22] and [25] to the one-side Lipschitz nonlinear systems under the input saturation constraint. Note that the scenario of input saturation has been relatively less considered for the observer-based control synthesis. Such a decoupling condition is important as it establishes that observer and controller can be designed using separate conditions and their gains can be determined straightforwardly using convex routines. An extension for Lipchitz nonlinear time-delay systems without input saturation was developed in [23]; however, only a sufficient condition was provided. In contrast, the proposed scheme exhibits control result for a generic class of nonlinear systems by considering the saturation nonlinearity at the input signal.

The constraints of Theorem 2 are relatively simple than Theorem 1; however, these constraints contain nonlinear terms. The following theorem addresses the management of nonlinear term $P_1\rho\tilde{\varepsilon}_1 P_1 + P_1\beta\tilde{\varepsilon}_2 P_1$ without loss of generality (by introducing any conservatism).

Theorem 3: *A solution to the conditions in Theorem 1 exists, if and only if there exist matrices* $P_1 \in \Re^{n\times n}$, $P_2 \in \Re^{n\times n}$, $Z \in \Re^{m\times n}$, $S \in \Re^{m\times n}$, *and* $M \in \Re^{n\times p}$, *a diagonal matrix* $U \in \Re^{n\times n}$, *and scalars* $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2$, $\tilde{\varepsilon}_3$ *and* $\tilde{\varepsilon}_4$ *such that the matrix inequalities in (31), (32), (33), (35), and either of the following conditions hold true:*

(i) If $\rho\tilde{\varepsilon}_1 + \beta\tilde{\varepsilon}_2 > 0$, *then*

$$\begin{bmatrix} P_1A^T + AP_1 + BZ + Z^TB^T & I - \frac{P_1\tilde{\varepsilon}_1 I}{2} + \frac{P_1\alpha\tilde{\varepsilon}_2 I}{2} & -BU + S^T & \sqrt{\rho\tilde{\varepsilon}_1 + \beta\tilde{\varepsilon}_2}P_1 \\ * & -\tilde{\varepsilon}_2 I & 0 & 0 \\ * & * & -2U & 0 \\ * & * & * & -I \end{bmatrix} < 0. \quad (51)$$

(ii) If $\left(\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2\right)=0$*, then*

$$\begin{bmatrix} P_1A^T+AP_1+BZ+Z^TB^T & I-\dfrac{P_1\tilde{\varepsilon}_1 I}{2}+\dfrac{P_1\alpha\tilde{\varepsilon}_2 I}{2} & -BU+S^T \\ * & -\tilde{\varepsilon}_2 I & 0 \\ * & * & -2U \end{bmatrix}<0. \tag{52}$$

(iii) If $\left(\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2\right)<0$*, then*

$$\begin{bmatrix} P_1A^T+AP_1+BZ+Z^TB^T-T_1 & I-\dfrac{P_1\tilde{\varepsilon}_1 I}{2}+\dfrac{P_1\alpha\tilde{\varepsilon}_2 I}{2} & -BU+S^T \\ * & -\tilde{\varepsilon}_2 I & 0 \\ * & * & -2U \end{bmatrix}<0. \tag{53}$$

Here $T_1=-P_1\left(\rho\tilde{\varepsilon}_1 I+\beta\tilde{\varepsilon}_2 I\right)P_1$*. Various gains for the proposed observer-based control approach can be computed via* $F=ZP_1^{-1}$*,* $J=SP_1^{-1}$ *and* $L=P_2^{-1}M$ *.*

Proof. For the case $\left(\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2\right)>0,$ the constraints (34) and (51) are equivalent by employing the Shur complement. The inequality in (34) renders (52) by assigning $\left(\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2\right)=0$. Finally, for the case $\left(\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2\right)<0,$ we select $T_1=-\overline{P}_1\left(\rho\tilde{\varepsilon}_1 I+\beta\tilde{\varepsilon}_2 I\right)\overline{P}_1$, which provides (53). Hence, it is observed that the matrix inequalities (51), (52) and (53) are equivalent to (34) for $\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2>0,$ $\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2=0$ and $\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2<0,$ respectively, which completes the proof of Theorem 3. Solution of constraints provided in Theorem 3 can be obtained in similar manner as in [25]. □

Remark 8: In comparison to [25], Theorem 3 highlights the necessary and sufficient condition for the existence of a solution to the nonlinear constraints for the OSL systems under input saturation. This approach can be used to provide a solution of nonlinear constraints in various scenarios of $\overline{\varphi}=\rho\tilde{\varepsilon}_1+\beta\tilde{\varepsilon}_2$, so that $\overline{\varphi}$ may comprehend any positive, negative or zero value. It is worth noting that the condition in Theorem 3 for different cases of $\bar{\varphi}$ can deal with the nonlinear term $P_1\left(\rho\tilde{\varepsilon}_1 I + \beta\tilde{\varepsilon}_2 I\right)P_1$. The first two cases in Theorem 3 are based on linear matrix inequalities for given values of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. The condition in Theorem 3 for $\rho\tilde{\varepsilon}_1 + \beta\tilde{\varepsilon}_2 < 0$ is computationally complex and can be resolved by application of the cone complementary linearization algorithm.

By taking $\tilde{\varepsilon}_1 = 0$, $\tilde{\varepsilon}_3 = 0$, $\alpha = 0$, and $\beta = \Lambda^2$, a novel condition can be obtained as a specific scenario of Theorem 3 for the observer-based control of Lipschitz systems under input saturation. The following corollary provides a straightforward observer-based controller design condition, compared to Corollary 1, for the Lipschitz nonlinear systems.

Corollary 2: *There exists a solution to the condition in Corollary 1 if and only if there exist matrices $P_1 \in \Re^{n\times n}$, $P_2 \in \Re^{n\times n}$, $Z \in \Re^{m\times n}$, $S \in \Re^{m\times n}$, and $M \in \Re^{n\times p}$, a diagonal matrix $U \in \Re^{n\times n}$, and scalars $\tilde{\alpha}$ and $\tilde{\beta}$ such that the matrix inequalities (32), (33),*

$$P_1 = P_1^T > 0,\ P_2 = P_2^T > 0,\ U > 0,\ \tilde{\varepsilon}_2 > 0,\ \tilde{\varepsilon}_4 > 0, \tag{54}$$

$$\begin{bmatrix} P_1 A^T + AP_1 + BZ + Z^T B^T & I & -BU + S^T & \sqrt{\tilde{\varepsilon}_2 \Lambda} P_1 \\ * & -\tilde{\varepsilon}_2 I & 0 & 0 \\ * & * & -2U & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{55}$$

$$\begin{bmatrix} A^T P_2 + P_2 A + MC + C^T M^T + \tilde{\varepsilon}_4 \Lambda^2 I & P_2 \\ * & -\tilde{\varepsilon}_4 I \end{bmatrix} < 0, \tag{56}$$

are satisfied. Various gains for the proposed observer-based control approach can be computed via $F = ZP_1^{-1}$, $J = SP_1^{-1}$ and $L = P_2^{-1}M$.

The inequalities in Theorem 3 can be solved as linear matrix inequalities by application of the cone complementary linearization algorithm ([20, 25, 46, 47, 48]). For this purpose, we can employ the minimization of objective function $trace(P_1\bar{P}_1 + 0.5T_1\bar{T}_1 + 0.5P_1\varphi P_1\bar{T} + Y\bar{Y} + P_1P_2P_1\bar{Y})$ subject to $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $\bar{P}_1 = \bar{P}_1^T > 0$, $T_1 = T_1^T > 0$, $\bar{T}_1 = \bar{T}_1^T > 0$, $Y = Y^T > 0$, $\bar{Y} = \bar{Y}^T > 0$, $U > 0$,

$$\begin{bmatrix} T_1 & I \\ * & \bar{T}_1 \end{bmatrix} \geq 0, \begin{bmatrix} -\varphi & P_1 \\ * & \bar{T}_1 \end{bmatrix} \geq 0, \tag{57}$$

$$\begin{bmatrix} Y & I \\ * & \bar{Y} \end{bmatrix} \geq 0, \begin{bmatrix} P_2 & \bar{P}_1 \\ * & \bar{Y} \end{bmatrix} \geq 0, \tag{58}$$

$$\begin{bmatrix} Y & -Z_{(i)}^T + S_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geq 0, \begin{bmatrix} P_1 & I \\ * & \bar{P}_1 \end{bmatrix} \geq 0, \tag{59}$$

$\forall i = 1,\dots,m,$ and constraints in Theorem 3, where $T_1 = -P_1\varphi P_1$ and $\varphi = \left(\rho\varepsilon_1 I + \beta\varepsilon_2 I\right)$. It can be noted that the relation between matrices and their inverses are ensured via minimization of the objective function and by considering the constraints like (57)-(59). For instance, the second constraint in (59) leads to $P_1 - \bar{P}_1^{-1} \geq 0$, which further can be written as $P_1\bar{P}_1 \geq I$. To attain results (closer to $P_1\bar{P}_1 = I$), we can minimize the trace of $P_1\bar{P}_1$ for attaining the matrix inverse relation. The computational complications in Theorem 1 are further addressed in Theorem 3 owing to the proposed decoupling method and the cone complementary linearization approach for solving the constraints. The controller and observer gains are computed off-line; therefore, the computational issues due to the size of input vector and the cone complementary linearization algorithm do not affect the real-time implementation of the proposed control strategy.

4. Simulation results

To explore features of the proposed methodologies and to provide comparison with the existing approaches, two practical nonlinear systems, namely, a single-link flexible-joint robot and a complex Chua's circuit are considered in our study.

4.1 Application to flexible-joint robot

Consider the model of a single-link flexible-joint robot [49], given as

$$\begin{aligned}
\dot{\theta}_m &= \omega_m, \\
\dot{\omega}_m &= \frac{k}{J_m}(\theta_l - \theta_m) - \frac{L}{2J_m}\omega_m + \frac{k_\tau}{J_m}\psi(u), \\
\dot{\theta}_l &= \omega_l, \\
\dot{\omega}_l &= \frac{k}{J_l}(\theta_m - \theta_l) - \frac{mgh}{J_l}\sin\theta_l,
\end{aligned} \tag{60}$$

where θ_m and θ_l denote the angles of rotations of the motor and the link, respectively, and ω_m and ω_l describe the angular velocities of the motor and the link, respectively. The description of other physical quantities has been elaborated in Table I. The motor can be operated through a DC power supply ranging between -15 V and 15 V. The values of various constants employed in the present study and the saturation bound are also provided in Table I. By using the values provided in Table I into (60) and by incorporating the nonlinear model (1), we can select the system matrices and nonlinearity as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, \tag{61}$$

$$B=\begin{bmatrix}0\\21.6\\0\\0\end{bmatrix},\ f(t,x)=\begin{bmatrix}0\\0\\0\\-3.33\sin x_3\end{bmatrix}, \tag{62}$$

$$C=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix}. \tag{63}$$

Note that $x=\begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}^T=\begin{bmatrix}\theta_m & \omega_m & \theta_l & \omega_l\end{bmatrix}^T$ and $\hat{x}=\begin{bmatrix}\hat{x}_1 & \hat{x}_2 & \hat{x}_3 & \hat{x}_4\end{bmatrix}^T$. The state estimation error vector is given by the relation

$$x=\begin{bmatrix}e_1 & e_2 & e_3 & e_4\end{bmatrix}^T=\begin{bmatrix}x_1-\hat{x}_1 & x_2-\hat{x}_2 & x_3-\hat{x}_3 & x_4-\hat{x}_4\end{bmatrix}^T. \tag{64}$$

For designing controller (4) and observer (6) by using Theorem 3, we consider the cone complementary linearization approach for the minimization of the objective function given as $trace(P_1\bar{P}_1+\tilde{\alpha}\varepsilon I+\beta\delta I+Y\bar{Y}+P_1P_2P_1\bar{Y})$ by using the methods of [20], [46] and [47]. By solving the constraints in Theorem 3 for six iterations of the cone complementary linearization algorithm, we obtain the following results.

$$P_1=\begin{bmatrix}1.26 & -1.456 & 0.445 & -3.691\\ -1.456 & 43.96 & 1.261 & 7.014\\ 0.445 & 1.261 & 0.652 & -2.410\\ -3.691 & 7.014 & -2.410 & 17.228\end{bmatrix}, \tag{65}$$

$$P_2=\begin{bmatrix}3.474 & 0.317 & -0.937 & 0.669\\ 0.317 & 0.0875 & -0.391 & 0.086\\ -0.937 & -0.391 & 4.358 & -0.197\\ 0.669 & 0.086 & -0.197 & 0.389\end{bmatrix}, \tag{66}$$

$$Z=\begin{bmatrix}-0.4914 & -4.5212 & -0.3889 & 0.3442\end{bmatrix}. \tag{67}$$

By employing $F=ZP_1^{-1}$ and $L=P_2^{-1}M$, the proposed observer-based control gains for the single-link flexible-joint robot are determined as

$$F=\begin{bmatrix}-3.106 & -0.320 & -1.506 & -0.986\end{bmatrix}, \tag{68}$$

$$L^T = \begin{bmatrix} 1.579 & 45.412 & 7.470 & 1.656 \\ -22.201 & 416.034 & 31.145 & -37.487 \end{bmatrix}. \quad (69)$$

The initial condition for the robotic system is taken to be

$$x^T(0) = \varpi\begin{bmatrix} 13 & -11 & 12 & -13 \end{bmatrix}. \quad (70)$$

The parameter ϖ can be selected for different simulation studies. Figs. 1 and 2 show the closed-loop response of the flexible-joint robot and the state estimation errors, respectively, for $\varpi = 1$. By application of the proposed observer-based control scheme in (4) and (6), all states and state estimation errors are converging to the origin. The corresponding control signal is also plotted in Fig. 3. It is notable in Fig. 3 that even for a large initial condition, the control signal does not saturate and remains between the lower and upper saturation limits. Hence, the proposed design has a capability to stabilize the nonlinear systems under saturation of the control signal.

Now, the proposed observer-based control scheme is tested under the effect of a large initial condition. We take $\varpi = 2$ to test the proposed control strategy with $x^T(0) = \begin{bmatrix} 26 & -22 & 24 & -26 \end{bmatrix}$. The closed-loop response, estimation error, and the saturated control input are provided in Figs. 4, 5, and 6, respectively, for $\varpi = 2$. Fig. 4 shows that the response of the closed-loop system is stable and the robot states are converging to the origin. The estimation error behavior in this case, shown in Fig. 5, is almost similar to the case of $\varpi = 1$. The estimation errors are also converging to the origin. The saturated control signal plotted in Fig. 6 undergoes saturation due to the larger initial conditions for $\varpi = 2$. However, the control signal for the proposed method has capability to recover from the saturation. It is observed from the plot that the control signal recovers from the saturation in a short period of time. This property demonstrates the ability of the proposed controller to deal with the windup effect due to saturation nonlinearity.

A similar behavior is observed for the case of $\varpi = 5$, when initial conditions are taken five times larger than the first case (that is, $x^T(0) = 5 \times \begin{bmatrix} 13 & -11 & 12 & -13 \end{bmatrix}$). The resultant closed-loop trajectories and the saturated control signal are provided in Figs. 7 and 8, respectively, for this initial condition. It is notable that the state trajectories of Fig. 7, even in the worst case of initial condition, are converging to the origin. While, the control signal of Fig. 8 recovers from the saturation in 2.5 seconds for the control of the robotic system. Hence, the proposed observer-based control methodology has capacity to deal with the saturation effects and windup consequences for the nonlinear systems either by avoiding the saturation or by ensuring recovery from the saturation, without employing the exact states for feedback.

4.2 Application to Chua's circuit

Let us consider the following chaotic Chua's circuit (please refer to [10] and the references therein):

$$A = \begin{bmatrix} -2.548 & 9.1 & 0 \\ 1 & -1 & 1 \\ 0 & -14.2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \tag{71}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = 0, \tag{72}$$

$$f(t,x) = \begin{bmatrix} 9.11|x_1 + 1| - 9.11|x_1 - 1| \\ 0 \\ 0 \end{bmatrix}. \tag{73}$$

The saturation limit is accounted as $\bar{u} = 2$. The nonlinearity $f(t,x)$ is considered as a complex function with respect to the theory of circuits and systems due to two reasons: First, this function represents a nonlinear negative resistance characteristics, which is always difficult to handle in electrical and electronic circuits. Second, this nonlinear negative resistance can cause complex chaotic oscillations in the Chua's circuit, which behavior is very sensitive to the initial conditions and parametric variations. The control of such complex nonlinear systems is an interesting and a challenging research issue.

In the present study, we compare the present approach with the existing methods, considering the nonlinear compensation and state feedback methods. The response of only first state (output) is considered for the comparison purpose. By applying Theorem 3, the proposed observer-based control parameters have been computed as

$$F = \begin{bmatrix} -1.121 & -8.495 & -1 \end{bmatrix}, \tag{74}$$

$$L^T = \begin{bmatrix} 0.879 & 2.067 & -2.335 \end{bmatrix}. \tag{75}$$

We choose the initial condition as

$$x(0) = \begin{bmatrix} 1 & 0.2 & -1 \end{bmatrix}^T. \tag{76}$$

A comparison of various approaches has been provided in the present study for the output stabilization. The closed-loop responses using nonlinear proportional integral (PI) control (ignoring saturation) of [10] and [45], linear compensation with nonlinear control of [26], and nonlinear compensation using states for feedback (see in [10] and [45]) along with the response using the proposed method for the observer-based control are shown in Fig. 9. The corresponding control signal for the proposed observer-based control scheme is demonstrated in Fig. 10, which reflects an ability of the constrained input signal to recover from saturation nonlinearity. The responses in Fig. 9 using the nonlinear PI and linear compensation are unable to stabilize the complex behavior of the Chua's circuit. Oscillations of increasing amplitude are observed using these approaches. The closed-loop response of the nonlinear compensation using states is fast; however, it has an overshoot and, further, it employs exact states for the saturation compensation. The proposed methodology response, in contrast, is not as fast as the nonlinear compensation; however, it eliminates the undesirable overshoot without employing the exact states of the system. The slower response of the proposed method is due to the estimation of states, required to deal with the practical situations, when additional sensors for state measurement are not feasible. While the nonlinear compensation uses the exact states of the plant for feedback, which may not be possible in the practical control systems.

An analysis has been carried out for determining the maximum allowable value of Lipschitz constant Λ for the proposed control method. It was observed that feasible results were obtained for $\Lambda \leq 5.72$ in the robotic manipulator case. For chaotic Chua's circuit, the solution of the proposed observer-based control strategy was attained for $\Lambda \leq 11.14$. Although, the proposed results are based on a generalized local approach; however, the developed control strategy still can be improved for higher values of Lipschitz constants, which can be addressed in future by means of linear parameter varying formulations of nonlinear functions.

The implementation of the proposed control signal is a non-trivial research task. Our method can be implemented similar to the classical observer-based control schemes. The observer in (6) will receive the output $y(t)$ and will employ it to estimate the state plant state $x(t)$. The estimated state $\hat{x}(t)$ will be used to control the plant (1). For implementation using a digital technology, discretization of (4) and (6) can be considered because digital technology is preferred over analog circuits due to its cheaper price and re-configurable nature. Hence, the proposed methodology is better or more practical than the existing nonlinear techniques due to its reasonable performance, simplicity in implementation, and applicability to the practical control systems, whenever states are not available. It should be noted that we have considered practical case studies in the simulation results for the electro-mechanical robotic system and the electronic chaotic circuit to demonstrate applicability of proposed methods. In future, experimental studies of the proposed control can be considered over digital technologies by considering the practical implementation aspects.

5. Conclusions

This paper studied an observer-based control of the OSL nonlinear systems under input saturation under the condition of an unknown state vector. An observer was employed to estimate the system states and the estimated states were employed for the feedback purpose. A closed-loop system was obtained in terms of the closed-loop system's state dynamics and estimation error dynamics. A condition using matrix inequalities ensuring the asymptotic convergence of the closed-loop state and the state estimation error for the given values of the observer and controller gains, under input saturation, was derived. A decoupling approach to degenerate the main condition into computationally less complicated constraints was developed for computing the controller and observer gains through the convex routines. The proposed decoupled conditions for nonlinear systems with input saturation and a generalized nonlinearity can be solved by employing the cone complementary linearization algorithm. Novel results of the proposed methodology for the Lipschitz case were also provided and several features of the resultant approaches for the Lipschitz nonlinear systems were discussed. In contrast to the conventional methods, the proposed observer-based control approach can be applied to design observer-based controllers, without using the actual states for feedback, for the OSL nonlinear systems in the presence of the input saturation constraint. A numerical example on stabilization of a single-link flexible-joint robot and a chaotic Chua's circuit were detailed. The simulation results demonstrate that the proposed approach can be employed to the nonlinear electro-mechanical plants and electrical circuits, when the system states are not available and the control signal is subjected to the saturation nonlinearity. In future, the proposed method can be considered for faster and finite-time convergence, robustness against disturbances and parametric uncertainties, and measurement and input delays. In addition to theoretical contributions, the developed scheme can experimentally tested for nonlinear practical control systems under unmeasurable states and input saturation.

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Declaration of Interest: None

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Table I: Parameters of the robot model

Model parameters	*Values*
J_m (inertia of motor)	3.7×10^{-3} kg m^2
J_l (inertia of link)	9.3×10^{-3} kg m^2
k (torsional spring constant)	1.8×10^{-1} Nm rad^{-1}
L (length of link)	3.1×10^{-1} m
k_τ (amplifier gain)	8×10^{-2} Nm V^{-1}
m (point mass)	2.1×10^{-1} kg
g (gravity constant)	9.8 m/s^2
h (center of gravity height)	1.5×10^{-2} m
$\bar{u}$ (input saturation limit)	15 V

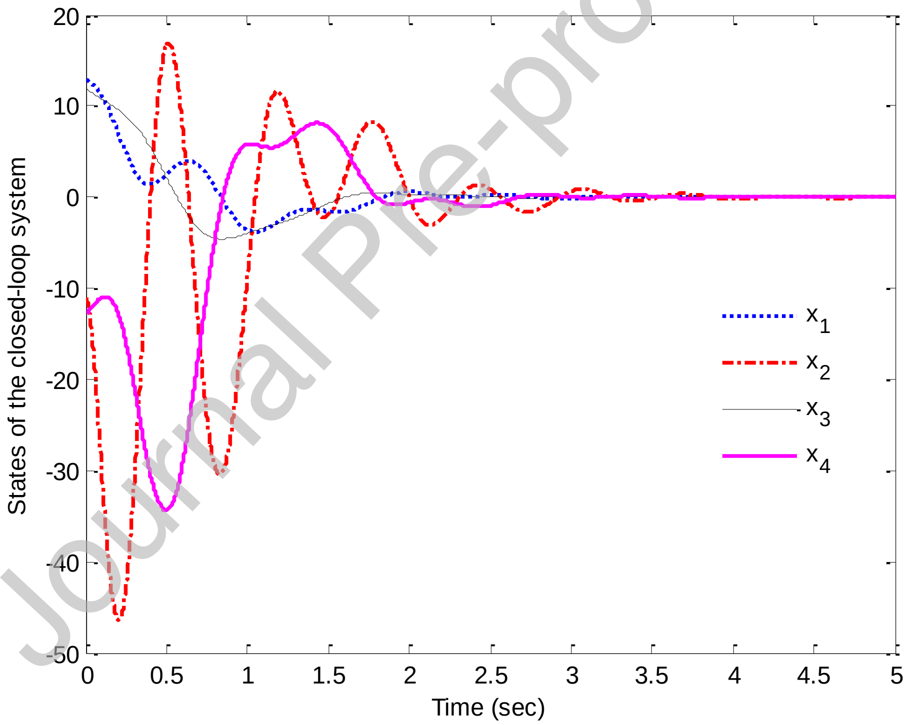

Fig. 1. Closed-loop response under input saturation for $\varpi = 1$

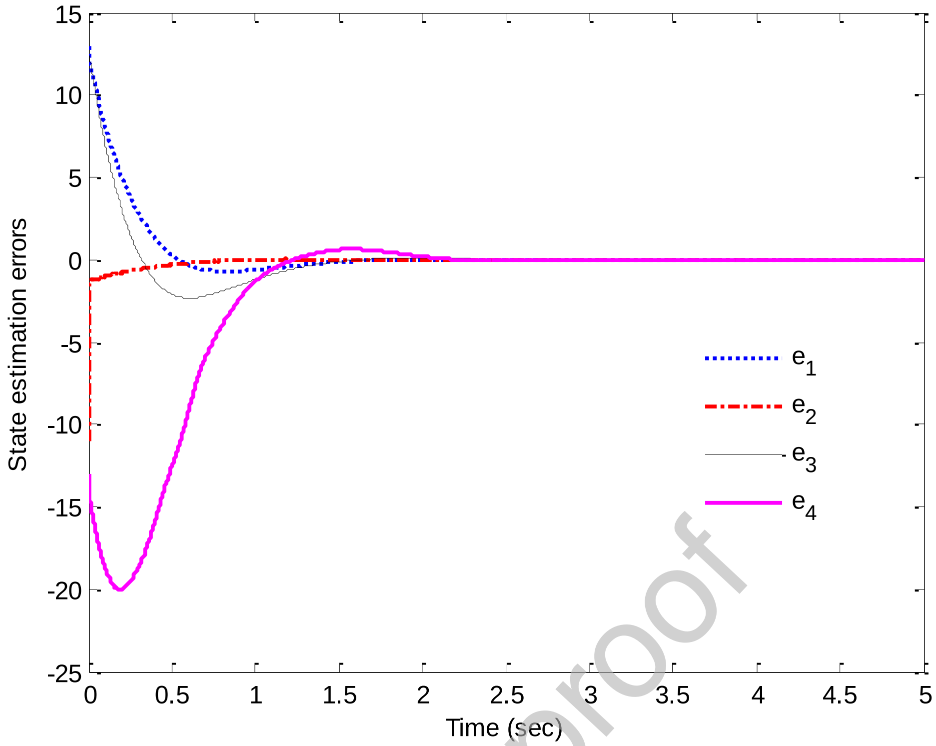

Fig. 2. Convergence of the state estimation errors to the origin for $\varpi = 1$

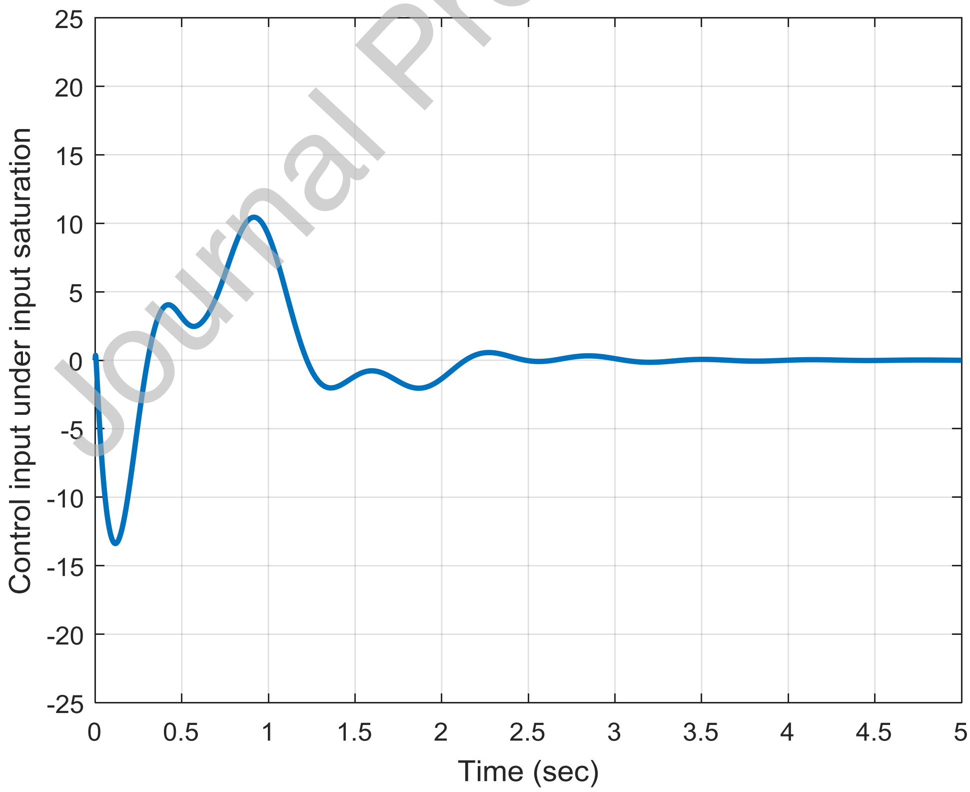

Fig. 3. Control signal under input saturation for $\varpi = 1$

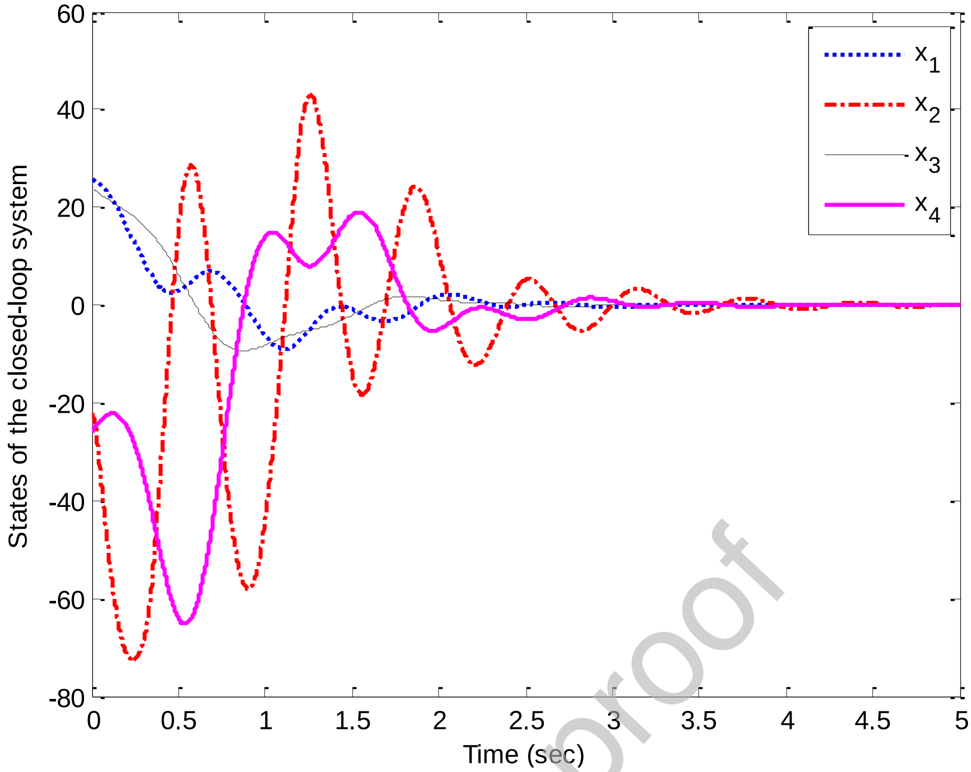

Fig. 4. Closed-loop response under input saturation for $\varpi = 2$

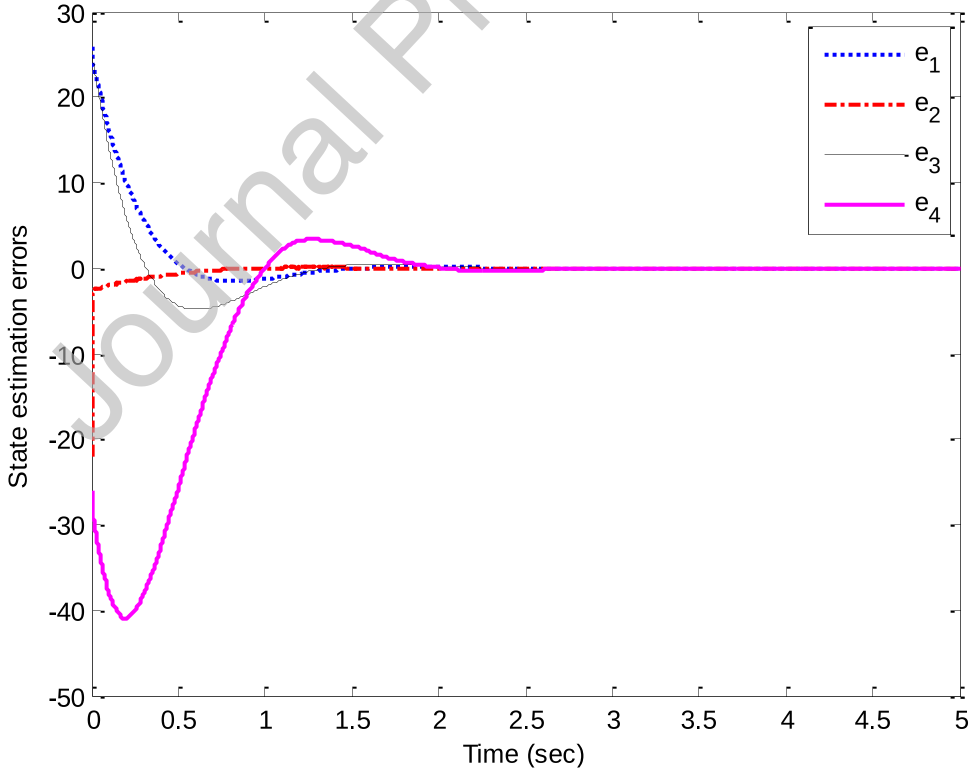

Fig. 5. Convergence of the state estimation errors to the origin for $\varpi = 2$

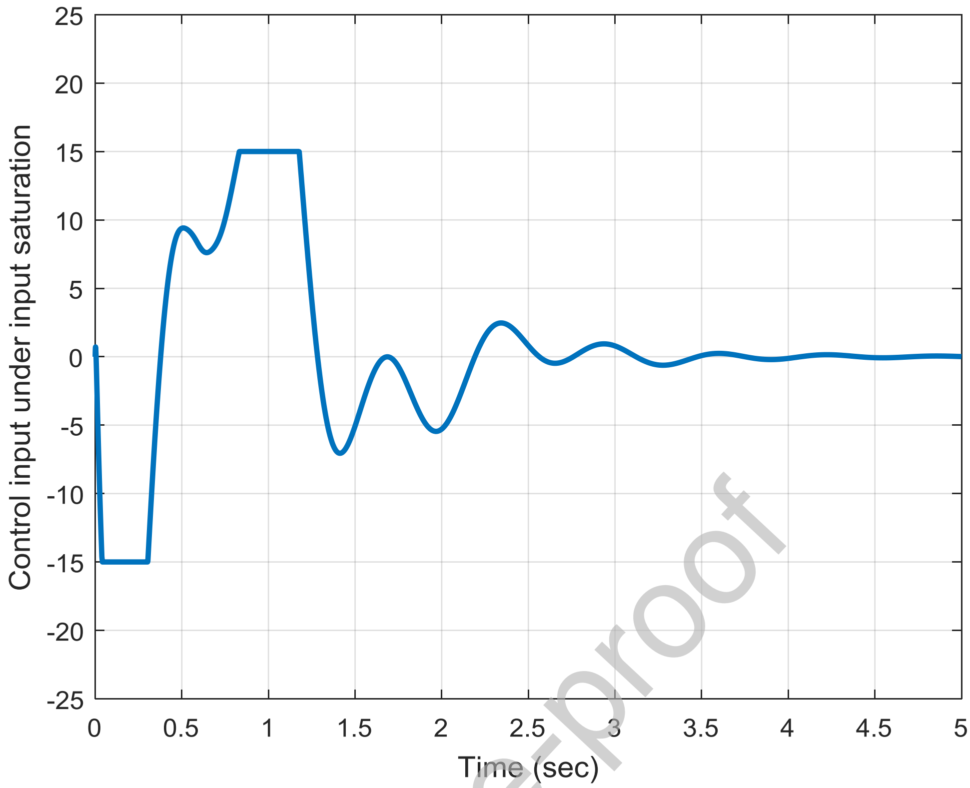

Fig. 6. Control signal under input saturation for $\varpi = 2$

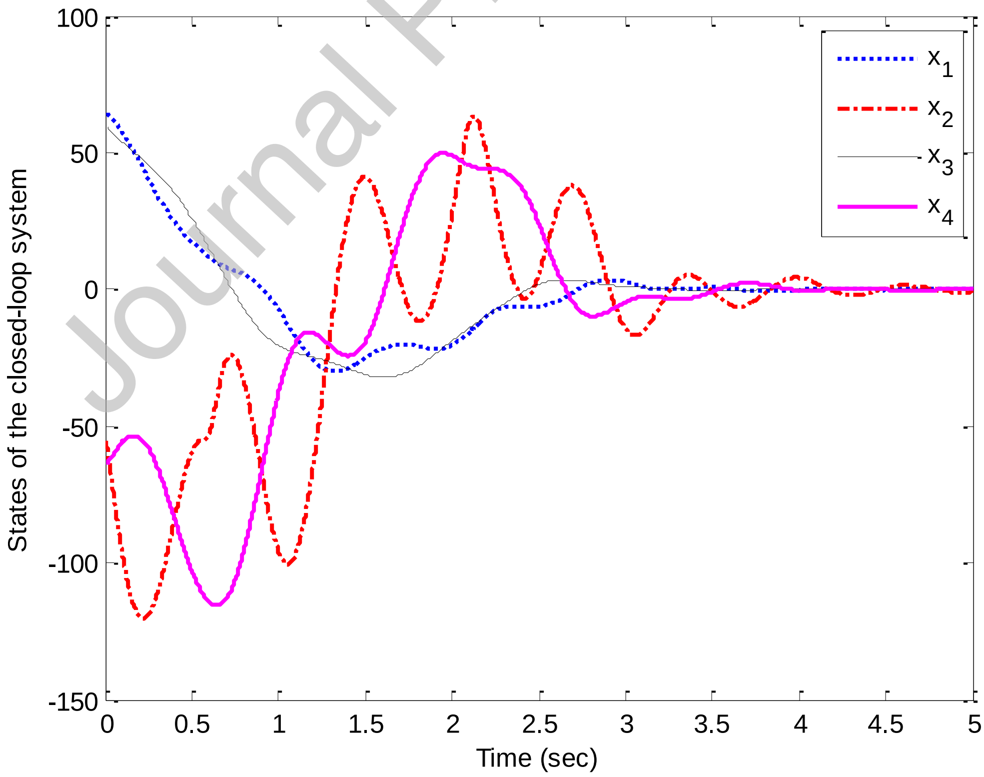

Fig. 7. Closed-loop response under input saturation for $\varpi = 5$

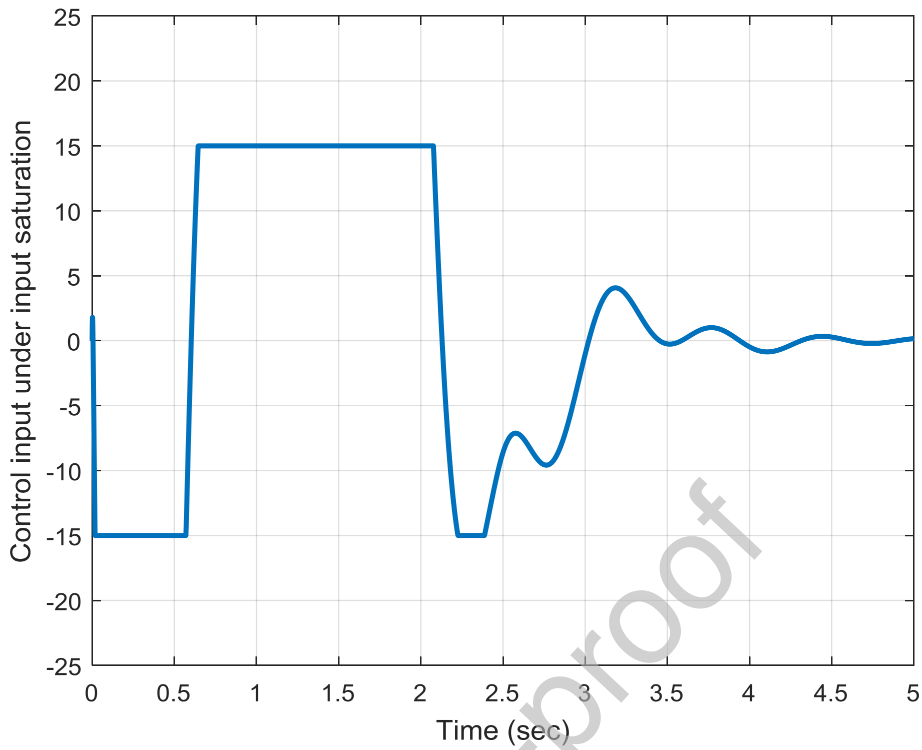

Fig. 8. Control signal under input saturation for $\varpi = 5$

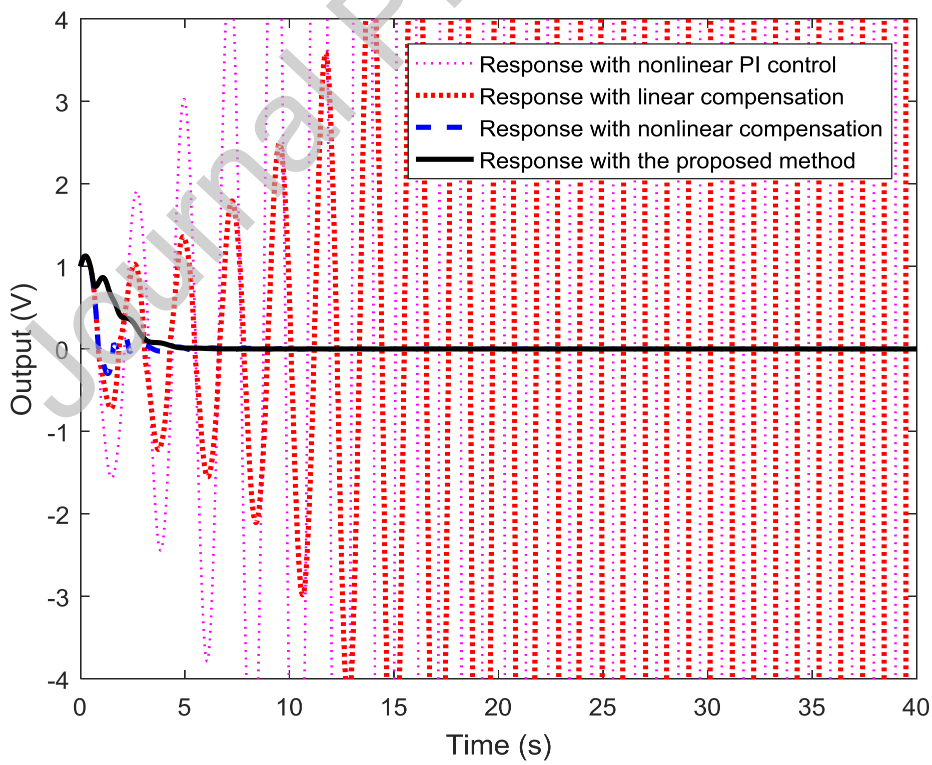

Fig. 9. Comparison of the existing control schemes with the proposed method for stabilization of Chua's circuit

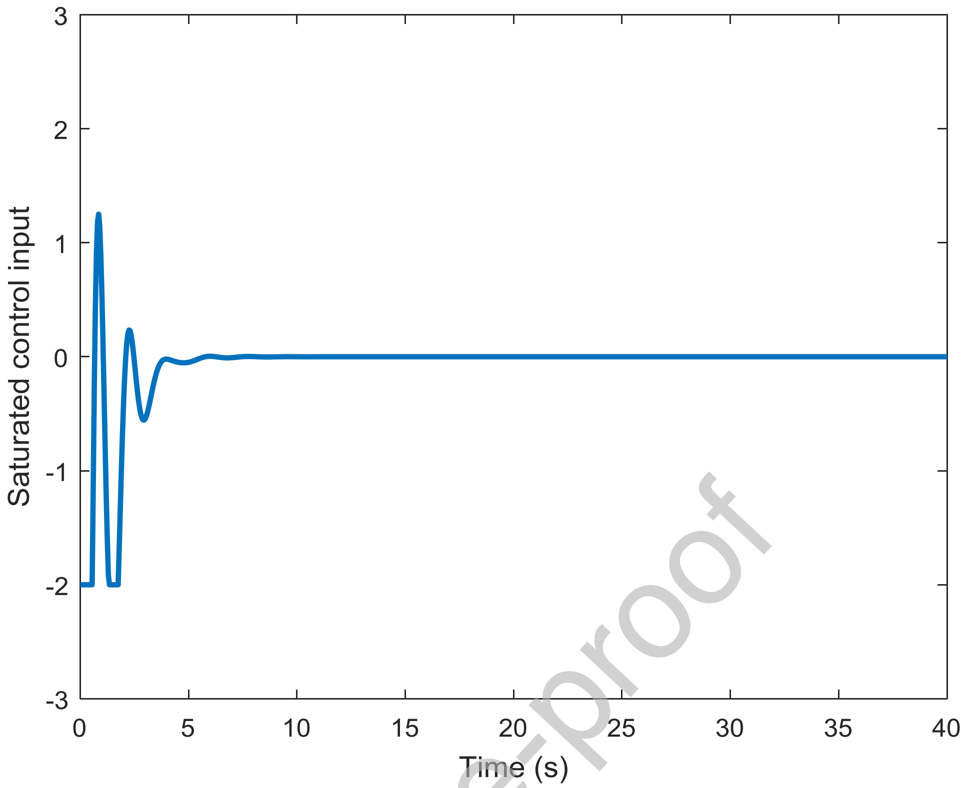

Fig. 10. Saturated control input signal using the proposed method for stabilization of Chua's circuit