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Review

Vortex-induced vibrations and control of marine risers: A review

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ABSTRACT

This paper reviews the dynamics and vibration control techniques for marine riser systems. The riser pipes are modeled as Euler-Bernoulli beams that vibrate under the effects of ocean loads and the movements of the surface vessel, resulting in hybrid ODE-PDE equations. Chronological development of such hybrid models is first discussed, and their approximated ODE models for simulation are examined. Theoretical and experimental techniques for instability and fatigue analyses on the riser systems are also summarized. To increase the fatigue life against ocean currents, passive vibration suppression devices (e.g., strakes and spoilers) were mounted on the surface of the riser. Whereas to tackle the instability problem caused by sea waves, active control techniques utilizing the movements of the vessel were employed. In Conclusions, as future riser technologies, seven research issues are identified.

1. Introduction

A marine riser, an essential component of subsea oil/gas production systems, acts as a conductor pipe between an offshore platform floating in an ocean (i.e., a storage platform or a vessel) and a well on the seabed (Chakrabarti and Frampton, 1982). As shown in Fig. 1, the riser is connected to the well through the blowout preventer (BOP) valve that prevents leakages of the fluid transported through the riser. The riser system is exposed to harsh environmental loads like ocean currents and waves. The currents act along the length of the riser, whereas the waves inflict the movements of the vessel. A severe current and a large movement of the vessel can result in a large deflection of the riser, which causes a disconnection of the riser from the BOP valve or a failure due to fatigue in the riser system. In the past fifty years, to ensure the safety and to enhance the productivity of riser systems, several investigations on the dynamics analyses and vibration controls of various riser systems have been reported in the literature.

The main objective of this paper is to review the works on the dynamics and control of the riser itself comprehensively under the influence of ocean currents and waves. The issues of laying and reentry of the riser and its contact with the seabed are important (Elosta et al., 2016; Jensen et al., 2009a, 2010; Wang et al., 2016); however, those will not be touched due to page limitation. At the end of the review, future research directions to improve riser technologies will be identified. This paper intends to portray a comprehensive picture on the evolution of mathematical modeling, analyses, and the vibration control strategies applicable to riser systems to the present state-of-the-art.

Riser systems can be classified into production and drilling risers from the operational view point. A production riser transports oil/gas from the seabed to the vessel; a drilling riser provides a pathway for drilling and transports drilling mud (Chakrabarti, 2005; He et al., 2014). Risers can be further classified into flexible or rigid according to the material used in their construction. A rigid or straight top-tensioned riser is usually utilized for shallow water operations, whereas a flexible or steel catenary riser is typically used for deep ocean operations (i.e., >2000 m) (Mekha, 2001; Meng and Zhu, 2015).

When a riser moves in the presence of currents, vortices are shed along its surface, resulting in the formation of an unstable wake region around it. Vortex shedding takes place at different frequencies and amplitudes, thus effecting different vortex patterns. According to Xu et al. (2009), the formation of different patterns of vortices depends upon the Reynolds number (Re) of the incident flow. Fig. 2 shows three different patterns of the vortices (i.e., 2S, 2P, and P + S) shed by the structure of the riser upon its interaction with the incident flow. The 2S pattern is a combination of two single vortices shed in opposite directions relative to each other during one cycle (i.e., Von Karman vortex shedding); the 2P pattern represents two pairs of vortices shed in one cycle, in which each pair consists of vortices in opposite direction relative to each other; and the P + S pattern denotes the asymmetric combination of a pair of vortices with a separate single vortex during each cycle (Williamson and Govardhan, 2004). Vortices inflict a periodically varying transverse force on the riser (i.e., perpendicular to the direction of the flow), resulting in

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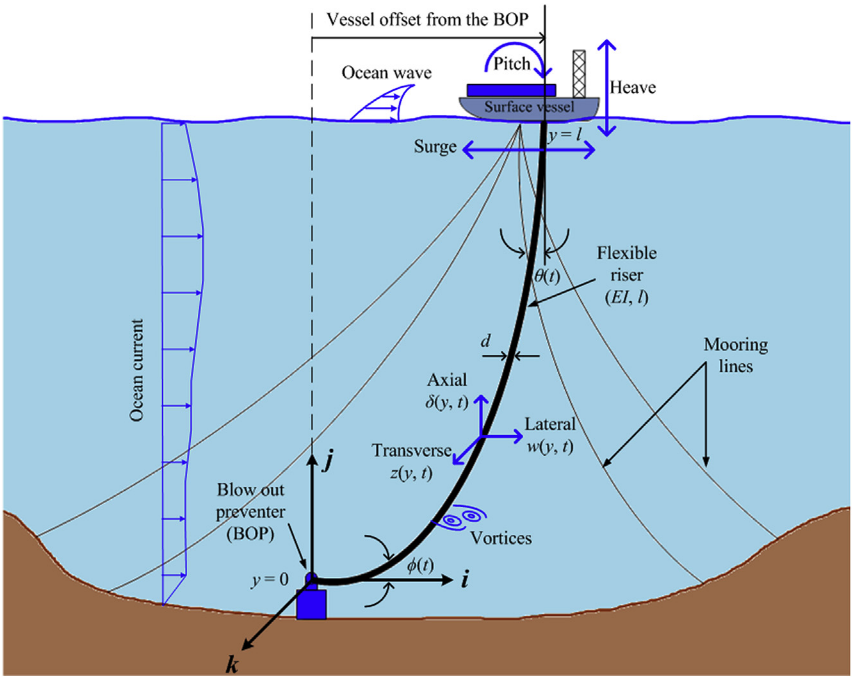

Fig. 1. Schematic of a subsea production system.

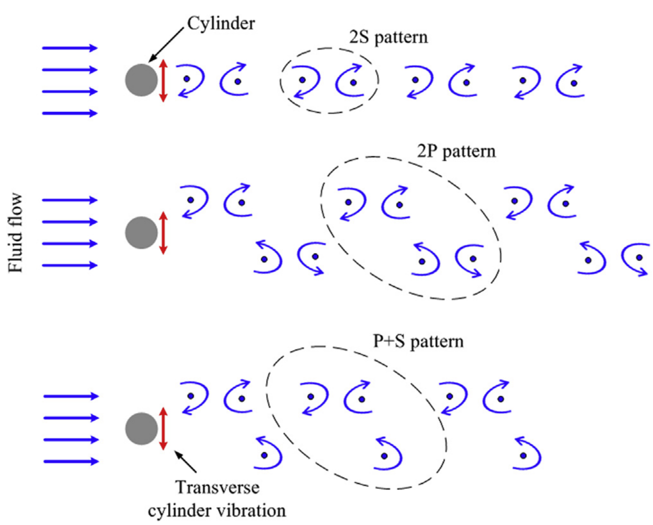

Fig. 2. Different patterns of vortex shedding from a cylinderical object to inline forces.

periodic transverse vibrations known as the vortex-induced vibrations (VIVs) (Wu et al., 2012b). According to Modarres-Sadeghi et al. (2011), the vortex-induced response of a riser is a complex phenomenon, and continuously shifts between a stationary state and a chaotic state. Also, when the frequency of the vortices (i.e., the vortex-shedding frequency) is close to one of the natural frequencies of the riser, the so-called lock-in phenomenon occurs, whereby the riser manifests large vibrations. Xue et al. (2008) noted that the riser can experience either single- or multi-mode (i.e., excitation of multiple modes) lock-in responses. For more details, interested readers can refer to the papers on the topic of VIV (Sarpkaya, 1979; Vandiver et al., 2009).

Risers ought to confront continuous loads (Jeans et al., 2003), which are the main cause of fatigue damage (Collette, 2011; Collette and Incecik, 2006; Elosta et al., 2014; Hodapp et al., 2015; Jensen, 1990, 2015; Marsh et al., 2016; Yuan et al., 2014; Zhu and Collette, 2017). Fatigue, the main concern reflecting the structural integrity of a riser, is caused by both the inline (i.e., in the direction of a flow) and the transverse (i.e., perpendicular to the direction of the flow) forces (Xue et al., 2014). Early works ignored the contribution of the inline forces to fatigue damage; however, later investigations revealed that the fatigue-causing effects of the inline forces and the transverse forces are equally important (Lie and Kaasen, 2006; Trim et al., 2005). According to Baarholm et al. (2006), the contribution of the inline forces to fatigue damage is particularly evident for the lower modes of vibration (i.e., the first and second modes), whereas that of the transverse forces is clear at the higher modes. Several subsequent investigations (Iranpour et al., 2008; Song et al., 2011) were concurrent with those results. More recent findings of Gao et al. (2011) and Zhang and Tang (2015) have reported that the maximum fatigue damage usually incurs at the boundary ends of the riser.

Along with the damage induced by the currents, the one resulting from the movements of the vessel is also a concern with regard to the integration and safety of the riser and the production system. According to Kuiper et al. (2008), the heave (i.e., vertical) motion of the vessel can result in i) instability caused by the periodic variation of the tension at the top portion of the riser and ii) a local dynamic buckling of the riser. The latter can be potentially resulted from the occurrence of high bending stresses that can exceed the yield stress of the riser. Vessel movements will also change the contact angles at the top and bottom ends of the riser with the vessel and the BOP of the well, respectively. According to Nguyen et al. (2010), if the end angles increase beyond a specific threshold (usually 2°), the connection between the riser and the BOP can be broken.

The control problem of a riser system is mainly concerned with the suppression of its vibrations caused by the currents and the movements of the vessel (caused by the waves). The two main control objectives, accordingly, are the prevention of both the fatigue and instability of the riser. In a deep sea, the damage caused by the movements of the vessel is significantly less than that caused by the currents, as they act over the entire submerged length of the riser (Trim et al., 2005). At a shallow depth, however, the instability of the riser caused by the movements of the vessel is the main concern (Perunovic and Jensen, 2003). The heave motion of a vessel is normally suppressed by mooring the ship to the seabed (see the mooring lines in Fig. 1), which is a passive method. Rotations such as pitch and yaw, meanwhile, are more effectively suppressed by active control strategies utilizing thrusters to maintain the desired vessel orientation (How et al., 2009). Regarding fatigue prevention, one strategy is to design the riser with a large safety factor. Leira et al. (2005) performed reliability analyses to investigate the effects of different safety factors for controlling fatigue damage of the risers for large depths. In relation to the suppression of the VIVs, passive devices such as spoilers (Abbassian and Moros, 1996) and strakes (Gao et al., 2015a) can be mounted on the surface of the riser to extend its fatigue life. Active methods for suppressing the VIVs include the proportional-integral control (Rustad et al., 2008), proportional-integral-derivative control (Jensen et al., 2009b), optimal control (Mazzilli and Sanches, 2011), boundary control (Choi et al., 2004; He et al., 2017; Kim et al., 2005a,b; Kim and Hong, 2009; Nguyen and Hong, 2010; Yang et al., 2004, 2005a,b; Zhang et al., 2015; Zhao et al., 2017), sliding mode control (Ngo and Hong, 2012a,b), and adaptive control (Hong, 1997; Hong et al., 2002; Hong and Bentsman, 1994a,b).

This review discusses experimental and theoretical investigations of the complex responses of the riser systems under harsh environmental conditions (see Section 3). Most experimental studies have evaluated the responses of the riser models, albeit only under simplified (e.g., uniform-flow) loading conditions. Moreover, the utility of an experimental setup for simulating highly turbulent flows is limited. Nonetheless, several works reported immensely valuable insights into the responses of the riser system (see Subsection 3.2). Theoretical analyses that predict time responses of the riser by solving the relevant equations of motion alternatively can yield more rich data. Several studies presenting a mathematical formulation of the dynamics, hydrodynamic forces, and the resultant fluid-structure interaction have been reported (see Section 2). This review includes deterministic (i.e., in both time and frequency domains) and non-deterministic (i.e., stochastic) methods for determining the response of the riser (see Subsection 3.1). After discussing the dynamics of riser systems, vibration control methods based on both passive and active strategies are presented (see Section 4). In Section 5, the following seven research issues are identified: i) A Timoshenko beam model to consider the shear forces, ii) time-varying parameters to incorporate the varying hydrodynamic forces, iii) 6 DOF vessel model to cope with extreme ocean conditions, iv) a new numerical scheme for real-time parameter estimation, v) enhanced experimental facilities to consider real hydrodynamic loads, vi) hybrid control techniques combining passive damping devices and active controls, and vii) control strategies for preventing failure at the end points of the riser due to the movements of the vessel.

This paper is organized as follows: Section 2 presents the equations of motion of the riser systems and the mathematical formulations of the hydrodynamic forces acting on them; Section 3 reviews the experimental and theoretical techniques to analyze the response of the riser systems; Section 4 provides a detailed account of the numerous passive and active control strategies developed for suppressing the vibrations of the risers operating in harsh ocean environments; finally, Section 5 discusses future research issues in relation to the dynamics and control of offshore marine risers.

2. Riser dynamics

A riser system usually is modeled as an Euler-Bernoulli beam that vibrates under the influence of the waves and currents in the ocean and the movements of the vessel. Fig. 1 shows a schematic of a flexible marine riser system, wherein d is the diameter, EI is the flexural rigidity, l is the length, $u(x, t)$ is the lateral deflection, $w(x, t)$ is the transverse deflection, and $\delta(x, t)$ is the axial deflection of the riser at position x and time t. As indicated, the bottom and top ends of the riser are pin-jointed to the BOP on the seabed and to the surface vessel, respectively, where $\phi(t)$ and $\theta(t)$ are the bottom and top angles of the riser, respectively. The vessel is considered to undergo three motions: heave, surge, and pitch (i.e., movements in the vertical $\widehat{ij}$-plane), which are mainly caused by the waves and the wind. To restrict such movements, the vessel can be moored to the seabed as depicted in Fig. 1. Ocean currents meanwhile, which are shown to act along the length of the riser, are the main cause of the VIVs in the riser.

In the following subsections, various mathematical formulations describing the responses of the riser to different hydrodynamic forces are reviewed. For consistency and ease of comparison, the symbols appearing in the literature have been unified. Also, the derivatives with respect to t and x are denoted by $\dot{}$ and $'$, respectively.

2.1. Lateral vibrations

Huang and Dareing (1968) is recognized as the first work in the derivation of the equations of motion of the riser system. For analyzing the lateral dynamics (i.e., the motions only in the $\widehat{ij}$-plane, in which the sea current flows) of a flexible drilling riser, the following partial differential equation (PDE) was developed.

$$EIu''''(x,t) - ((m_r - m_i)gx - F_v)u''(x,t)(m_r - m_i)gu'(x,t) + m\ddot{u}(x,t) = 0, \tag{1}$$

where m_r and m_i denote the masses per unit length of the riser and the drilling fluid flowing inside the riser, respectively, g is the gravitational acceleration, m is the total mass per unit length of the riser (including the masses of the drilling fluid and the external fluid moving with the pipe), and F_v is the vertical thrust force acting at the bottom of the riser (due to drilling). In Eq. (1), the effects of the ocean currents and waves on the dynamics of the riser were not considered. However, it certainly leads to a more detailed formulation on the interaction between the riser and the surrounding fluid.

The inline hydrodynamic force $F_I(x, t)$ acting along the $\hat{j}$-axis is modeled by the following nonlinear Morison equation (Morison et al., 1950).

$$\begin{aligned} F_I(x,t) &= F_m(x,t) + F_d(x,t) \\ &= (\pi/4)\rho_w d^2 C_m(x)\dot{v}_y(x,t) + (1/2)\rho_w d C_d(x)\big(v_y(x,t) - \dot{u}(x,t)\big)\big|v_y(x,t) \\ &\quad - \dot{u}(x,t)\big|, \end{aligned} \tag{2}$$

where $F_m(x, t)$ and $F_d(x, t)$ represent the inertial and the drag forces, respectively, ρ_w denotes the density of the water, $v_y(x, t)$ signifies the horizontal water particle velocity, C_m is the inertial coefficient, and C_d stands for the drag coefficient. In Young et al. (1978), the following equation was developed to analyze the lateral vibrations of the riser under the effects of the inline hydrodynamic force F_I.

$$\begin{aligned} &EIu''''(x,t) - (T'(x) + \gamma_i A_i - \gamma_o A_o)u'(x,t) - (T(x) + p_o A_o - p_i A_i)u''(x,t) \\ &\quad + m\ddot{u}(x,t) = F_I(x,t), \end{aligned} \tag{3}$$

where T denotes the actual riser tension, A represents the area of the riser, γ stands for the specific weight, and p is the fluid pressure, and the subscripts i and o signify the properties of the quantities considered inside and outside of the riser, respectively. In Eq. (3), a simplified form of the drag force in Eq. (2) was alternatively proposed assuming that the drag force is caused by a constant velocity of the current, v. Then, the drag force is obtained in the following form.

$$F_d = (1/2)\rho_w d C_d v|v|. \tag{4}$$

Later, Sarpkaya and Isaacson (1981) replaced the quadratic term (i.e., $(v_y - \dot{u})|v_y - \dot{u}|$) of the drag force in Eq. (2) with a linear expression involving the root mean square value of the relative velocity between the riser and the incident current, σ_v, resulting in the following linear formulation.

$$F_d(x,t) = (\pi/4)\rho_w d^2 C_m(x)\dot{v}_y(x,t) + (1/2)\rho_w d C_d(x)\sqrt{8/\pi}\sigma_v\left(v_y(x,t) - \dot{u}(x,t)\right) \quad (5)$$

Chakrabarti and Frampton (1982) derived the following equation of motion of the riser considering the fact that the hydrodynamic forces act along both the horizontal and the vertical axes.

$$EIu''''(x,t) - T_e'(x)u'(x,t) + T_e(x)u''(x,t) + m\ddot{u}(x,t) = F_I(x,t), \quad (6)$$

where

$$T_e(x) = T(x) + A_o(x)p_o(x) - A_i(x)p_i(x) \quad (7)$$

and

$$T(x) = T_c + \int_0^l \rho_r (A_o(x) - A_i(x))dx - \int_0^l F_x(x,t)dx. \quad (8)$$

In Eqs. (6)–(8), Te denotes the effective tension, T is the axial tension acting at the center of the riser cross-section (which varies continuously along the length), Tc is a constant tension due to the inertial loading of the drilling mud, ρr is the weight density of the riser material, and Fx is the vertical component of the hydrodynamic force caused by the currents and the internal fluid. Recently, Fan et al. (2017) has developed a new formulation to calculate the effective tension Te of a free hanging riser (i.e., when the bottom end of the riser is not connected to the seabed), which is given as follows.

$$T_e(x) = W_{BOP} + (W_s - (\rho_w A_o - \rho_i A_i)g)(l - x), \quad (9)$$

where W_{BOP} denotes the weight of the BOP valve, W_s represents the submerged weight per unit length of the riser, ρ_i signifies the density of the fluid flowing inside the riser, and g is the gravitational acceleration.

Later, Niedzwecki and Liagre (2003) in consideration of both linear (i.e., viscous) and nonlinear (i.e., due to drag) fluid damping have derived the following equation of motion of the riser for the lateral deflection (i.e., along the $\hat{j}$-axis).

$$\begin{aligned} &EIu''''(x,t) - T_e(x)u''(x,t) + m\ddot{u}(x,t) \\ &+\left(c(x,t) + (1/2)\rho_w d C_d(x)\sqrt{8/\pi}\,\sigma_v\right)\dot{u}(x,t) \\ &= (\pi/4)\rho_w d^2 C_m(x)\dot{v}_y(x,t) + (1/2)\rho_w d C_d(x)\sqrt{8/\pi}\sigma_v v_y(x,t), \end{aligned} \quad (10)$$

where c denotes the linear viscous damping coefficient.

2.2. Vortex-induced vibrations

For analyzing the transverse vibrations (or VIVs) of the riser, the transverse force F_T, acting along the $\hat{k}$-axis, has to be included in the forcing function. Chakrabarti and Frampton (1982) initially modeled the transverse force as

$$F_T(x,t) = (1/2)\rho_w d C_l(x,t)\left(v_y(x,t) - \dot{u}(x,t)\right)^2, \quad (11)$$

where C_l denotes the lift coefficient. The transverse force is a consequence of the vortex shedding phenomenon. Therefore, to incorporate the oscillating effect, due to vortex shedding, of the transverse force, Faltinsen (1990) proposed the following equation.

$$F_T(x,t) = (1/2)\rho_w d C_l(x,t) v_y(x,t)^2 \cos\left(2\pi f_v t + \varphi_z\right), \quad (12)$$

where φ_z is the phase angle along the $\hat{k}$-axis, and f_v is the vortex-shedding frequency represented by the following equation.

$$f_v = \mathrm{St}\, v_y(x,t)/d, \quad (13)$$

where St is the Strouhal number.

Furnes (2000) investigated the VIVs of the riser subject to the transverse force by using the following PDE.

$$EIw''''(x,t) - T_e'(x)w'(x,t) - T_e(x)w''(x,t) + c\dot{w}(x,t) + m\ddot{w}(x,t) = F_T(x,t). \quad (14)$$

In Sagatun et al. (2002), the PDE used to investigate the VIVs of the riser also included the term $c_r(v - \dot{w}(x,t))$ to consider the damping effects due to the relative velocity between the riser and the current.

$$\begin{aligned} &EIw''''(x,t) - T_e'(x)w'(x,t) - T_e(x)w''(x,t) + c\dot{w}(x,t) + c_r(v - \dot{w}(x,t)) \\ &+ m\ddot{w}(x,t) = F_T(x,t), \end{aligned} \quad (15)$$

where c_r denotes the linear damping parameter associated with the dissipation of energy due to the relative velocity between the riser and the water.

2.2.1. Wake-oscillator model

Another way to investigate the VIVs of the riser is to consider two coupled differential equations that model the dynamics of the riser and the wake separately. To model the VIVs of a riser, Facchinetti et al. (2004) utilized the following single-degree-of-freedom spring--mass-damper equation, considering an elastically mounted rigid cylinder under the influence of wakes.

$$m\ddot{w}(t) + c\dot{w}(t) + kw(t) = F_T(t), \quad (16)$$

where k is the stiffness of the riser. The equation describing the wake dynamics was given as follows.

$$\ddot{q}(t) + \eta\Omega_v\left(q(t)^2 - 1\right)\dot{q}(t) + \Omega_v^2 q(t) = \beta\ddot{w}(t), \quad (17)$$

where η is the van der Pol parameter, $\Omega_v = 2\pi \mathrm{St} v/d$ is the Strouhal circular frequency, $q(t)$ is the local fluctuating lift coefficient, and β is the linear coupling coefficient. In Eq. (17), $q(t) = 2C_l(t)/C_{l0}$ where $C_{l0} = 0.3$ is a constant lift coefficient, $\eta\Omega_v(q(t)^2 - 1)\dot{q}(t)$ is the viscous damping term, and $\beta\ddot{w}(t)$ is the acceleration coupling term for modeling the effect of the riser structure on the fluid in the wake region. In earlier models (Le Cunff et al., 2002; Sarpkaya, 1979), a velocity coupling term $\beta\dot{w}(t)$ was considered instead of $\beta\ddot{w}(t)$. The coupling between the structure and the wake has been established by modeling the transverse force as $F_T(t) = (1/2)\rho_w d v^2 (C_{l0}/2) q(t)$ (Xu et al., 2008). Eqs. 16 and 17 are collectively known as the wake-oscillator model (Keber and Wiercigroch, 2008; Li et al., 2010; Lin et al., 2009; Nishi and Doan, 2015; Xue et al., 2011).

Violette et al. (2007) proposed a distributed wake-oscillator model in which the dynamics of the wake and the riser, respectively, were accounted for by the following PDEs.

$$\ddot{w}(x,t) + \left(\chi\Omega_f/r_m\right)\dot{w}(x,t) - \lambda^2 w''(x,t) + \kappa^2 w''''(x,t) = \Omega_f^2 M q(x,t), \quad (18)$$

$$\ddot{q}(x,t) + \eta\Omega_f\left(q(x,t)^2 - 1\right)\dot{q}(x,t) + \Omega_f^2 q(x,t) = \beta\ddot{w}(x,t), \quad (19)$$

where $\chi = C_d/4\pi\mathrm{St}$ is the stall coefficient, $\Omega_f = \Omega_v/\Omega_{ref}$ is the non-dimensional shedding frequency (where $\Omega_{ref} = 2\pi\mathrm{St}v_{ref}/d$ is the Strouhal circular frequency at an arbitrary reference flow velocity v_{ref}), $r_m = m/\rho_w d^2$ is the mass ratio, $\lambda^2 = (T/m)/(d\Omega_{ref})^2$ is the non-dimensional tension, $\kappa^2 = (EI/m)/(d^2\Omega_{ref})^2$ is the non-dimensional bending stiffness, and $M = (C_{l0}/2)/(8\pi^2\mathrm{St}^2 r_m)$ is a coefficient of the forcing term acting on the structure due to the wake dynamics. Distributed-wake-dynamics model Eq. (19) was later utilized by Meng and Chen (2012) and Srinil et al. (2009) to model the unsteady hydrodynamic forces for the analysis of the vibrational behavior of a catenary riser in the 3D space. Also, to incorporate the cellular vortex-shedding feature caused by a sheared flow in the wake dynamics equation, Srinil (2011) added the diffusion term $-\tau\dot{q}''$ to the left-hand side of Eq. (19), where τ is the diffusion parameter. In all the wake-oscillator models discussed to this point, the parameters η, β, C_{l0}, and C_d were considered to be constants. Contrastingly, in

Mukundan et al. (2009), to model the randomness shown in the experimental data provided by the Norwegian Deepwater Program (see Trim et al. (2005)), those parameters were considered to be randomly varying along the length of the riser.

2.3. Effect of internal fluid

Investigations by Chatjigeorgiou (2017) and Wu and Lou (1991) revealed that the internal fluid has a significant effect on the tension of the riser, whose effect was early described in the equations of the lateral motion of the riser developed by Young et al. (1978), Eq. (3), and Chakrabarti and Frampton (1982), Eq. (6). However, those formulations assumed the internal fluid to be at rest. Therefore, to incorporate the effect of the speed of the internal fluid, Seyed and Patel (1992) modified the equation for the effective tension (T_e), Eq. (7), as follows.

$$T_e(x) = T(x) + A_o(x)p_o(x) - A_i(x)p_i(x) - \rho_i v_i^2 A_i, \tag{20}$$

where ρ_i and v_i denote the density and the average velocity of the internal fluid, respectively. Considering Eq. (20), Kuiper and Metrikine (2005) modified Eq. (3) to incorporate the effect of the velocity of the internal fluid in the following PDE.

$$\begin{aligned} &EIu''''(x,t) - (\partial/\partial x)(T(x)u'(x,t)) + \rho_r A_i\big(v_i^2 u''(x,t) - 2v_i\dot{u}'(x,t) \\ &+ \ddot{u}(x,t)\big) - (\partial/\partial x)((A_o p_o(x) - A_i p_i(x))u'(x,t)) + \rho_r(A_0 - A_i)\ddot{u}(x,) = F_I(x,t). \end{aligned} \tag{21}$$

Later, Kuiper et al. (2007) revealed that the fluid transported by the riser also contributes to the drag part in the inline force. The following nonlinear equation was formulated to depict their finding.

$$F_d(x,t) = \alpha_1\mu_i l\dot{u}(x,t) + (1/2)\alpha_2\rho_w l d\dot{u}(x,t)|\dot{u}(x,t)|, \tag{22}$$

where α_1 and α_2 are the two unknown dimensionless constants, and μ_i denotes the dynamic viscosity of the fluid conveyed by the riser.

In investigating the impact of the internal fluid on the VIVs, Guo et al. (2004) found that the effect of the internal fluid velocity on the VIVs is strong when the natural frequency of the riser is close to the vortex-shedding frequency, whose effects however can be decreased by increasing the tension at the top of the riser. Dai et al. (2014b) developed the following PDE to investigate the effect of the internal flow on the VIVs of the riser.

$$\begin{aligned} &m\ddot{w}(x,t) + \Big(m_i\big(\dot{v}_i(l-x,t) + v_i(x,t)^2\big) - (EA/2l)\textstyle\int_0^l w'(x,t)^2 dx\Big)w''(x,t) \\ &+ 2m_i v_i(x,t)\dot{w}'(x,t) + EIw''''(x,t) = F_T(x,t), \end{aligned} \tag{23}$$

where m_i denotes the mass per unit length of the fluid transported by the riser. Later, Eq. (23) was updated by Dai et al. (2014a) to include the effect of base excitations on the VIVs of the riser. A new term, $m\ddot{w}(l,t)$, was added to the forcing function to model the base excitations. The new model reveals that the base excitations can be utilized to control the amplitude of the VIVs.

2.4. Multi-dimensional riser models

All of the formulations discussed thus far were developed to investigate the lateral and transverse vibrations of the riser, separately. However, in reality, the lateral and transverse vibrations are coupled due to the vortex shedding (i.e., the vortex shedding generates oscillating forces along both the $\hat{k}$ and $\hat{j}$ axes) (Yuan et al., 2017). The transverse force along the $\hat{k}$-axis ($F_{T,z}$) is given by Eq. (12). Whereas that along the $\hat{j}$-axis is given as follows (Faltinsen, 1990).

$$F_{T,y}(x,t) = a_c\cos\big(4\pi f_v t + \phi_y\big), \tag{24}$$

where a_c is the amplitude of the oscillating current and φ_y is the phase angle along the $\hat{j}$-axis.

Srinil and Zanganeh (2012) developed a two-dimensional (2D) riser model for representation and analysis of the coupled lateral and transverse vibrations of the riser, where the riser was treated as a flexibly mounted circular cylinder (i.e., a spring-mass-damper system) subject to a uniform flow. Equation (16) was modified so as to obtain the following two coupled ODEs for modeling the lateral and transverse vibrations of the cylinder.

$$\begin{aligned} &m_y\ddot{u}(t) + c_y\dot{u}(t) + k_y\big(u(t) + \alpha_{1y}u(t)^3 + \alpha_{2y}u(t)w(t)^2\big) \\ &= (1/2)\rho_w dv^2(C_d - C_l\dot{w}(t)/v), \end{aligned} \tag{25}$$

$$\begin{aligned} &m_z\ddot{w}(t) + c_z\dot{w}(t) + k_z\big(w(t) + \alpha_{1z}w(t)^3 + \alpha_{2z}w(t)u(t)^2\big) \\ &= (1/2)\rho_w dv^2(C_l + C_d\dot{w}(t)/v), \end{aligned} \tag{26}$$

where α_{1y}, α_{2y}, α_{1z} and α_{2z} are the geometrical coefficients, the subscripts y and z appearing in the parameters m, c, and k define their properties along the lateral ($\hat{j}$-axis) and the transverse ($\hat{k}$-axis) directions, respectively. Equations 25 and 26, which are Duffing oscillators, have cubic nonlinear terms (u^3 and w^3) that model the axial stretching feature of the riser, along with the terms uw^2 and wu^2 that model the coupling between the lateral and transverse deflections. Nguyen et al. (2013) later developed a more realistic 2D model based on the PDEs that considered the riser as a flexible Euler-Bernoulli beam. In this model, the coupling between the transverse and lateral vibrations due to environmental forces was also considered:

$$\begin{aligned} &m\ddot{u}(x,t) - (EA/2)\big(3u'(x,t)^2u''(x,t) + w'(x,t)^2u''(x,t) \\ &+ 2u'(x,t)w'(x,t)w''(x,t)\big) - Tu''(x,t) + EIu''''(x,t) \\ &= F_{I,y}(x,t) + F_{T,y}(x,t), \end{aligned} \tag{27}$$

$$\begin{aligned} &m\ddot{w}(x,t) - (EA/2)\big(3w'(x,t)^2w''(x,t) + u'(x,t)^2w''(x,t) \\ &+ 2w'(x,t)u'(x,t)u''(x,t)\big) - Tw''(x,t) + EIw''''(x,t) \\ &= F_{I,z}(x,t) + F_{T,z}(x,t), \end{aligned} \tag{28}$$

where $F_{I,y}$ and $F_{I,z}$ denote the inline forces and $F_{T,y}$ and $F_{T,z}$ signify the transverse forces along the $\hat{j}$ and $\hat{k}$ axes, respectively. The hydrodynamic forces in Eqs. 27 and 28 are given as follows.

$$F_{I,y}(x,t) = (\pi/4)C_m\rho_w d^2\dot{v}_y(x,t) - \Big(c_y + (1/2)C_d\rho_w d\sqrt{8/\pi}\,\sigma_y\Big)\ \dot{u}(x,t), \tag{29}$$

$$F_{I,z}(x,t) = (\pi/4)C_m\rho_w d^2\dot{v}_z(x,t) - \Big(c_z + (1/2)C_d\rho_w d\sqrt{8/\pi}\,\sigma_z\Big)\ \dot{w}(x,t), \tag{30}$$

$$F_{T,y}(z,t) = (1/2)C_d\rho_w d\sqrt{8/\pi}\sigma_y(x,t)v_y(x,t), \tag{31}$$

$$F_{T,z}(x,t) = (1/2)C_d\rho_w d\sqrt{8/\pi}\sigma_z(x,t)v_z(x,t), \tag{32}$$

where c_y and c_z denote the linear viscous damping coefficients, v_y and v_z represent the water particle velocities, and σ_y and σ_z signify the root mean square values of the corresponding water particle velocities along the $\hat{j}$ and $\hat{k}$ axes, respectively.

In Do and Pan (2009), Meng and Chen (2012), and Srinil et al. (2009), PDEs for investigating the transverse, lateral, and axial vibrations of the riser in the 3D space were formulated. The equations were of the form already discussed above. However, the coupling parts between the three different types of deflections of the riser, see Eqs. 27 and 28, were not considered.

2.5. Numerical models

For lumped mass systems (Chai et al., 2002; Shah and Hong, 2014), the equations of motion were given in the form of ordinary differential equations (ODEs), which can easily be solved analytically. However, for distributed-parameter systems, it is not always feasible to obtain an exact or an analytical solution of the system (particularly, nonlinear PDEs). Therefore, in such cases, numerical methods, for example, the finite element method (FEM), the rigid finite element method (RFEM), the finite difference method (FDM), etc. are employed to obtain approximate solutions. Such methods are used to first discretize the domain of the system in terms of spatial entities (i.e., lengths, areas, or volumes) and then based on the discretization scheme, an approximate model of the system is formulated, which can easily be solved numerically.

FEM is the most widely used numerical approach (Cuamatzi-Melendez et al., 2017), wherein the riser is discretized into a finite number of elements first, then ODEs are developed for each element considering the mass and the external/internal forces concentrated on the node (i.e., the center of individual elements or the connection point between the elements), and finally the equations of individual elements are stacked together to obtain the global equation of motion of the riser in a matrix form. For example, Patel and Jesudasen (1987) developed the following global equation for the lateral motion of the riser under the effect of waves and the movements of a vessel.

$$\mathrm{M}\ddot{\mathrm{U}} + \mathrm{C}\dot{\mathrm{U}} + \mathrm{KU} = \mathrm{F}_y + \mathrm{F}_v, \tag{33}$$

where M, C, and K denote the global matrices of the mass, the damping, and the stiffness of the riser; $\ddot{\mathrm{U}}, \dot{\mathrm{U}}$, and U represent the vectors of the accelerations, velocities, and deflections of the nodes of the riser, respectively; F_y signifies the vector of the forces due to buoyancy and the weight of the riser; and F_v is the vector for the excitations of the vessel, which were modeled in the following form.

$$\mathrm{F}_v = -\mathrm{M}\ddot{\mathrm{Y}} - \rho(C_m - 1)\mathrm{V}\ddot{\mathrm{Y}}, \tag{34}$$

where $\ddot{\mathrm{Y}}$ and V denote the vectors of the accelerations of the vessel (derived from the transfer function between the waves and the movements of the vessel) and the volumes of the elements of the riser, respectively. Later, Chen and Lin (1989) modified the global equation of motion, Eq. (33), to include the effect of the movements of the vessel in a different way. The following formulation was introduced.

$$\mathrm{M}\ddot{\mathrm{U}} + \mathrm{C}\dot{\mathrm{U}} + \mathrm{KU} = \mathrm{F}_y - \mathrm{C}_0\dot{\mathrm{U}}_0 - \mathrm{K}_0\mathrm{U}_0, \tag{35}$$

where C_0 and K_0 represent the matrices of the influence coefficients and U_0 and $\dot{\mathrm{U}}_0$ are the vectors of the deflection and the velocity at the top end of the riser, respectively.

To simulate the VIVs in consideration of the effects of the internal fluid, Yamamoto et al. (2004) utilized the FEM to convert the previously developed PDE, Eq. (14), into the following global equation for the transverse motion of the riser.

$$\mathrm{M}\ddot{\mathrm{W}} + \mathrm{C}\dot{\mathrm{W}} + \mathrm{KW} = \mathrm{F}_z, \tag{36}$$

where W represents the vector of transverse deflections of the riser and F_z denotes the vector for the hydrodynamic force acting along the $\hat{k}$-axis. In Eq. (36), the global mass matrix M also includes the effects of the internal fluid. Later, in simulating the VIVs, Huera-Huarte et al. (2006) modified Eq. (36) by considering the damping matrix C as a linear combination of the global mass matrix M and stiffness matrix K (i.e., Rayleigh damping), which are given in the following form.

$$\mathrm{C} = \alpha\mathrm{M} + \beta\mathrm{K}, \tag{37}$$

where $\alpha = 0.0555$ and $\beta = 0.0002$, whose values were obtained by comparing their simulation results with the data acquired from the experiments conducted by Delta Flume in Holland, during May 2003. Further developments in simulating the VIVs were reported in Chen et al. (2012, 2016), where the force vector F_z in Eq. (36) includes the components of both the transverse and the drag forces along the $\hat{k}$-axis.

So far, we have discussed the FEM-based formulations developed for investigating only the lateral or only the transverse responses of the riser. For simulating the responses of the riser in the 3D space, one way is to discretize the riser into a finite number of linear spring elements, each element consisting of two nodes, with each node having three degrees of freedom (i.e., lateral u_i, transverse w_i, and axial δ_i nodal deflections, where the subscript i stands for the i-th node). This results in the following nodal equations of motion (Taylor, 1977).

$$m_i\mathrm{I}\ddot{\mathrm{S}}_i = \mathrm{T}_i + \mathrm{F}_i + \mathrm{Q}_i, \tag{38}$$

where I denotes the 3×3 identity matrix, $\mathrm{S}_i = [u_i, w_i, \delta_i]$ represents the nodal position vector, T_i is the vector of the nodal tension, F_i stands for the force vector consisting of the drag force, the inertia force, and the riser's weight concentrated at the node, and Q_i signifies the nodal vector representing the constant shear force. Later, Yazdchi and Crisfield (2002) developed a more detailed 3D formulation for the riser, considering nodal rotations $\vartheta_{i,x}$, $\vartheta_{i,y}$, and $\vartheta_{i,z}$ about the $\hat{i}, \hat{j}$, and $\hat{k}$ axes, respectively, in addition to the lateral, transverse, and axial deflections of the nodes, thereby resulting in six degrees of freedom at each node (i.e., $\mathrm{S}_i = [u_i, w_i, \delta_i, \vartheta_{i,x}, \vartheta_{i,y}, \vartheta_{i,z}]$). Later, a new 3D formulation was proposed by Athisakul et al. (2011) to investigate the bending, twisting, and axial deformations of a flexible riser caused by the internal fluid, considering $\mathrm{S}_i = [u_i, w_i, \varepsilon_i]$ where ε_i denotes the nodal strain.

Other than FEM, the RFEM has also been utilized in modeling the riser systems in which the riser is discretized into a finite number of rigid finite elements that are connected with each other by means of massless spring-damper elements. Based on the RFEM, Adamiec-Wojcik et al. (2015) investigated the bending and elongation of the riser in the $\hat{i}\hat{j}$-plane upon the movements of the vessel. First, the riser was discretized into small rigid elements having four degrees of freedom (i.e., the position coordinates x_e and y_e of the element along the $\hat{i}$ and $\hat{j}$ axes, respectively, the axial deflection δ_e of the element (i.e., along the $\hat{k}$-axis), and the rotation of the element ϑ_e about the $\hat{k}$-axis: $\mathrm{S}_e = [x_e, y_e, \delta_e, \vartheta_e]$). The following form of the global equation of motion of the riser was achieved.

$$\mathrm{M}\ddot{\mathrm{S}} - \mathrm{DR} = \mathrm{F}, \tag{39}$$

where M is the global mass matrix, $\mathrm{S} = \begin{bmatrix} \mathrm{S}_1^T & \mathrm{S}_2^T & \ldots & \mathrm{S}_n^T \end{bmatrix}^T$ is the vector of the generalized coordinates of the system, R is the vector of the reaction forces due to the constraints of the connections between the elements, D is the matrix of the coefficients associated with the constraints, and F is the force vector consisting of buoyancy, drag, and inertial forces. Later, Drag (2017) modified the formulation in Eq. (39) to make each element to have five degrees of freedom (i.e., the position coordinates x_e, y_e, and z_e along $\hat{i}, \hat{j}$, and $\hat{k}$ axes, and the rotations $\vartheta_{e,x}$ and $\vartheta_{e,z}$ about the $\hat{i}$ and $\hat{k}$ axes, respectively: $\mathrm{S}_e = [x_e, y_e, z_e, \vartheta_{e,x}, \vartheta_{e,z}]$).

Another approach, similar to the RFEM, for modeling the offshore structures is the finite segment method (FSM) (Xu and Wang, 2012). In FSM, a slender body (e.g., the marine riser) is first discretized into a series of flexible segments and then the deflections of the segment and the external forces acting on the segment are independently analyzed. Consequently, the deformation of the whole structure is decomposed into micro-deformations of the flexible segments and the governing equations are listed according to the equilibria of the moments on the individual segments (Xu et al., 2013).

Besides the FEM/RFEM, the FDM is the secondly most widely used numerical method for solving the equations of motion of the riser systems. In the FDM, the length of the riser is first discretized into a finite number of small spatial segments Δx, resulting in N number of nodal points (i.e., $x_1, x_2, \ldots, x_N$) along the length of the riser. Then, to obtain the response of the riser in time domain, the PDE is discretized both in space and time by converting its derivative terms (both in space and time) into algebraic approximations, which are functions of Δx and the discrete time interval Δt, by using the Taylor series expansion. The resulting equations are then solved for obtaining the responses at all spatial points. For example, Jain (1994) developed the following FDM-based formulation to perform the static analysis of the riser system (i.e., only the spatial discretization was performed).

$$\begin{aligned}&(1/(\Delta x^4))(EI_{i+1}(w_{i+2}-2w_{i+1}+w_i)-2EI_i(w_{i+1}-2w_i+w_{i-1})+EI_{i-1}(w_i\\&-2w_{i-1}+w_{i-2}))-(T_i/(\Delta x^2))(w_{i+1}-2w_i+w_{i-1})\\&=F_{Ti},\end{aligned} \tag{40}$$

where the index I denotes the discretization in space. Later, in analyzing the dynamic response of a catenary riser, the following formulation based on FDM was developed by Chatjigeorgiou (2008).

$$\begin{aligned}(S_i^{j+1}+S_i^j-S_{i-1}^{j+1}-S_{i-1}^j)/(2\Delta x)=(S_i^{j+1}+S_{i-1}^{j+1}-S_i^j-S_{i-1}^j)/(2\Delta t)+(F_i^{j+1}\\+F_{i-1}^{j+1}+F_i^j+F_{i-1}^j)/4,\end{aligned} \tag{41}$$

where the index j denotes the discretization in time and $S=[T,Q,\dot{u}_n,\dot{u}_t,\Phi,\psi]$ represents the vector of the unknowns (i.e., the tension T, the shear force Q, the normal $\dot{u}_n$ and the tangential $\dot{u}_t$ components of the lateral velocity of the riser, the curvature Φ of the riser, and the angle ψ between the riser's tangent and the $\hat{j}$-axis). Equation (41) investigates the response of the riser in 2D. To simulate the vortex induced vibrations of the riser, Huang et al. (2011a) discretized Eq. (14) by implementing the centered space and forward time finite difference scheme to obtain the following equations of motion of the riser along the $\hat{k}$-axis.

$$\begin{aligned}&(EI/\Delta x^4)w_{i-2}^j-((T_i/\Delta x^2)-(T_i'/2\Delta x)+(4EI/\Delta x^4))w_{i-1}^j+(EI/\Delta x^4)w_{i+2}^j\\&+((2T_i/\Delta x^2)+(6EI/\Delta x^4)+(m/\Delta t^2)+(c/\Delta t))w_i^j-((T_i/\Delta x^2)\\&+(T_i'/2\Delta x)+(4EI/\Delta x^4))w_{i+1}^j\\&=F_{Ti}^j+((2m/\Delta t^2)+(c/\Delta t))w_i^{j-1}-(m/\Delta t^2)w_i^{j-2}.\end{aligned} \tag{42}$$

To investigate the response of the riser in the 3D space, Chatjigeorgiou (2008) developed an FDM-based formulation considering $S=[T,Q_n,Q_t,\dot{u},\dot{w},\dot{\delta},\Phi_n,\Phi_t,\vartheta,\psi]$, where Q_n and Q_t denote the components of the shear force whereas Φ_n and $\Phi_t$represent the components of the curvature of the riser along the normal and tangential directions, respectively. The discretized governing equations of motion were obtained in a matrix form as follows.

$$\begin{aligned}&(M_i^{j+1}+M_i^j)((S_i^{j+1}-S_i^j)/\Delta t)+(M_{i-1}^{j+1}+M_{i-1}^j)((S_{i-1}^{j+1}-S_{i-1}^j)/\Delta t)+(K_{i-1}^{j+1}\\&+K_i^{j+1})((S_i^{j+1}-S_{i-1}^{j+1})/\Delta t)+(K_{i-1}^j+K_i^j)((S_i^j-S_{i-1}^j)/\Delta x)+(F_i^{j+1}+F_{i-1}^{j+1}\\&+F_i^j+F_{i-1}^j)\\&=0,\end{aligned} \tag{43}$$

where M and K denote the global mass and stiffness matrices of the system, and F signifies the global force vector. Later, Doan and Nishi (2015) utilized the same formulation, given by Eq. (43), to investigate the response of the riser in the 3D space considering the wake oscillator model for generating the vortex-induced force.

The fluid-structure modeling techniques discussed thus far neglect the interaction of the fluid with the entire external surface of the riser, which can be accounted for by modeling the complete 3D flow around the riser using the Navier-Stokes equation. Because the aspect ratio of the riser is very large and the flow around it is very complex, a complete 3D analysis

Table 1
A summary of important contributions in the field of mathematical riser modeling.

Area	Reference	Contribution
Hydrodynamic force	Morison et al. (1950)	Morison's equation for modeling of sea currents and waves, see Eq. (2)
	Chakrabarti and Frampton (1982)	A nonlinear hydrodynamic transverse force model, see Eq. (11)
	Young et al. (1978)	A simplified model of drag force caused by a constant velocity current, see Eq. (4)
	Faltinsen (1990)	An oscillating transverse force model, see Eq. (12)
	Faltinsen (1990)	An oscillating drag force model, see Eq. (24)
	Niedzwecki and Liagre (2003)	A linearized drag force model, considering the RMS value of the relative velocity between the riser and the water, see Eq. (5)
	Kuiper et al. (2007)	A nonlinear drag force model, considering the effect of internally flowing fluid, see Eq. (22)
Lateral vibrations	Huang and Dareing (1968)	Equations of motion of a drilling pipe, for investigation of buckling and lateral-free vibrations, see Eq. (1)
	Young et al. (1978)	Riser model for investigation of the influence of riser tension, buoyancy, and fluid pressure on lateral vibrations, see Eq. (3)
	Chakrabarti and Frampton (1982)	Riser model for investigation of the effect of lateral and axial hydrodynamic loadings on the lateral response, see Eq. (6)
	Chakrabarti and Frampton (1982)	Mathematical model of the riser's tension, see Eqs. (7)–(8)
	Seyed and Patel (1992)	A new model of the riser's tension incorporating the effect of an internally flowing fluid, see Eq. (20)
	Niedzwecki and Liagre (2003)	Riser model for simulation of lateral vibrations, considering linear (viscous) and nonlinear (drag) damping forces, see Eq. (10)
Transverse or vortex-induced vibrations	Furnes (2000)	Riser model for simulation of VIVs, considering linear damping, see Eq. (14)
	Sagatun et al. (2002)	Riser model for simulation of VIVs, considering both the linear and nonlinear damping forces, see Eq. (15)
	Dai et al. (2014b)	Riser model for investigation of the effect of internal fluid on VIVs, see Eq. (23)
	Facchinetti et al. (2004)	A wake-oscillator model for simulation of VIVs considering the riser as a rigid cylinder, see Eqs. 16 and 17
	Violette et al. (2007)	A distributed wake-oscillator model considering the riser as a distributed parameter system, see Eqs. 18 and 19
Multi-dimensional riser models	Srinil and Zanganeh (2012)	A 2D model for the coupled lateral-transverse riser vibrations, considering the riser as a rigid cylinder, see Eqs. 25 and 26
	Nguyen et al. (2013)	A 2D model for the coupled lateral-transverse riser vibrations, considering as a flexible riser, see Eqs. 27 and 28
	Do and Pan (2009) Meng and Chen (2012) Srinil et al. (2009)	Development of riser models for investigation of the lateral, transverse, and axial vibrations of a flexible riser in the 3D space, without considering the coupling between riser movements

is infeasible. Rather, a simpler approach, known as the strip theory (Meneghini et al., 2004; Willden and Graham, 2001, 2004; Vidic-Perunovic and Jensen, 2005) is utilized, which applies the hydrodynamic forces (modeled in a 2D plane or strips) on the surface of the riser. Then, the solution for the combined system (i.e., the flow and the riser model) is obtained by implementing a computational fluid dynamics (CFD) program. Some important works in the field of mathematical riser modeling is summarized in Table 1.

3. Riser analysis

Extensive analyses have been performed, both experimentally and theoretically (i.e., through computer simulations), to understand the behavior of the riser system operating in harsh environmental conditions. A thorough review of those analyses is presented below.

3.1. Theoretical analysis

To analyze a riser system theoretically, the equations of motion must be solved either analytically (to obtain the exact solution) or numerically (to obtain an approximate solution). In analytical methods (Aranha et al., 2001a; Atadan et al., 1997; Chucheepsakul et al., 2003; Gu et al., 2013b; Ramos and Pesce, 2003; Sparks, 2002; Yang and Xiao, 2014), a mathematical form of the solution (i.e., the deflection of the riser) of the riser's equation of motion was usually assumed. For example, in Kirk et al. (1979), the deflection was assumed to be a sinusoidal series. Another approach for obtaining an analytical solution is by applying the normal mode method (i.e., modal superposition; e.g., Furnes (2000)). In numerical techniques in contrast, approximation methods (e.g., FEM, FDM, and the finite volume method) are used to obtain an approximate solution of the equations of motion of the system. Since the works involving analytical solutions are limited (Patel and Seyed, 1995), this review will focus mainly on the numerical techniques for analyzing the riser systems.

3.1.1. Static and dynamic analyses

Theoretical solutions to the equations of motion of a riser system can be obtained by either static or dynamic analyses. In this subsection, a detailed review on the static and dynamic analysis methods is presented. Some earlier reviews on the said topic may be found in Chen et al. (2006), Larsen (1992) and Patel and Seyed (1995). Static riser analysis is performed to obtain the deflected shape of the riser under a constant load distribution (Jain, 1994; Santillan and Virgin, 2011). In such cases, the inertia term $m\ddot{w}$ in the equations of motion of the riser (for example, Eq. (14)) is not considered, and the hydrodynamic force given by Eq. (4) (i.e., a steady current profile) is considered as the forcing function. In dynamic analyses meanwhile, the deflected shape of the riser is obtained in consideration of the inertia term, the time varying load distribution, and the movements of the vessel (Chakrabarti and Frampton, 1982). Dynamic analyses can be categorized into three types (Kirk et al., 1979): i) time-domain or time-history analysis, ii) frequency-domain analysis, and iii) stochastic or non-deterministic analysis. Both the time- and frequency-domain techniques are based on deterministic solutions of the riser equations. Time-domain analyses, in comparison with those in the frequency domain, provide more general solutions; for example, the nonlinear drag term and the variation in buoyancy near the surface can be included in the analysis. However, frequency-domain analyses usually are performed to obtain a steady-state solution to the fatigue problem, where the wave forces and the motions of the vessel are assumed to be harmonic (Young et al., 1978). Whereas fatigue analyses have been performed mostly in the frequency domain, several time-domain fatigue analyses have been reported recently (Thorsen et al., 2015; Wang et al., 2015c; Xue et al., 2015). On the other hand, stochastic analyses (based on non-deterministic solutions) are based on the modal superposition method (random vibration), which requires the linearization of time-dependent loads including nonlinear drag forces (Jensen, 2011).

3.1.1.1. Frequency- and time-domain analyses. Young et al. (1978) performed a frequency-domain analysis for analyzing the stresses and fatigues on the riser subject to excitations from the waves and the movements of the vessel. The response of the riser caused by random waves was utilized to predict the stresses, while the response caused by the regular waves was utilized to predict the fatigue. Later, in Kirk et al. (1979), a frequency-domain analysis was performed to predict the dynamic and static stresses of the riser subject to the movements of the vessel. It was concluded that the movements of the vessel, in comparison to the drag forces induced by the currents, play a more vital role in generating the bending stresses. Benassai and Campanile (2002) performed a frequency-domain analysis using the modal priority technique in order to predict the multi-mode VIVs of a flexible riser under the actions of waves and currents. The employed technique was based on the amplitudes of the active vibration modes, rather than their frequencies, to predict the multi-modal oscillations of the riser under a sheared-flow condition. Modarres-Sadeghi et al. (2010) performed a frequency-domain analysis to investigate the effect of higher modes on the fatigue life of a flexible riser. They found that consideration of higher modes in the analysis resulted in a significant decrease in the fatigue life of the riser.

Mukundan et al. (2010) performed a combined time and frequency domain analysis using the wavelet transform technique for reconstructing the vortex-induced response of a flexible riser. They then utilized the reconstructed response to analyze the fatigue damage on the riser. Response reconstruction was achieved for two separate cases: i) when a large number of sensors were placed along the length of the riser, and ii) when only a small number of sensors was used. Shi et al. (2012) employed the weighted wave-form analysis (WWA) technique to reconstruct the response of a flexible marine riser, and concluded that for that purpose, sensor positioning is more important than the number of sensors used. Shi et al. (2014) also demonstrated the effectiveness of the WWA technique, as combined with field measurements, to predict the long-term fatigue damage to the riser system considered. Recently, a new approach based on a variance reduction technique for analyzing the long-term fatigue damage in conjunction with the time domain simulations has been proposed in Gao and Low (2016).

Patel and Jesudasen (1987) performed a time domain dynamic analysis to simulate the lateral response of a flexible riser subject to the movements of the vessel and disturbance loads due to the currents and waves. Using the FEM, they obtained a numerical solution to the equations of motion. However, using their proposed method, they could not simulate the VIVs of the riser. A simulation of the VIVs was later reported in Chen and Lin (1989), who developed a numerical scheme based on the Newmark method to simulate the responses in consideration of the effects of the static and dynamic offsets of the surface vessel and disturbance loads due to waves and currents. Sagatun et al. (2002) presented a time domain analysis for determining the VIVs of two adjacent risers moving relative to each other in a steady flow. The main purpose of their study was to predict whether the two risers would collide while moving in the presence of wakes. Later, a numerical investigation by Chen and Kim (2010) revealed that the velocity of the external current and the distance between two adjacent risers are the most significant factors in influencing the VIVs of the considered two adjacent risers.

A nonlinear FEM-based scheme was developed in Kaewunruen et al. (2005) to evaluate the nonlinear free vibrations of a flexible marine riser conveying internal fluid. According to their results, flexural rigidity has the most significant impact on the response: The higher the flexural rigidity is, the stiffer the vibrational responses are. In Huera-Huarte et al. (2006), Eq. (14) was solved using the FEM to investigate the VIVs of a flexible cylinder and to determine the hydrodynamic forcing function from the experimental data of Chaplin et al. (2005a). A more detailed FEM-based algorithm subsequently was developed by Monprapussorn et al. (2007) to investigate the effects of the internally flowing fluid on the response of an extensible marine riser. That study revealed that the impulsive acceleration of the internal fluid affected the stability of the riser significantly, specifically by shifting the equilibrium positions.

Later, Guo et al. (2008) utilized the Galerkin approximation method to analyze the effects of the internally flowing fluid on the VIVs of a top-tensioned riser. An advanced FEM-based simulation scheme for investigating the VIVs of a flexible riser was reported in Chen et al. (2012), who additionally considered variations in the structural parameters along the length of the riser in their formulation. Their results revealed that for lower modes, a high-amplitude riser response occurred at the position where the tension was small, whereas for higher modes, the same high-amplitude response occurred at the riser position where the bending stiffness was low. Chen et al. (2014) applied the FEM to their study of the impact of the heave motion of the vessel on the VIVs of the riser. They found that the heave motion has a significant impact on the displacements of the riser, particularly, in the lower modes. Soon thereafter, Chen et al. (2015) performed an FEM-based time domain dynamic analysis of a flexible riser, isolating the occurrence of a nonlinear response amplification phenomenon signifying the response amplification at the top end with the propagation along the length of the riser. In another recent study, Adamiec-Wojcik et al. (2015) developed an FEM-based algorithm to simulate the bending and longitudinal displacements of a freely hanging riser subject to vessel movements and ocean loads. They were able to demonstrate the capability of the developed algorithm to avoid a contact of the riser tip with an obstacle on the seabed by controlling the movements of the vessel. Recently, a FEM-based dynamic analysis of the marine riser conducted by Mao et al. (2016b) revealed that increasing the top tension of the riser can significantly reduce the large lateral deflection and the bending movement of the riser under extreme environmental loads. Also, Chen et al. (2016) reported an FEM-based time domain analysis for investigating the multi-mode VIVs of a flexible marine riser subject to stepped and shear flows. The FEM-based investigation of Meng et al. (2017) has revealed that the multi-mode VIVs can be caused by the changes in the velocity of the internal fluid. In Teixeira and Morooka (2017), an FEM-based analysis of the VIVs of the risers was performed: In their analysis, the estimation of the hydrodynamic coefficients was achieved in real time. Recently, in Khan and Ahmad (2018), a 3D nonlinear dynamic analysis of a riser was performed in the time domain using the finite element solver ABAQUS. The response history was then utilized in performing a fatigue reliability assessment by using Monte Carlo simulations.

One of the earliest studies on simulating the responses of risers in the 3D space is Ghadimi (1988), who utilized a numerical scheme combining the tangent stiffness incremental approach with the Wilson-theta numerical integration algorithm to obtain the lateral, transverse, and axial deflections of the riser under the effect of the hydrodynamic forces, modeled by Morison equation. Later, Herfjord et al. (1999) employed the CFD computations to simulate the VIVs of a flexible riser in the 3D space by utilizing the strip theory (as discussed in Subsection 2.5) to impose the hydrodynamic forces on the surface of the riser. Then, Yazdchi and Crisfield (2002) developed a nonlinear scheme, based on the FEM, to simulate the responses (namely bending, axial, and shear deformations) of a flexible riser in the 3D space. Meneghini et al. (2004) performed a CFD-based analysis to simulate the VIVs of a flexible cylinder subject to both uniform and sheared-flow conditions. They simulated the hydrodynamic forces by means of the distributed vortex method (DVM), which is a Lagrangian numerical technique for modeling of 2D incompressible and viscous fluid flows. Yamamoto et al. (2004) adopted the same simulation scheme to analyze the VIVs of a vertical riser. The CFD analysis of a vertical riser subject to a uniform current also was reported in Willden and Graham (2004), where it was demonstrated that the simulation of multi-modal VIVs can be achieved if the vortex-shedding frequency varies along the length of the riser. Athisakul et al. (2011) used the FEM to analyze both the static and dynamic responses of a fluid-transporting riser in the 3D space. Sun et al. (2012) developed a new strip-wise discrete vortex method to simulate the VIVs of flexible risers, also in the 3D space, and calculated the VIVs at each strip, using the finite volume method combined with the incremental method to simulate the fluid-structure interaction. Zhang et al. (2013) analyzed the experimental data of the VIVs by utilizing the FEM, specifically for estimating the tangential, transverse, and longitudinal hydrodynamic forces acting on them. Recently, Madani et al. (2016) utilized the strip theory in simulating the multi-mode VIVs of flexible risers.

Although the riser analyses have mostly used the FEM, several works have utilized the FDM. Wanderley and Levi (2005) presented a finite difference numerical scheme based on Beam and Warming approximation formula for simulating the VIVs of a flexible marine riser. Chatjigeorgiou (2008) used the FDM in developing an algorithm for analyzing the stability of the riser subject to vessel movements in the 2D space. Later, the same finite difference scheme was updated by Chatjigeorgiou (2010a,b) to investigate the response of the riser in the 3D space. The proposed scheme could also analyze the effect of internally flowing fluid on the response of the riser. Another algorithm, based on the FDM, was developed in Huang et al. (2011a) for investigating the 3D response of a top-tensioned riser considered as a tensioned beam. Considering the riser as a tensioned beam, a 3D FDM-based algorithm was utilized in Jung et al. (2012) to investigate the VIVs of the riser under the influence of both the inline and transverse hydrodynamic forces. Wang et al. (2014b) developed a numerical scheme, based on the FDM, for analyzing the stresses and deformations of a flexible marine riser under the combined effect of the axial and lateral hydrodynamic forces. They found that the stresses in the structure of the riser increase with the drag speed while bending deformation increases with the depth of water and decreases with the size of the riser. Recently, Doan and Nishi (2015) employed an FDM-based scheme combined with the wake-oscillator model to simulate the VIVs of a flexible riser. They discovered, however, that their proposed method needed to be improved for predicting more accurate amplitudes and frequencies of the responses. Torres et al. (2015) developed an FDM-based algorithm for application to a nonlinear analysis in estimating the structural parameters of the riser. Kamble and Chen (2016) utilized the finite-analytical Navier-Stokes technique, based on the finite volume method, in predicting the VIVs and fatigue damages to the riser upon inline and cross flows. Recently, a FDM-based numerical analysis was performed in Lei et al. (2017) to investigate the transverse deflections and bending stresses in the riser exposed to the coupled axial and transverse loads, considering a time-varying tension caused by the heave movements of the vessel.

3.1.1.2. Stochastic analysis. Sagrilo et al. (2000) performed a stochastic analysis to predict the tension at the top end of the riser under the influence of the heave-excited motions of the surface vessel. In Aranha and Pinto (2001), an analytical approximation of the probability density function of the dynamic tension in riser systems was developed, which requires an estimation of the statistical parameters of the dynamic tension using numerical simulation. In another work, Aranha et al. (2001b) estimated random excitations in the structure of the riser caused by the movements of the vessel. Mukundan et al. (2009) developed a stochastic algorithm based on a wake-oscillator model with randomly varying parameters to predict the minimum fatigue life of marine risers, and used experimental data provided by the Norwegian Deepwater Program (Trim et al., 2005) for the validation of their proposed prediction algorithm. Srivilairit and Manuel (2009) statistically investigated the acceleration of the riser and the coincident velocity profiles of the current in formulating the relationship between those profiles and the response of the riser. They applied the proper orthogonal decomposition method (i.e., a numerical scheme for statistical analysis) to analyze full-scale field data obtained from a deepwater drilling site. Sun et al. (2014) performed a stochastic analysis of the inline and transverse vibrations of a flexible riser, considering the vortex-induced response as a stationary random process and calculating it by the pseudo-excitation method. Low and Srinil (2016) proposed a stochastic approach (based on the point estimate method) to conduct fatigue analyses, which revealed that the randomness of wake coefficients (in the wake oscillator model) leads to large variability in fatigue.

3.2. Experimental analysis

Thus far, we have discussed a number of mathematical models and analysis techniques employed in investigating riser systems. To ensure the validity of these models and techniques by comparison of the theoretical results with experimental data, a proper experimental paradigm/setup needs to be established. A special attention to the design of such experimental setup is required, which often requires, in turn, the design of a test rig having eigen frequencies lower than that of the first eigenfrequency of the riser (Trim et al., 2005), so that the effect of the rig to the responses of the riser can be minimized. Furthermore, the experimental model must be able to predict the behavior of the actual system, which can be achieved by implementing the similarity theory (Fontaine et al., 2006; Guo et al., 2006; Queau et al., 2013; Shah et al., 2017). Also, the experimental data obtained from the sensors (i.e., accelerometers and angular rate devices) attached to the riser must be processed into the form suitable for validating the theoretical models. Kaasen and Lie (2003) applied the methods of modal decomposition and the least squares combined with frequency domain calculations to extract useful experimental data related to the lateral vibrations of the riser under the influence of steady currents.

Experimental investigations on the inline and transverse vibrations of a flexible riser subject to uniform currents have been reported in the literature several times (Chaplin et al., 2005b; Gu et al., 2013a; Guo and Lou, 2008a; Mao et al., 2016a; Trim et al., 2005), where the uniform current typically having been generated by towing a cylinder in a tank containing still water. Chaplin et al. (2005a) designed a special experimental facility to investigate the transverse and lateral vibrations of a vertical tensioned model riser subject to a stepped current. They were able to measure the responses to vortex excitations up to the eighth mode in the transverse direction and to the fourteenth in the lateral direction. The obtained experimental data thereby were later used by Huera-Huarte et al. (2006) to numerically calculate the inline and transverse forces on a flexible cylinder. Chaplin et al. (2005b) also performed experiments to measure the transverse and lateral vibrations of a vertical model riser subject to a stepped current. They compared their experimental measurements with the simulated data for the transverse and lateral vibrations of eleven different empirical and CFD-based studies. The comparison of experimental and numerical predictions revealed that the empirical models were more successful in predicting the VIVs of the riser than the CFD-based ones. The experimental data thus obtained was subsequently utilized by Ma et al. (2014) to validate their algorithm for simulating the VIVs. Trim et al. (2005) performed experiments to analyze the inline and transverse responses of a long flexible riser model considering different arrangements of the helical strakes, which are mounted on the surface of the riser for suppressing the VIVs (i.e., passive suppression). It was revealed that the length of the riser covered with helical strakes, as well as the nature of flow, are the two main factors to be considered in the design of a riser from the stand point of fatigue prevention. Wang et al. (2014a) performed experiments in an ocean basin to investigate both the inline and transverse vibrations of a steel catenary model riser subject to the movements of the vessel. They were able to confirm that the movements of the vessel cause VIVs and that the fatigue damage is more sensitive to the amplitude rather than the period of the movements. Later, the experimental investigation of Wang et al. (2015a) on the VIVs of a flexible riser subject to an oscillatory flow caused by the heave motion of the surface vessel revealed a strong dependence of fatigue damage on the Keulegan–Carpenter (KC) number of the flow. In fact, an inverse relation between fatigue damages and KC numbers was observed: Fatigue damage was found to be greater at a smaller value of KC number rather than at larger ones. In Wang et al. (2015b) meanwhile, an empirical relationship among the dominant frequency of the riser, the maximum equivalent velocity of the flow, and the KC number was obtained. The ratio of the dominant frequency of the riser and the frequency of the oscillating flow was found to be equal to the product of the St and the KC numbers corresponding to the maximum flow velocity.

During operation, a long flexible riser can undergo either the single- or multi-mode lock-in phenomenon. Multi-mode VIVs (Fan et al., 2015; Huang et al., 2011b) of risers have been analyzed in a number of studies. In investigating the multi-mode lock-in phenomenon, Marcollo and Hinwood (2006) performed experiments entailing simultaneous excitation of the VIVs of a flexible riser model at two different resonant modes, which resulted in an unexpected phenomenon of single-mode lock-in only at a higher frequency. Experimental investigations of Song et al. (2011) revealed that their flexible riser model underwent a multi-mode lock-in behavior while subject to both inline and transverse vibrations under a uniform current profile. However, the vibration amplitude in the inline direction was found to be considerably smaller than that of the transverse direction. Also, the frequency of the inline vibrations was found to be approximately twice that of the transverse vibrations. In all these studies mentioned above, the VIVs of the riser were analyzed under the effect of a uniform current profile. Mao et al. (2014), in contrast, performed experiments to investigate the multi-mode lock-in behavior of a flexible model riser under the effect of shear flow, wherein the experiments were conducted in a deepwater offshore basin rather than a water tank. The tests were performed at two different Reynolds numbers (i.e., $Re = 1105$ and 2761). At $Re = 1{,}105$, the frequencies of the inline and transverse vibrations during lock-in were found to be the same (which was not the case for the uniform current). However, for $Re = 2{,}761$, the frequency of inline vibrations was found to be twice that of the transverse vibrations (i.e., the same result reported earlier by Song et al. (2011) for a uniform current). Mao et al. (2015) later demonstrated experimentally that under shear flow, the vortex-shedding frequency differs at different points of the riser (i.e., along its length) but that the frequency of the VIVs remains the same.

The experimental works addressed above were undertaken to address the most commonly occurring problems related to the complex vibrations in the risers. There are also several other studies that have addressed some of the uncommon yet very important issues. Fontaine et al. (2006) experimentally investigated the vibrations due to interference between two risers in tandem configurations. They found that the root mean square value of the motion (deflection) of the downstream (rear) riser was more than twice the value of the upstream (front) riser. This was due to the fact that the motion of the rear riser was influenced not only by its own wake but also by the vortices generated by the front riser (i.e., wake-induced oscillations). Guo and Lou (2008b) performed the first experiment analyzing the effect of the internal flow on the response of the riser, determining that increasing the internal flow resulted in an increased strain (in both the inline and transverse directions) and a decreased natural frequency. Liu et al. (2014) performed experiments to investigate the effects of the tension of the drill-pipe on the VIVs, revealing that the VIVs can be suppressed by increasing the tension of the drill-pipe. Recently, Xu et al. (2018) performed experiments to investigate the multi-mode vibration behavior of two side-by-side flexible cylinders (risers) by towing them in a water tank. Their investigations revealed that the risers exhibited large oscillations in the inline direction as compared to those in the transverse direction.

4. Riser control

As discussed in the preceding section, a number of studies have analyzed the riser responses under various load conditions. In this section, the schemes specifically related to the prevention or suppression, by both passive and active control strategies, of harmful vibrations of the risers and especially fatigue damage will be reviewed.

4.1. Passive control

Passive control strategies include modifications of the riser's structural rigidity, internal fluid's velocity, the top tension, structural damping, and other means (Meng and Zhu, 2015). The most applicable passive control procedure for the suppression of the vibrations, however, entails covering the riser along its length with vortex-suppression devices, for example, spoilers (Abbassian and Moros, 1996), control rods (Wu et al., 2012a), splitter plates (Lou et al., 2016), and helical strakes (Baarholm et al., 2005; Holland et al., 2017). Helical strakes are the most commonly used vibration suppression devices; indeed, it is experimentally proven that the use of helical strakes can significantly reduce fatigue damage (Gao et al., 2016, 2015b).

Abbassian and Moros (1996) conducted a feasibility study of the use of air-bubble spoilers as a vortex suppression device for a drilling riser subject to uniform flows. Their work was based on the observation that injection of air bubbles into the flow reduces the vortex-induced force but has little effect on the frequency of the riser. They found, however, that air-bubble spoilers are not economically feasible for actual risers undergoing large deflections. Wu et al. (2012a) carried out an experimental investigation analyzing the capability of control rods for suppressing the VIVs of a long tensioned riser. They performed the experiments with four slender control rods in parallel with the riser and arranged them at a uniform angle interval of 90°. They concluded that by decreasing the spacing between the control rods, an effective suppression of the vibration could be achieved, provided that the complete length of the riser is covered by the control rods. Borges et al. (2014) explored the use of visco-elastic layers, which can be assembled along the length of a steel catenary riser, resulted in an increased structural damping and, thereby, reduced vibrations, without employing any vibration suppression devices. Nishi and Doan (2015) proposed a method that utilizes only a small number of damping devices for suppression of the transverse vibrations of a flexible marine riser. They used the modal control technique to first identify the mode that contributes most to the VIVs of the riser, and then suppressed it using the damping devices. Tan et al. (2015) compared the responses of composite and steel catenary risers in simulations, finding the composite risers to be more vulnerable to VIVs and fatigue damage, due specifically to their lower structural frequencies. It was also shown that assembling small-diameter long-buoyancy modules at the bottom of the riser resulted in superior vibration suppression as compared to the top of the riser.

Baarholm et al. (2005), in an FEM-based simulation study, demonstrated that the use of helical strakes increased the fatigue life of risers. In their analysis, they used different existing empirical values for the lift and damping coefficients. Further, they performed experiments on a model of the riser to validate the empirical values for the coefficients used in their analysis. In Trim et al. (2005), various configurations of helical strakes were used to evaluate their performances in suppressing the VIVs. It was revealed that the VIVs of the riser were reduced by increasing the coverage area of the strakes on the riser. The greater strake coverage also reduced the VIVs due to inline forces as compared with those due to the transverse forces. Lubbad et al. (2011) carried out an experimental investigation of the efficiency of round-edged helical strakes in suppressing the vibrations of a rigid riser (circular cylinder). Three different parameters, the number of starts (i.e., the individual strakes used), the pitch (the distance along the length of the riser required for one complete turn of the strake), and the diameter were varied to obtain twenty-eight different configurations of the strakes. It was concluded that regardless of the pitch and the diameter of the strake, the configuration of three starts was the most efficient among the tested configurations in reducing the amplitudes of vibrations in both the inline and transverse directions. In Quen et al. (2014), an experimental analysis of the effects of the pitch and the height of helical strakes in suppressing the VIVs of long risers was carried out. It was concluded that although the pitch does not play a vital role in suppressing the amplitude of vibrations, a higher pitch nonetheless delays lock-in. An increased height, moreover, was found to improve vibration suppression relative to the contribution of pitch. Experimental investigations reported by Gao et al. (2015b, 2016) likewise concluded that as compared to pitch, the height of the strakes has a greater influence on the amplitude of VIVs and consequently on the fatigue life of risers, under both uniform and sheared-flow conditions.

4.2. Active control

Besides the passive controls above, the vibrations also can be controlled through active means. For example, Rustad et al. (2008) implemented a proportional-integral control to prevent the collisions of two risers in tandem arrangement and subject to uniform currents. Their control law was developed to adjust the top tensions of the two risers individually by implementing a feedback control. Their motivation was to propose a solution to the problem to prevent a collision of two risers in tandem configuration that would be more cost-effective than maintaining the top tension at a high constant value or increasing the space between two risers. The numerical investigation of Mazzilli and Sanches (2011), undertaken to address the instability problem of a riser, developed a system of optimal control incorporating a linear quadratic regulator (LQR) for stabilizing the large-amplitude vibrations of a steel-catenary riser undergoing VIVs. In their results, only a single mode (i.e., the twenty-sixth one), which has been determined to be the most excited during the vortex-induced response of the riser, was evaluated, and the case of multi-mode lock-in was not considered. Fortaleza (2013) designed a P-type control law for controlling the displacement at the top boundary of a riser model to suppress the VIVs. For control design, the study also evaluated only one (the most excited) vibration mode, as determined via a modal analysis. Applying the control law experimentally to a reduced-scale riser model, a 30% reduction in VIVs was achieved.

Boundary control (Do, 2017a; Do and Pan, 2008) has been the most widely employed active control method for suppressing the vibrations of flexible marine risers. How et al. (2009) developed a boundary control law for limiting the upper joint angle of a tensioned riser (i.e., the joint connecting the riser and the surface vessel), where the recommended maximum value of the angle was considered to be 2°. A boundary control was applied to operate the torque actuator located at the top end of the riser, which not only restricted the upper joint angle of the riser but also suppressed the VIVs due to the unknown time-varying disturbance caused by ocean currents. Equations (4) and (11) were used to define the oscillating drag and transverse forces, respectively, in their mathematical model of the riser. Do and Pan (2008) developed a boundary control law for suppressing the transverse vibrations of a flexible marine riser (considered as a 2D system) as driven by a hydraulic system and subject to waves, winds, and ocean currents. Lyapunov direct method and the back-stepping technique were utilized in developing the control law to drive the hydraulic system at the top end of the riser. The dynamics of the hydraulic actuator also were included in the designed control law. Do and Pan (2009) later extended their control technique to stabilize a coupled 3D riser system (i.e., axial, lateral, and transverse deflections of the riser were considered) subject to inline and transverse drag force components and in consideration of the effective weight of the riser itself. Ge et al. (2010) developed a boundary control law for suppressing the vibrations of a flexible riser in the axial and lateral directions utilizing two actuators at the top end of the riser (i.e., for generating control force in the axial and lateral directions). Nguyen et al. (2013) designed a boundary control law for controlling the displacement and velocity of the top end of the riser to suppress its lateral and transverse vibrations, considered to be coupled due to bending. Their control law was implemented by obtaining a negative feedback of the top angle of the riser and the corresponding angular velocity. Later, in Nguyen et al. (2014), a boundary control scheme utilizing force and torque inputs at the top end of a flexible riser was developed to suppress the rotational movements of the riser under the effect of transverse and drag (inline) forces. Recently, He et al. (2016) designed a boundary control law to control the lateral vibrations of a flexible marine riser subject to ocean currents and in consideration of the saturation phenomenon of the actuator. The boundary control law was implemented at the top boundary of the riser, where two cases of feedback (i.e., full-state feedback and output feedback) were considered. In most works discussed above, the riser was considered inextensible. However, Do and Lucey (2017) developed a boundary control scheme to stabilize the lateral and axial deflections of the riser under the influence of ocean currents, considering the riser to be extensible. Their proposed boundary control laws were implemented at the top boundary of the riser and required a feedback of the deflections and velocities of the top end of the riser. Recently, a new boundary control scheme has been developed by Do (2017b) to stabilize the extensible marine riser in the 3D space (i.e., considering the deflections of the riser along $\hat{i}$, $\hat{j}$, and $\hat{k}$ axes).

4.2.1. Active suppression of vessel-induced vibrations

Most works discussed so far dealt with the suppression of the vibrations caused by the ocean's currents. However, the movements of the vessel (caused by the sea waves) also have a significant impact on the response of the riser (Cabrera-Miranda and Paik, 2017; Kuiper et al., 2008). Therefore, to suppress the vibrations of the riser caused by the movements of the vessel, several active control schemes have been proposed. Zhang and Li (2015) proposed a linear quadratic Gaussian controller for suppressing the axial stresses of a tensioned riser subject to wave-induced motions of the vessel. They utilized a strain feedback in implementing their control law for controlling the tension of the riser by using a winch. In Nguyen et al. (2010), the position of the surface vessel was controlled to maintain the bottom end angle of a rigid drilling riser within 2°. The proposed control scheme consisted of two parts: i) keeping the vessel in close proximity to a reference position, which was achieved by implementing an integral control of the lengths of mooring lines, and ii) implementing the proportional-integral-derivative control law for thruster-based heading control of the vessel. An experimental floating production storage and offloading vessel model was utilized in validating the proposed control scheme, which was found to be feasible only for moderate sea states.

He et al. (2015b) developed a boundary control law based on an integral-barrier Lyapunov function for controlling the top tension of the riser installed in a vessel-riser system. The control law was designed to control the movements of the surface vessel and to minimize the top end angle of the riser (in the case of very large vessel movements) to restrict the top tension within a prescribed limit. Later, He et al. (2011) developed an adaptive boundary control law for suppressing the transverse vibrations of a flexible riser so as to handle the uncertainties in the parameters of the model and the time variation of ocean currents. Vessel dynamics also were included in the control model, the control input being applied to control the position of the vessel for suppressing the vibrations of the riser. He et al. (2015a) also designed a boundary control law for suppressing the vibrations of both the mooring lines and the riser with the help of a force actuator installed in a vessel-mooring-riser system. Table 2 summarizes some of the important works published in the field of riser vibration control.

Table 2
A summary of important contributions in the field of riser control.

Area	Reference	Contribution
Passive control methods	Abbassian and Moros (1996)	Usage of air-bubble spoilers for suppressing the VIVs
	Baarholm et al. (2005)	Usage of helical strakes for suppressing the VIVs
	Trim et al. (2005)	An increase in the coverage of helical strakes increases the suppression of the VIVs caused by inline forces
	Quen et al. (2014)	A higher pitch of the strakes results in the delay of the occurrence of lock-in
	Gao et al. (2016)	The height of the strakes plays a more significant role in suppressing the VIVs than does the pitch of the strakes
	Wu et al. (2012a)	Usage of control rods for suppressing the VIVs
	Borges et al. (2014)	Usage of visco-elastic layers for suppressing the VIVs
Active control methods	Rustad et al. (2008)	Development of a PI control law for prevention of collision between two risers in tandem arrangement
	Mazzilli and Sanches (2011)	Development of a LQR control law for stabilization of large amplitude vibrations of a steel-catenary riser
	Fortaleza (2013)	Development of a P-type control law for control of riser displacement at the top boundary and consequent suppression of VIVs
	Zhang and Li (2015)	Development of an LQ Gaussian control law for suppressing the axial stresses of a riser subject to the vessel's movements
	Nguyen et al. (2010)	Development of integral and PID control laws for controlling the lengths of mooring lines and the vessel's heading
	Do and Pan (2008)	Development of a boundary control law for suppressing the VIVs of a flexible marine riser
	Do and Pan (2009)	Development of a boundary control law for suppressing the coupled axial-lateral-transverse vibrations of a flexible riser
	How et al. (2009)	Development of a boundary control law for limitation of the upper joint angle of a tensioned riser
	Ge et al. (2010)	Development of a boundary control law for suppressing the axial and lateral vibrations of a flexible marine riser
	He et al. (2011)	Development of an adaptive boundary control law for suppressing the VIVs of a flexible marine riser
	Nguyen et al. (2013)	Development of a boundary control law for suppressing the coupled lateral-transverse vibrations of a flexible marine riser
	Nguyen et al. (2014)	Development of a boundary control law for suppressing the riser's movements caused by the transverse and drag forces
	He et al. (2015a)	Development of a boundary control law for suppressing the vibrations of both the riser and mooring lines
	He et al. (2015b)	Development of a boundary control law for control of the top tension of a riser
	He et al. (2016)	Development of a boundary control law for suppressing lateral vibrations of the riser, considering the actuator's saturation

5. Discussion and future prospects

In this paper, an extensive review of the literature related to the mathematical modeling, analysis, and vibration control of marine risers was presented. In Section 2, the discussions on different formulations of a riser system included the modeling of different hydrodynamic load functions, the lateral and transverse responses of the riser, the effects of internal fluid on the responses of the riser, damping functions, and the coupling between the lateral and transverse vibrations of the riser. Section 3 introduced numerous reports of experimental and theoretical analyses on riser systems. Most of those investigations were mainly concerned with fatigue damage in risers. The effects of sea currents (uniform and sheared), internal flow and the movements of the vessel on the response of the riser (especially vortex-induced vibrations) were discussed. Finally, in Section 4, the control strategies (both active and passive) applied for suppressing the vibrations of the riser were discussed. Based upon this review, the following seven aspects of key technologies for improvement have been identified in the areas of modeling, analysis, and control of a riser system.

i) Timoshenko beam theory: First, in the modeling part, most formulations so far have modeled the riser system as an Euler-Bernoulli beam. But such simplification cannot describe the response of the riser upon shear forces. Also, most works assumed the cross section of the riser to be constant. However, in a harsh environment in the ocean, the riser can exhibit a complex deformation including twisting and elongation (resulting in a change in the shape/cross-section of the riser). Therefore, in the future, a more complete beam formulation, for example, based on the Timoshenko beam theory, can be developed.

ii) Varying parameters: Also, most existing works regarded that the parameters associated with the hydrodynamic forces (for example, the damping and added-mass coefficients) are constant along the lateral, transverse, and axial directions. However, such parameters should be modeled as time varying entities.

iii) Six DOF motion of the vessel: Another important aspect in the formulation of a riser system, which needs to be improved, is the modeling of the movements of the surface vessel. Most formulations in the literature considered only the pitch, yaw, and heave motions of the vessel. However, considering the real environment, a complete motion of the vessel should be considered (i.e., the six degrees of freedom, as was done for a mobile harbor system in Hong and Ngo (2012)). A recent investigation by Wang et al. (2017) has revealed that the movements of the vessel also result in the VIVs of the risers, which phenomenon should also be addressed in the future formulations of the riser-vessel system.

iv) Numerical scheme for real-time estimation: For the analysis of a riser system in the future, more emphasis should be given to the development of a numerical scheme that can estimate the parameters associated with the hydrodynamic forces in real time. This will eventually lead to the development of a more efficient control scheme.

v) Experimental facility: Also, there is a need to develop more advanced experimental facilities for investigating the responses of the riser under complex flow situations. It is evident that the existing experimental setups possess a limited capability in simulating realistic hydrodynamic load conditions for the riser.

vi) Hybrid control: Regarding the control of vortex-induced vibrations, this review found that both passive and active control methods are being applied to riser systems. However, there are shortcomings in both methods: For example, the passive techniques, which require mounting of vibration suppression devices on the riser, are expensive. Whereas the active control methods, which tend to control the vibrations of the riser by applying control inputs at the top end of the riser, are not much effective in suppressing the vibrations at larger depths (i.e., the compensating movements of the riser cannot occur at larger depths due to the limited controlled movements of the vessel). To overcome these problems, more emphasis should be given to the development of hybrid active-passive control techniques, in which case, the number of passive devices mounted on the riser can be reduced (for cost effectiveness) and, rather than applying control inputs only at the top boundary, the sensor/actuator pairs can be mounted at several points along the length of the riser, which may require application of the wireless technology for data/signal transmission for implementation of a control scheme.

vii) Failure prevention at the connections of the riser: Lastly, considering the effects of the movements of the vessel on the riser, this review admits that the movements of the vessel should be restricted to avoid an accidental disconnection between the riser and the BOP valve (Nguyen et al., 2010). However, not much work has been reported to constrain the movements of the vessel to prevent possible damages at the riser-vessel interface. Recently, in Adamiec-Wojcik et al. (2016), an optimization technique was proposed to restrict the bending moments at the riser-vessel interface by utilizing the vertical displacements of the top end of the riser (by using a winch). In their study, only the heave motion of the vessel was considered in generating the bending moments at the top end of the riser. Therefore, in the future, more emphases should be given in developing control techniques, for example artificial neural networks, for avoiding a failure at the riser-vessel interface due to six DOF movements of the vessel.

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