Road-frequency based optimisation of damping coefficients for semi-active suspension systems

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Abstract: In this paper, a sequential quadratic programming method for determining the optimal damping coefficients of a semi-active suspension system is investigated. Two objective functions (i.e., mean squares of the sprung-mass absolute acceleration and the dynamic load) are minimised under four constraints. By splitting the road frequency range into four regions, the optimal damping coefficients in individual regions are obtained. Simulation results of three cases (passive, semi-active with optimal static damping coefficients, and semi-active with optimal dynamic damping coefficients) show that the semi-active suspension system significantly improve the ride comfort, road holding, and reduce the noise and harshness.

Keywords: suspension control; semi-active damping; road frequency; ride comfort; road holding; optimisation; modified skyhook control.

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1 Introduction

Vehicle suspension systems function mainly to isolate passengers from road disturbances to provide ride comfort, to maintain the traction force between the tyre and the road surface (road holding), and to keep the suspension deflection (rattle space) small. With a fixed suspension-spring constant, the sprung-mass can be better isolated from road disturbances by soft damping, which allows larger suspension deflections. However, better road contact is achieved using hard damping, which prevents unnecessary suspension deflections. These requirements are mutually conflicting in the control system design of suspension systems. To meet both these requirements, many types of suspension systems, ranging from passive (Lu et al., 1984; Tamboli and Joshi, 1999; Gobbi and Mastinu, 2001), semi-active (Choi and Sung, 2008; Besinger et al., 1995; Hong et al., 2002; Shen et al., 2006; Akutain et al., 2007; Liu et al., 2008; Valasek et al., 1998; Hong and Bentsman, 1994; Hong, (1997); Retting and Stryk, 2005; Bourmistrova et al., 2005; Wang and Hu, 2005; Choi and Kim, 2000; Vassal et al., 2008; Nguyen et al., 2010; Turnip et al., 2010), to active suspensions (Michelberger et al., 1993; Sohn et al., 2004; Hrovat, 1993; Zaremba, 1997; Hada et al., 2007; Turnip et al., 2009; Kamada et al., 2010), are currently being employed and studied. Among the three types of suspension control, the semi-active suspension has recently received much attention, since it provides the best compromise between cost and performance. This paper focuses on the (offline) optimisation of damping coefficients with respect to the frequency ranges of various roads. The proposed optimisation is carried out in association with the scheme of a modified skyhook control, which provides a solution to the trade-off between ride comfort and road holding capacity.

Research in the area of semi-active suspensions has been conducted since the early seventies. The most common type of semi-active control policy is based on the skyhook control, in which a fictitious damper is inserted between the sprung-mass and the stationary sky to suppress the vibratory motion of the sprung-mass and to act as a tool for computation of the desired damping force. Various kinds of innovative control strategies have been proposed to implement this type of semi-active control. The original skyhook control can reduce the resonant peak of the sprung-mass quite significantly to yield good ride quality (Liu et al., 2008; Hada et al., 2007). But to improve both the ride quality and the handling performance of a vehicle, both resonant peaks of the sprung and the unsprung masses need to be reduced (Kim et al., 2003; Lu and Zeng, 2008; Valentini, 2009; Balike et al., 2010; Subic et al., 2010). It is known, however, that the skyhook damper alone cannot reduce both resonant peaks at the same time. In this respect, Besinger et al. (1995) proposed a modification of the skyhook control, which includes a variable damper as well as a skyhook damper, for the computation of the desired control inputs. Hong et al. (2002) investigated a procedure for determining the optimal values of the skyhook damper and the variable damper for all frequencies of various ISO road classes. Shen et al. (2006) compared the performance of the three control methods; namely the limited relative displacement method, the modified skyhook method, and the modified Rakheja–Sankar method by using experimental results obtained from a test bed. The hybrid skyhook and the extended groundhook were investigated and compared by Akutain et al. (2007), who also quantified the performance and the limitations of the suspensions. Recently, Liu et al. (2008) used the skyhook model together with a sliding mode control to improve the frequency response of the vehicle in the low frequency region.

The basic ideas of the optimisation problem in this paper are as follows. Road roughness becomes a source of vibrations and effects on the Dynamic Loads (DLs) of the individual wheels. Since the profile of a road is random in nature, so are the vibrations and DLs. The ride comfort and road holding of a vehicle can be described in Root Mean Squares (RMS) values of the vertical accelerations of the sprung and the unsprung masses, respectively, and the DLs (Lu et al., 1984; Akutain et al., 2007; Hada et al., 2007), in which the accelerations of the sprung and unsprung masses must be minimised. Also, the suspension must operate without hitting the bumper stops (i.e. oscillates within the rattle-space). The DLs between the tyres and the road surface must be kept low for good directional control during the cornering of a vehicle at high speed. Therefore, the semi-active suspension needs to be able to provide damping coefficients that will meet such constraints.

Previous investigations essentially treated the suspension design as an optimal control problem over the all-frequency range, often with feedback capabilities. The obtained single fixed gain(s) throughout the all-frequency range can reduce the two resonance peaks and may be optimal over the all-frequency range, but may not be optimal for a specific sub-range, for instance, 4–8 Hz, which is a sensitive range to the human body. Therefore, the all-frequency range is split into a number of sub-regions (in this paper, four regions are used), and the optimal gains in the four individual regions are obtained. Using a multi-objective parameter optimisation approach, Valasek et al. (1998) investigated an extended groundhook control logic to reduce the dynamic tyre forces. Bourmistrova et al. (2005) applied an evolutionary algorithm to determine the control parameters for a quarter-car model, while optimising a multi-objective fitness function, which is the weighted sum of the rate-of-change of acceleration of the sprung-mass and the suspension deflection.

Compared with the various optimisation techniques in the literatures, the optimisation technique proposed in this paper addresses issues related to the frequency ranges and the types of roads. The dynamic responses of the vehicle model, which are functions of damping coefficients, excitation frequency, and road roughness, are derived. Sprung-mass acceleration and DLs are parameters used to evaluate the performances of ride comfort, road holding, and noise and harshness. This study aims to find the optimal damping coefficients that give the minimum values of the sprung-mass accelerations and the DLs under the constraints of maximum and minimum damping forces, rattle space, and tyre deflection. Using a sequential quadratic programming method based on the derivation of simultaneous nonlinear equations, the optimal damping coefficients with respect to the excitation frequency ranges (i.e., body motion 0 – 4 Hz, ride comfort 4 – 8 Hz, road holding 8 – 12 Hz, and noise and harshness 12 Hz and higher) and the various types of roads (i.e., asphalt, concrete, and rough) are optimised.

The contributions of this paper are as follows. First, the generation of the damping force as a function of damping coefficients, road excitation frequencies, and road types is proposed. A modified skyhook control, with two independent variables to be specified by designers, is developed and tuned to precisely yield the desired dynamic response of the vehicle. Second, the optimal damping coefficients are obtained, and the adaptation procedure based on the frequency ranges and the type of road is developed.

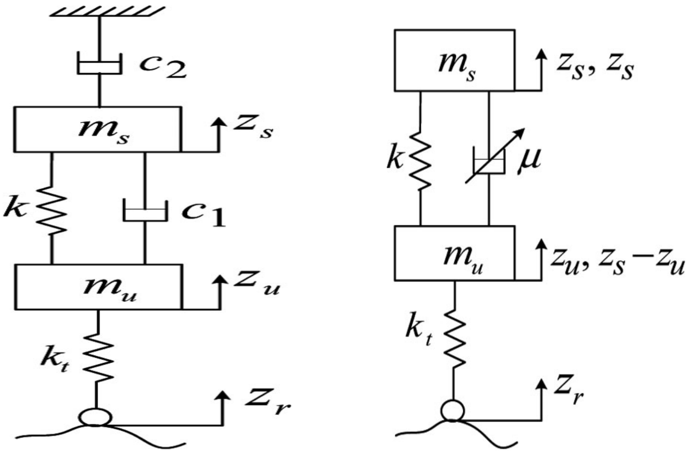
The structure of this paper is as follows. The proposed semi-active control law consisting of two tunable parameters is developed in Section 2. Vehicle transfer functions and the road profile characterisations are derived in Section 3. Formulation of the optimisation problem subject to four constraints is explored in Section 4. Simulations and discussions of the optimisation results based on the proposed control strategy are discussed in Section 5. Conclusions are given in Section 6.

2 Control strategy: modified skyhook control

In semi-active suspensions, the conventional suspension-spring is kept, but the damper with a fixed orifice is replaced with a controllable one. The controllable damper offers a wide range of damping forces because it can change its damping characteristics either by changing the size of an orifice (i.e., a continuously variable orifice damper) or by adjusting the viscosity of the fluid passing through an orifice (i.e., a Magneto-Rheological (MR) damper). In this paper, an MR damper is used. This class of dampers provides fast, smooth and continuously varying damping forces with low power consumption. An MR damper consists of a hydraulic cylinder containing a fluid, which is composed of micron-sized magnetically polarisable particles dispersed in a carrier medium such as water or synthetic oil. The presence of a magnetic field causes the fluid particles to align and oppose themselves to the fluid movement, and such behaviour, therefore, results in the increase of the damping coefficient. The detail of MR damper can be found in Koo et al. (2006) and Ahn et al. (2009). Furthermore, whereas an active suspension system requires an external power source to energise an actuator that controls the vehicle, a semi-active system uses external power only in adjusting the damping levels and in operating the embedded controller and a set of sensors. The controller determines the level of damping based on the control strategy and then automatically adjusts the damper to achieve that damping level.

The most common type of semi-active control policy is the modified skyhook control shown in Figure 1(a), which includes a skyhook damper with coefficient c_2 connected to the inertial frame and a variable damper with coefficient c_1. Figure 1(b), which depicts an equivalent model of Figure 1(a), shows a variable coefficient μ. In Figure 1, z_s, z_u, and z_r are the vertical displacements of the sprung-mass, the unsprung-mass, and the road profile, respectively; $\dot{z}_s$ and $\dot{z}_s - \dot{z}_u$ are the absolute and the relative velocities, respectively. m_s is the quarter-car body mass (sprung-mass); m_u is the unsprung-mass that represents the mass of the wheel, tyre, brake, and suspension linkages. k and k_t are the spring coefficients of the suspension-spring and the tyre, respectively. Table 1 shows the system parameters used in this paper based on a typical passenger car.

Figure 1 Quarter-car models: (a) the modified skyhook control considered; and (b) an equivalent model of the modified skyhook control



(a) (b)

Table 1 System parameters

Parameter	*Value*
Sprung mass (m_s)	412.5 kg
Unsprung mass (m_u)	51 kg
Suspension stiffness (k)	43.3 kN/m
Tire stiffness (k_t)	210 kN/m
Rattle space ($z_s - z_u$)	91 mm

For suspension systems, two mean square indices have to be taken into account. The first is related to the comfort of the passengers, and this mean square index depends on the acceleration of the sprung-mass; the second is related to the road holding, and this mean square index depends on the DLs or vertical fluctuations of the wheels. A compromise between these two indices has to be found using only one coefficient, which is μ, as shown in Figure 1(b). The passive suspension system does not offer a solution to this trade-off, since the two conflicting conditions are independent of each other.

On a conceptual basis, this trade-off problem can be solved by increasing the number of parameters available, that is, by making the damping force depend on more than one parameter. This is possible if the skyhook damping strategy is adopted, because it imposes not only a force proportional to the relative velocity between the sprung and the unsprung masses, but also a force proportional to the absolute velocity of the sprung-mass itself. A direct measurement of the relative and the absolute velocities is usually not possible. Therefore, a displacement sensor is used, and the velocities are estimated using the measured relative displacement data after suitable filtering.

The equation of motion of the sprung-mass is given as follows.

$$m_s\ddot{z}_s + c_1(\dot{z}_s - \dot{z}_u) + c_2\dot{z}_s + k(z_s - z_u) = 0 \text{ (Figure 1(a))}, \tag{1}$$

$$m_s\ddot{z}_s + \mu(\dot{z}_s - \dot{z}_u) + k(z_s - z_u) = 0 \text{ (Figure 1(b))}. \tag{2}$$

where $\mu = \mu(c_1, c_2)$ is the design parameter. The equation of motion of the unsprung-mass (after using either (1) or (2)) is given by

$$m_u\ddot{z}_u + m_s\ddot{z}_s + k_t(z_u - z_r) = 0\,. \tag{3}$$

In comparing Figure 1(a) and (b), the damping force must be provided as

$$\mu = c_1 + c_2 \frac{\dot{z}_s}{(\dot{z}_s - \dot{z}_u)}, \tag{4}$$

where c_1 and c_2 are chosen in consideration of ride comfort, road holding and road conditions. Based on the idea of making the damping force depend on more than one parameter, the following control law is proposed.

$$\begin{aligned} u &= c_1(a,\ f)(\dot{z}_s - \dot{z}_u) + c_2(a,\ f)(\dot{z}_s) \\ &= \mu(\dot{z}_s - \dot{z}_u), \end{aligned} \tag{5}$$

where u is the control input of the system, a is the road roughness coefficient, and f is the excitation frequencies of the road. The detailed derivation of c_1 and c_2 as a function of a and f will be discussed in Section 3.2. It is noted that c_1 and c_2 are the parameters to be optimised.

The control strategy for semi-active suspension systems is therefore similar to that for active suspensions, except for the constraints given by the following equations.

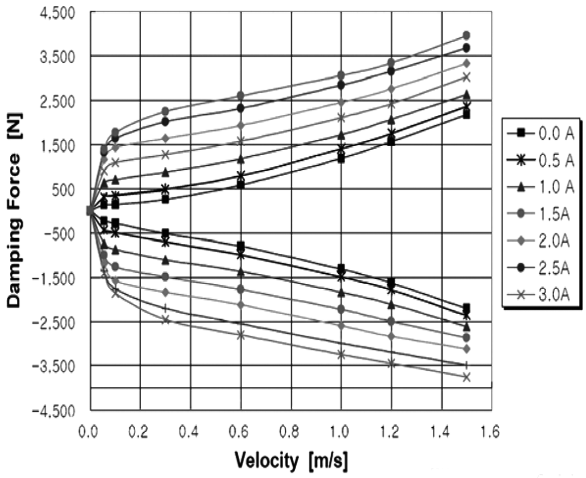
$$u = \begin{cases} c_1(a,f)(\dot{z}_s - \dot{z}_u) + c_2(a,f)(\dot{z}_s), & \text{if } \dfrac{\dot{z}_s}{\dot{z}_s - \dot{z}_u} > 0 \\ c_{\min}(\dot{z}_s - \dot{z}_u), & \text{if } \dfrac{\dot{z}_s}{\dot{z}_s - \dot{z}_u} < 0 \end{cases}. \tag{6}$$

It is worth emphasising that when the absolute and the relative velocities are in the same direction, then the semi-active control input is proportional to both velocities; otherwise, the semi-active control input must provide the minimum value that it can afford. Even though the control input is calculated by equations (6), the actual control input should be generated based on the variable damper itself. From experimental results, as shown in Figure 2, the actual control input is limited as follows.

$$F_d = \begin{cases} F_d^{\max}, & \text{if } F_d^{\max} \le u \\ u, & \text{if } F_d^{\min} < u < F_d^{\max} \\ F_d^{\min}, & \text{if } F_d^{\min} \ge u \end{cases} \tag{7}$$

where $F_d^{\max}$ and $F_d^{\min}$ denote the maximum and the minimum actual damping force available at a given relative velocity.

Figure 2 Damping force characteristics of a typical MR damper



3 System responses and road characterisation

3.1 Transfer functions

Taking the Laplace transforms of equations (2) and (3), the transfer functions with respect to the road input are obtained as follows.

$$H_l(s) = \frac{\text{Output}_l(s)}{z_r(s)} = \frac{\text{num}_l}{\text{den}}, \quad l = 1, 2, \ldots, 6, \tag{8}$$

where the subscript l represents the output variable of interest: $l = 1$ refers to the vertical sprung-mass acceleration, $\ddot{z}_s$; $l = 2$ indicates the relative velocity between the sprung and the unsprung masses, $\dot{z}_s - \dot{z}_u$; $l = 3$ means the sprung-mass velocity, $\dot{z}_s$; $l = 4$ denotes the rattle space deflection, $z_s - z_u$; $l = 5$ indicates the tyre deflection between the unsprung-mass and the road input, $z_u - z_r$; and $l = 6$ denotes the DLs between the tyre and the road surface. The individual numerator polynomials (num$_l$) and the denominator polynomial (den) in equations (8) are given by

$$\text{num}_1 = \frac{c_1 k_t}{m_s m_u} s^3 + \frac{k k_t}{m_s m_u} s^2,$$

$$\text{num}_2 = -\frac{k_t}{m_u} s^3 - \frac{c_2 k_t}{m_s m_u} s^2,$$

$$\text{num}_3 = \frac{c_1 k_t}{m_s m_u} s^2 + \frac{k k_t}{m_s m_u} s,$$

$$\text{num}_4 = -\frac{k_t}{m_u} s^2 - \frac{c_2 k_t}{m_s m_u} s,$$

$$\text{num}_5 = -s^4 - \left(\frac{c_1}{m_u} + \frac{c_1 + c_2}{m_s}\right) s^3 - \frac{k\ (m_s + m_u)}{m_s m_u} s^2$$

$$\mathrm{num}_6 = \frac{k_t}{(m_s + m_u)g} s^4 + \frac{k_t}{m_s m_u g}\left(c_1 + \frac{m_u}{m_s + m_u} c_2 \right) s^3 + \frac{k k_t}{m_s m_u g} s^2,$$

$$\mathrm{den} = s^4 + \left(\frac{c_1}{m_u} + \frac{c_1 + c_2}{m_s} \right) s^3 + \left(\frac{k}{m_s} + \frac{k}{m_u} + \frac{k_t}{m_u} \right) s^2 + \frac{k_t}{m_s m_u}(c_1 + c_2)s + \frac{k k_t}{m_s m_u}. \quad (9)$$

3.2 Road profile and variances of the responses

A large amount of statistical measurement data on road roughness has been acquired (Lu et al., 1984; Gobbi and Mastinu, 2001; Hong et al., 2002; Michelberger et al., 1993; Hada et al., 2007; Gillespie, 2003; Brown, 1997; Sahin and Unlusoy, 2010). Investigations indicate that certain regularity exists behind the random behaviour of a road. The distribution of road roughness is essentially similar to a normal (Gaussian) distribution, and the frequency constitutions of various road surfaces are very similar among roads. The power spectral density of a road surface can be expressed by the following formula (Gobbi and Mastinu, 2001; Michelberger et al., 1993):

$$s_{z_r}(\omega) = \frac{2av\sigma_{z_r}^2}{(av)^2 + \omega^2}, \quad (10)$$

where a is the road roughness coefficient, v is the vehicle speed, σ_z is the variance of the road profile, and ω is the angular frequency of the road. Both a and σ_{z_r} depend on the shape of the road irregularity spectrum (see Table 2 in Michelberger et al., 1993).

Table 2 Coefficients of the first order model of typical roads, equations (10)

Type of road	a [*1/m*]	σ_{z_r} [*m*]
Asphalt	0.15	0.0033
Concrete	0.20	0.0056
Rough	0.40	0.0120

Source Michelberger et al. (1993)

A vehicle system characterised by transfer functions, which relate the input representing the surface irregularities of the road and the outputs representing the vibrations of some parts of the vehicle, is subject to road disturbances. Because of the random properties of road disturbances, the vibrations of the vehicle will be random as well. It has been proven for linear systems that when the input is a Gaussian random process, then the output of the system must be a Gaussian random process as well. By the definition in Brown (1997), the variance of a random variable described by a stationary and ergodic stochastic process is

$$\sigma_l^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_l(\omega)\mathrm{d}\omega. \quad (11)$$

The Power Spectral Density (PSD) $S_l(\omega)$ of the six outputs discussed in Section 3.1 of an asymptotically stable system can be computed as (Tamboli and Joshi, 1999; Gobbi and Mastinu, 2001; Zhang et al., 2002; Hada et al., 2007)

$$S_l(\omega) = \left|H_l(j\omega)^2\right| S_{z_r}(j\omega), \quad l = 1, \ldots, 6. \tag{12}$$

It is shown in Brown (1997) that an analytical solution exists for σ_l^2 if equation (12) can be written in the form

$$S_l(\omega) = \frac{P_l(j\omega)P_l(-j\omega)}{Q(j\omega)Q(-j\omega)} \tag{13}$$

where Q is a polynomial of degree n, and P_l is a polynomial of degree $n-1\,(n \geq 1)$. By equations (8)–(10), one can conclude that equation (12) can be spelled out as

$$\left|H_l(j\omega)\right|^2 = H_l(j\omega) \quad H_l(-j\omega) \tag{14}$$

and

$$S_{z_r}(j\omega) = 2av\sigma_{z_r}^2 \frac{1}{(av + j\omega)} \frac{1}{(av - j\omega)}. \tag{15}$$

By letting $s = j\omega$ and multiplying equation (14) and (15) in the form of equation (13), we obtain that the degree n becomes 5. Therefore, $Q(s)$ and $P_l(s)$ can be expressed, respectively, as

$$Q(s) = q_5 s^5 + q_4 s^4 + q_3 s^3 + q_2 s^2 + q_1 s + q_0, \tag{16}$$

where

$$q_5 = 1,$$

$$q_4 = av + \frac{c_1}{m_u} + \frac{c_1 + c_2}{m_s},$$

$$q_3 = \frac{k}{m_s} + \frac{k}{m_u} + \frac{k_t}{m_u} + av\left(\frac{c_1}{m_u} + \frac{c_1 + c_2}{m_s}\right),$$

$$q_2 = \frac{k_t}{m_s m_u}(c_1 + c_2) + av \quad \left(\frac{k}{m_s} + \frac{k}{m_u} + \frac{k_t}{m_u}\right),$$

$$q_1 = \frac{kk_t}{m_s m_u} + av \quad \frac{k_t}{m_s m_u}(c_1 + c_2),$$

$$q_0 = av\frac{kk_t}{m_s m_u},$$

and

$$P_l(s) = p_{4,l}s^4 + p_{3,l}s^3 + p_{2,l}s^2 + p_{1,l}s + p_{0,l} \tag{17}$$

for $l = 1, 2, \ldots, 6,$ and the coefficients in equation (17) (i.e., $p_{4,l}$, $p_{3,l}$, $p_{2,l}$, $p_{1,l}$, and $p_{0,l}$) are tabulated in Table 3, which means that the lth row in Table 3 will be picked when the lth output is considered. Finally, using the results of integration in equation (11) (Gobbi and Mastinu, 2001; Brown, 1997), the variances of the six outputs are expressed as follows.

$$\sigma_l^2 \equiv \sigma_l^2(c_1, c_2, a, v) = 2av\sigma_{z_r}^2 \frac{\sigma\_\mathrm{num}_l}{\sigma\_\mathrm{den}}, \quad l = 1, 2, \ldots, 6 \tag{18}$$

where

$$\begin{aligned} \sigma\_\mathrm{num}_l = {} & p_{4,l}^2(q_0q_1q_2q_3 - q_0q_1^2q_4 + q_0^2q_3^2 + q_0^2q_1q_5) \\ & + (p_{3,l}^2 - 2p_{2,l}p_{4,l})(q_0q_1q_2q_5 - q_0^2q_3q_5) \\ & + (p_{2,l}^2 + 2p_{0,l}p_{4,l} - 2p_{1,l}p_{3,l})(q_0q_1q_4q_5 - q_0^2q_5^2) \\ & + (p_{1,l}^2 - 2p_{0,l}p_{2,l})(q_0q_3q_4q_5 - q_0q_2q_5^2), \end{aligned} \tag{19}$$

$$\begin{aligned} \sigma\_\mathrm{den} = {} & 2q_0\{q_1q_2^2q_5^2 - q_0q_1q_4q_5^2 - q_0q_2q_3q_5^2 + q_0^2q_5^3 \\ & - q_1q_2q_3q_4q_5 + q_1^2q_4^2q_5 + q_0q_3^2q_4q_5 - q_0q_1q_4q_5^2\}. \end{aligned} \tag{20}$$

From equation (8), it is evident that the dynamic responses of the vehicle model are functions of the suspension system parameters and the road roughness. For a given vehicle speed, equation (18) can be converted to a frequency domain of road characterisation of a road by utilising the relationship between the wave number Ω and the frequency f, that is, $f = v\Omega$. The wave number Ω is estimated from the measured relative displacement signal.

Table 3 Coefficients of $P_l(s)$ in (17) (where l indicates the lth output)

l	$P_{0,l}$	$P_{1,l}$	$P_{2,l}$	$P_{3,l}$	$P_{4,l}$
1	0	0	$\frac{k\ \ k_t}{m_s m_u}$	$\frac{c_1 k_t}{m_s m_u}$	0
2	0	0	$\frac{-k_t c_2}{m_s m_u}$	$\frac{-k_t}{m_u}$	0
3	0	$\frac{k\ \ k_t}{m_s m_u}$	$\frac{c_1 k_t}{m_s m_u}$	0	0
4	0	$\frac{-k_t c_2}{m_s m_u}$	$\frac{-k_t}{m_u}$	0	0
5	0	0	$\frac{-k\ \ (m_s + m_u)}{m_s m_u}$	$\frac{-c_1}{m_u} - \frac{(c_1 + c_2)}{m_s}$	-1
6	0	0	$\frac{k\ \ k_t}{m_s m_u g}$	$\frac{k_t(m_s c_1 + m_u(c_1 + c_2))}{(m_s + m_u)m_s m_u g}$	$\frac{k_t}{(m_s + m_u)g}$

4 Optimisation problem formulation

4.1 Objective functions

This section describes the development of mathematical models, in the form of a Non-Linear Programming (NLP) problem, to be used for the optimisation process. The NLP model itself is based upon the model and analysis utilised to describe the dynamic behaviour of a vehicle. Ride performance can be described by various criteria, as mentioned in the introduction. Here, we use the mean square value of the vertical acceleration $\sigma_1^2 = \ddot{z}_s^2$ and the DL $\sigma_6^2 = DL^2$ as the primary design criteria to be used in the objective functions of the NLP formulation. The objective function DL^2 will be applied for frequency range 8 – 12 Hz, whereas the objective function $\ddot{z}_s^2$ will be applied for the other frequency ranges. The calculations of $\ddot{z}_s^2$ and DL^2 are based on the evaluation of the transfer functions of $\ddot{z}_s(s)/z_r(s)$ and $DL(s)/z_r(s)$ from equation (8) with $l = 1$ and $l = 6$, respectively, and the Laplace transform of the power spectral density of the road surface profile in equation (10). Thus we arrive at the following expressions.

$$\overline{\ddot{z}_s^2} = 2av\sigma_{z_r}^2 \frac{\sigma\_\text{num}_l}{\sigma\_\text{den}}, \quad \text{for } l = 1 \tag{21}$$

$$\overline{DL^2} = 2av\sigma_{z_r}^2 \frac{\sigma\_\text{num}_l}{\sigma\_\text{den}}, \quad \text{for } l = 6 \tag{22}$$

where σ_num_l and σ_den are the numerators and the denominator in equations (19) and (20), respectively. Hence equations (21) and (22) give the definition of the objective functions to be minimised, with c_1 and c_2 being the design variables and $a, v, \sigma_z, m_s, m_u, k,$ and k_t being the design parameters, respectively.

4.2 Constraints

The NLP model must be completed by including the design constraints, expressed as equalities or inequalities. In deriving the constraints, one should generally include some empirical information about the problem, so that the optimisation procedure can result in reasonable designs. Here, the first and second constraints considered deal with the maximum and the minimum damping forces, respectively, as shown in Figure 2. Substituting these values into equation (7), the constraints of the control force can be expressed as follows.

$$c_1 RMS(\dot{z}_s - \dot{z}_u) + c_2 RMS(\dot{z}_s) \le F_d^{\max}, \tag{23}$$

$$F_d^{\min} - c_1 RMS(\dot{z}_s - \dot{z}_u) + c_2 RMS(\dot{z}_s) \le 0, \tag{24}$$

where $\text{RMS}(\dot{z}_s - \dot{z}_u)$ and $\text{RMS}(\dot{z}_s)$ are calculated from equation (18) with $l = 2,$ and $l = 3,$ respectively. These constraints will be used for all frequency ranges. The third constraint deals with the need to limit the rattle space in accordance with the actual design. The maximum allowable suspension stroke has to be taken into consideration to prevent the bumper from hitting the car body, which can possibly result in deterioration of ride comfort and even structural damage. To minimise the chances of bumper hitting, the rattle space constraint can be expressed as

$$\mathrm{RMS}(z_s - z_u) \le (z_s - z_u)_{\max}, \tag{25}$$

where $\mathrm{RMS}(z_s - z_u)$ is calculated from equation (18) with $l = 4$, and $(z_s - z_u)_{\max}$ is the maximum rattle space, as shown in Table 1. The fourth constraint deals with the need to limit the tyre deflection (wheel-hop) so that the tyre will keep in contact with the road surface. To ensure firm uninterrupted contact of the wheels to the road, the DLs should not exceed the static ones (Gao et al., 2006), that is,

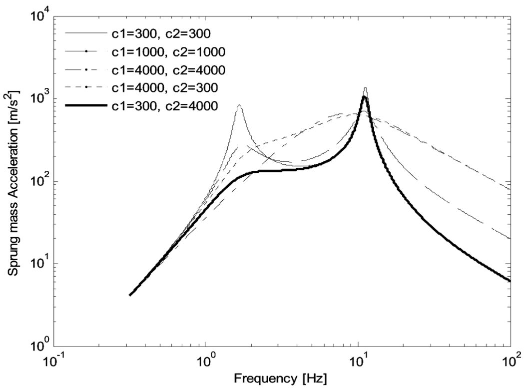
$$\mathrm{RMS}(z_u - z_r) \le 9.81(m_s + m_u)/k_t, \tag{26}$$

where $\mathrm{RMS}(z_u - z_r)$ is calculated from (18) with $l = 5$. The third and fourth constraints will be applied for frequency ranges 4 – 8 Hz, 8 – 12 Hz, and 12 Hz and higher. In other words, the strategy in designing control law for suspension system is to minimise the mean square value of the sprung-mass acceleration and the DLs while satisfying the other four requirements.

5 Simulations and discussions

Revisiting the proposed control law in equation (5), the variables c_1 and c_2 define the control gains that are used in computing u in proportion to $\dot{z}_s - \dot{z}_u$ and $\dot{z}_s$. Figure 3 shows that, as the damping coefficients c_1 and c_2 increase, the peaks of the accelerations of the sprung and the unsprung masses at their natural frequencies decrease, while the acceleration levels in the frequency range 4–8 Hz and in the frequency range 12 Hz and higher increase. Decreasing c_1 while increasing c_2 results in an increase of the amplitude of the unsprung-mass natural frequency, with gain in the reduction of the amplitude of other frequency ranges. Another notable observation is that the transmissibility to the sprung-mass acceleration with constant c_1 in the higher frequency range of over 12 Hz is constant for any value of c_2. The dynamic response of the system for different values of the damping coefficients is clearly a weighted combination of the damping coefficients of the modified skyhook control.

Figure 3 Comparison of the magnitude plots of the frequency responses of $\ddot{z}_s(s)/z_r(s)$ for various damping coefficients



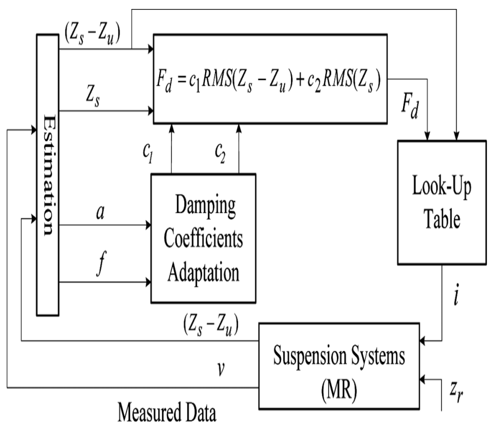
The optimal damping coefficients for the individual frequency ranges and road types are tabulated in Table 4. For a rough road with an excitation frequency greater than 4 Hz, the optimal damping coefficients do not satisfy the third and fourth requirements of the constraint. This means that for an excitation frequency greater than 4 Hz, the damper will hit the damper stopper and for an excitation frequency greater than 8 Hz, the tyre will leave the road surface. This information can be used in other control systems, such as ABS or ECS, to reduce the speed of the vehicle.

Table 4 Optimal damping coefficients

f	*0 – 4 Hz*		*4 – 8 Hz*		*8 – 12 Hz*		*12 Hz and higher*	
Type of road	c_1	c_2	c_1	c_2	c_1	c_2	c_1	c_2
Asphalt	3544.7	4000	1544.7	4000	4000	1180.9	873.95	4000
Concrete	3573.9	4000	1689.8	3782.7	3913.2	3313.8	1558.6	3891
Rough	3596.1	4000	1506.2	1819	2356.8	0	317.65	2561

Figure 4 shows the block diagram of the proposed control strategy, where *i* denotes the current input. Using measured data such as the vehicle speed *v* and the relative displacement $z_s - z_u$, the relative velocity $\dot{z}_s - \dot{z}_u$, the absolute velocity $\dot{z}_s$, the road roughness coefficient *a*, and the road excitation frequency *f* are estimated. Based on the values of *a* and *f*, the optimal damping coefficients are assigned. Using equation (7), the optimal damping force is calculated. The damping force generated from the MR damper depends on the current input to the solenoid valve and the relative velocity in the rattle space. To determine the current input to the solenoid valve, it is necessary to know the damping force characteristics of the valve vs. the current input at a given relative velocity. In this paper, the damping force characteristics for various current inputs for a specific relative velocity were measured in a test rig, see Figure 2. The experimental data are tabulated as a look-up table. Using the Pulse Width Modulation (PWM) generator, a pulse signal is generated based on the duty ratio of control input u with respect to the maximum and the minimum damping force. Finally, the PWM signal is converted into a current value through a damper driver.

Figure 4 Block diagram of the proposed modified skyhook control



The magnitudes of the frequency responses of the sprung-mass acceleration and the DLs with respect to the road disturbance (i.e., an asphalt road and the excitation frequency with and without frequency ranges) are shown in Figure 5 and 6, respectively. The solid line, dashed line, and the dotted line represent the magnitude plots of the frequency responses of the passive, the semi-active with static optimal values of c_1 and c_2 for all frequencies, and the semi-active with dynamic optimal values of c_1 and c_2 for each frequency range, as shown in Table 4, respectively. In Figure 5, the dotted line depicts a large improvement according to the other optimal values. However, the dashed line shows a deterioration of ride comfort in the frequency range 4–8 Hz and the deterioration of noise and harshness in the frequency range of 12 Hz and higher, when compared with the passive suspension. In Figure 6, the damping force could be designed to provide significant improvements of road holding (near 10 Hz) without degrading the ride quality performance (near 1 Hz). Comparison of the frequency responses in Figure 5 and 6 shows that the improved responses were achieved while the wheel-hop vibrations were being suppressed.

Figure 5 Comparison of three frequency responses: $\ddot{z}_s(s)/z_r(s)$

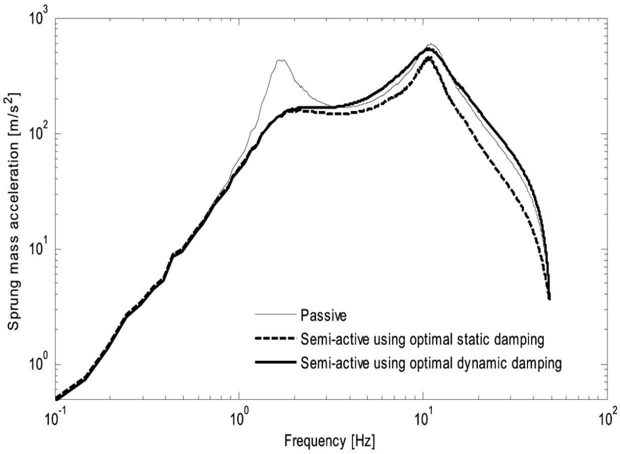
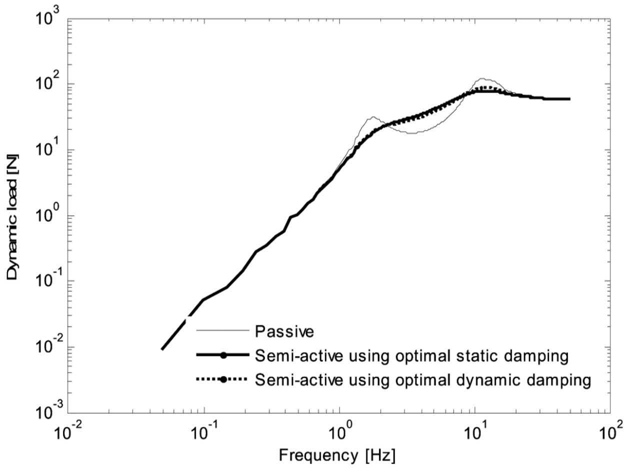


Figure 6 Comparison of three frequency responses: $DL(s)/z_r(s)$



6 Conclusions

To describe the dynamic responses of a vehicle model to random excitations generated by the vertical road irregularity, analytical formulae have been derived. The effects of road excitation frequency and road roughness, which are associated with ride comfort, road holding, and noise and harshness of a vehicle, were studied. To find the best compromise for the trade-off between these effects, an optimisation technique using sequential quadratic programming based on the derivation of simultaneous nonlinear equations was developed. A modified skyhook control, with two independent variables to be specified, is designed and tuned to precisely achieve the type of desired response. The simulation result showed that a semi-active suspension system with the proposed control strategy improved ride comfort, road holding, and noise and harshness significantly, compared with the conventional passive suspension systems as well with semi-active suspension for static optimal damping coefficients. However, when the vehicle went on a rough road characterised by an excitation frequency range of greater than 4 Hz, the optimal damping coefficients could not be obtained with respect to the rattle space and the tyre deflection constraints. One of the main advantages of the proposed control strategy is that it can be adapted to various roads, excitation frequencies, and vehicle speeds. An interesting issue in future work is the application of the developed control strategy to a full-car model which includes pitch, yaw, and roll motions.

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Nomenclature

c_1	Skyhook damping coefficient (Ns/m)
c_2	Variable suspension damping coefficient (Ns/m)
μ	Controllable suspension damping coefficient (Ns/m)
z_s	Vertical displacement of the sprung-mass (m)
z_u	Vertical displacement of the unsprung-mass (m)
z_r	Vertical displacement of the road profile (m)
$z_s - z_u$	Relative displacement (suspension travel) (m)
$z_u - z_r$	Tyre deflection (m)
$\dot{z}_s - \dot{z}_u$	Relative velocity between the sprung- and unsprung-mass (m/s)
m_s	Quarter-car sprung-mass (kg)
m_u	Quarter-car unsprung-mass (kg)

k	Suspension spring stiffness (N/m)
k_t	Tyre stiffness (N/m)
u	Control input (N)
a	Road roughness coefficient (1/m)
f	Excitation frequency of the road (Hz)
F_d	Actual damping force (N)
v	Vehicle speed (m/s)
σ_{z_r}	Variance of the road profile (m)
ω	Angular frequency of the road (rad/s)
Ω	Wave number of the road (1/m)