Robust Adaptive Control of a Time-Varying Heat Equation with Unknown Bounded Disturbance*

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In this paper, a robust model reference adaptive control of a radiative heat equation with unknown time-varying coefficients and spatiotemporally varying disturbance is investigated. In the adaptive control of time-varying plants, the derivative of a Lyapunov function candidate, which allows the derivation of adaptation laws, is not negative semi-definite in general. Under the assumption that the disturbance is uniformly bounded, the proposed robust adaptive scheme guarantees the boundedness of all signals in the closed loop system and the convergence of the state error near to zero. With an additional persistence of excitation condition, the parameter estimation errors are shown to converge near to zero as well. Simulation results are provided.

Key Words: Robust Model Reference Adaptive Control, Radiative Heat Equation, Parabolic Partial Differential Equation, Uniform Ultimate Boundedness, Stability, Persistence of Excitation

1. Introduction

In this paper a robust model reference adaptive control (MRAC) of a radiative heat equation with unknown time-varying coefficients and spatiotemporally varying disturbance is investigated. The heat equation is a distributed parameter system (DPS) governed by a linear parabolic partial differential equation (PDE). Such DPSs are described by operator equations on infinite dimensional Hilbert (or Banach) spaces. As in the adaptive control of finite dimensional systems a robust MRAC, under the assumption that the structure of the plant is known and only parameters in the system equation are unknown, is derived. Distributed sensing and actuation are also assumed[3),(18),(19)].

Compared to the adaptive control/identification of finite dimensional systems, that of infinite dimensional systems is not well developed and has been recently studied[(1),(2),(7),(10),(11),(14),(15),(17)]. In the case of adaptive control for time-varying infinite dimensional systems, the MRAC of a linear slowly time-varying parabolic system was presented in Hong et al.[(12)].

The mathematical models of physical plants that control engineers adopt for the purpose of designing control systems normally contain some uncertainty. This is due to imperfect knowledge on the system parameters and/or disturbances. Parameter time-variations may be due to unmodeled dynamics, for instance, neglected frictions, neglected high order dynamics, etc., and may also arise from linear approximations along different motions over a wide range of operating conditions. Studies on the control of linear parabolic PDEs with uncertainty include H^∞ algorithms developed in the frequency domain[(8),(9)], Lyapunov-based robust controller design methods[(5)], and adapative control[(4),(21)].

The objective of a MRAC scheme is to determine a feedback control law which forces the state of a

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plant to asymptotically track the state of a given reference model. At the same time, the unknown parameters in the plant model are estimated and used to update the control law. Typically, the whole adaptive system is represented as two error systems describing the evolution of the state error and the parameter estimation errors. In the infinite dimensional systems the state error and the parameter estimation errors take the forms of a partial differential equation and ordinary differential equations, respectively. The resulting closed loop system consisting of the plant, the reference model, and the estimator will be nonlinear. This is true even if the underlying plant and reference model, and the estimator are linear. The nonlinearity arises in the coupling of error dynamics. Consequently, the scheme requires a careful stability analysis to ensure that all signals, both input and output, remain in some sense bounded. It is also desirable, although not necessarily essential, that some sort of parameter convergence is achieved.

To conclude various stabilities in the sense of Lyapunov, the associated theorems require that the time derivative of a Lyapunov function candidate, $\dot{V}$, should be at least ≤ 0, i.e., negative semidefinite. Therefore, if $\dot{V}$ allows positive values near an equilibrium point, no stability can be asserted. In the MRAC of time-varying plants, the derivative of a Lyapunov function, which is introduced to derive adaptation laws, is not negative semi-definite in general.

The present paper makes the following contributions: To the authors' best knowledge, this paper is the first treatment of an infinite dimensional system with unknown time-varying parameters and additive spatiotemporally varying disturbance in the frame of robust MRAC. The unknown time-varying parameters are not required to be slow, which can be allowed to vary arbitrarily fast, and the disturbance is allowed to vary in both time and spatial variables. The well posedness of the closed loop system is established. Using an appropriate Lyapunov function candidate, the tracking error convergence near to zero is established. With the additional assumption of persistence of excitation, the convergence of parameter errors near to zero is established as well.

The paper has the following structure. In section 2, the standard MRAC of a radiative heat equation with time-varying coefficients is reviewed. In section 3, a robust MRAC algorithm in the presence of bounded disturbance is proposed. The derived control law guarantees its robustness with respect to the inaccessible disturbance and yields the desired equations of motion, thereby ensuring the adaptability of the controller. In section 4, with the persistence of excitation condition, the adjustable parameters in the adaptive controller are shown to admit convergence to their nominal values when an appropriate reference signal is used. In section 5, computer simulations are provided. Conclusions are given in section 6.

2. Problem Formulation

In this section, the standard MRAC algorithm for a linear, 1-dimensional, radiative heat equation with time-varying coefficients and bounded disturbance is formulated. As in the adaptive control of finite dimensional systems, under the assumption that the structure of the plant is known and only parameters in the system equation are unknown, the MRAC is investigated.

The heat equation is a DPS described by a linear parabolic PDE with time-varying coefficients as

$$\dot{\xi}(x,t)=a(t)\xi_{xx}(x,t)+b(t)\xi(x,t) + u(x,t)+\varphi(x,t) \quad (1)$$

where $x\in[0,1]$, $t>0$, $a(t)$ and $b(t)$ denote the conductivity coefficient and the radiation coefficient, respectively, and are unknown time-varying coefficients that are not necessarily slow-varying, $\xi(x,t)$ is the distributed state of the plant, i.e., the temperature at position x at time t, $\dot{\xi}=\partial\xi/\partial t$, $\xi_{xx}=\partial^2\xi/\partial x^2$, $u(x,t)$ is the control input function, and $\varphi(x,t)$ is the inaccessible external disturbance. Note that $\varphi(x,t)$ is a spatiotemporally varying function. Boundary conditions are given as

$$\xi(0,t)=h_1(t),\ \xi(1,t)=h_2(t).$$

Initial condition is given as

$$\xi(x,0)=\xi_0(x).$$

The output y of Eq. (1) in general is given by $y(x,t)=G\xi(x,t)$, where $G: C([0,1]\times R^+)\rightarrow C(\Omega\subset[0,1]\times R^+)$ is a linear bounded time invariant operator with the form depending on the characteristics of the sensor. The following assumptions are made.

Assumptions: (i) The structure of Eq. (1) (plant) is *a priori* known. (ii) Boundary conditions are *a priori* known and $h_1(\cdot), h_2(\cdot)\in C^\infty[0,\infty)$. (iii) Distributed sensing and actuation are available, and the observation operator G is *a priori* known ($G=I$ may be assumed, where I denotes the identity operator from $C([0,1]\times R^+)$ onto itself). (iv) Coefficients $a(t)$ and $b(t)$ are unknown and uniformly bounded with uniformly bounded derivatives. However, $a(t)>0$ is assumed, due to the parabolicity condition of the plant. (v) Disturbance $\varphi(x,t)$ is unknown but uniformly bounded.

Now, the reference model with the same boundary conditions is defined as

$$\dot{\xi}_m(x,t)=a_m\xi_{mxx}(x,t)+b_m\xi_m(x,t)+r(x,t), \quad (2)$$

$$\xi_m(0,t)=h_1(t),\ \xi_m(1,t)=h_2(t),$$
$$\xi_m(x,0)=\xi_{m0}(x)$$

where the subscript m indicates variables and parameters related to the reference model, and $r(x,t)$ is the reference input which is analytic on $[0,1]\times[0,\infty)$. It is assumed that $a_m>0$ and $b_m<a_m\pi^2$. It is known that if $r(\cdot,\cdot)$ is analytic in $[0,1]\times[0,\infty)$, the solution of Eq.(2) is analytic in $[0,1]\times\{0<t<T<\infty\}$[8].

The control objective for MRAC can now be stated as follows: Find a bounded control signal $u(x,t)$ that drives $\xi(x,t)$ to $\xi_m(x,t)$ asymptotically and keeps all signals in the closed-loop bounded. Now adopting the procedure in Hong and Bentsman[11], consider a control law of the form

$$u(x,t)=(a_m-\hat{a}(t))\xi_{xx}(x,t)+(b_m-\hat{b}(t))\xi(x,t)+r(x,t) \quad (3)$$

where $\hat{a}$ and $\hat{b}$ are adaptive estimates to be specified in the sequel. Substituting Eq.(3) into Eq.(1) yields the following closed loop plant equation

$$\dot{\xi}=(a_m-\tilde{a})\xi_{xx}+(b_m-\tilde{b})\xi+r+\varphi \quad (4)$$

where $\tilde{a}=\hat{a}-a$ and $\tilde{b}=\hat{b}-b$ are parameter estimation errors. Note that if $\tilde{a}=\tilde{b}=0$ and $\varphi=0$, then Eq.(4) is exactly the same as Eq.(2).

Introducing the state error $e=\xi-\xi_m$, the following state error equation is obtained.

$$\dot{e}=(a_m-\tilde{a})e_{xx}+(b_m-\tilde{b})e-(\tilde{a}\xi_{mxx}+\tilde{b}\xi_m)+\varphi,$$
$$e(0,t)=e(1,t)=0,\ e(x,0)=\xi_0(x)-\xi_{m0}(x). \quad (5)$$

Now, consider a functional $V: L_2(0,1)\times R^2\to R^+$ such that

$$V(e,\tilde{a},\tilde{b})=\frac{1}{2}\langle e,e\rangle+\frac{1}{2\gamma_a}\tilde{a}^2+\frac{1}{2\gamma_b}\tilde{b}^2 \quad (6)$$

where γ_a and γ_b are positive constants, which will become the adaptation gains later. $\langle\cdot,\cdot\rangle$ is the inner product in the space $L_2(0,1)$ defined as $\langle h,g\rangle=\int_0^1 h(x)g(x)dx$ and with the induced norm $\|\cdot\|$.

Differentiating (6) with respect to t along (5) yields:

$$\begin{aligned}\dot{V}(t)&=\langle e,\dot{e}\rangle+\frac{1}{\gamma_a}\tilde{a}\dot{\tilde{a}}+\frac{1}{\gamma_b}\tilde{b}\dot{\tilde{b}}\\&=a_m\langle e,e_{xx}\rangle+b_m\langle e,e\rangle\\&\quad+\tilde{a}\left(-\langle e,e_{xx}\rangle-\langle e,\xi_{mxx}\rangle+\frac{1}{\gamma_a}\dot{\tilde{a}}\right)\\&\quad+\tilde{b}\left(-\langle e,e\rangle-\langle e,\xi_m\rangle+\frac{1}{\gamma_b}\dot{\tilde{b}}\right)+\langle e,\varphi\rangle.\end{aligned} \quad (7)$$

Let the differential equations of the adaptive estimators in Eq.(3) be given as

$$\dot{\hat{a}}=\gamma_a(\langle e,e_{xx}\rangle+\langle e,\xi_{mxx}\rangle), \quad (8.a)$$
$$\dot{\hat{b}}=\gamma_b(\langle e,e\rangle+\langle e,\xi_m\rangle). \quad (8.b)$$

From Eqs.(8.a), (8.b) the differential equations of the errors of the adaptive estimates are given as

$$\dot{\tilde{a}}=\gamma_a(\langle e,e_{xx}\rangle+\langle e,\xi_{mxx}\rangle)-\dot{a}, \quad (9.a)$$
$$\dot{\tilde{b}}=\gamma_b(\langle e,e\rangle+\langle e,\xi_m\rangle)-\dot{b}. \quad (9.b)$$

Then, substituting Eqs.(9.a), (9.b) into Eq.(7) yields:

$$\dot{V}\le-(a_m\pi^2-b_m)\|e\|^2-\frac{1}{\gamma_a}\tilde{a}\dot{a}-\frac{1}{\gamma_b}\tilde{b}\dot{b}+\langle e,\varphi\rangle. \quad (10)$$

Now, from Eq.(10), the following observations are made.

Case (1) Assume that the system coefficients a and b are constant, i.e., $\dot{a}=\dot{b}=0$, and the disturbance $\varphi=0$. Then, Eq.(10) becomes $\dot{V}\le0$, i.e., negative semidefinite, which implies that Eq.(6) is a Lyapunov function. Therefore, the stability of an equilibrium point $(e,\tilde{a},\tilde{b})=(0,0,0)$ is guaranteed. Furthermore, the convergence of the state error e to zero is also guaranteed by the uniqueness and semigroup properties of the solution[10].

Case (2) Assume that $\varphi=0$, but a and b are time-varying. Then, Eq.(10) becomes

$$\dot{V}\le-(a_m\pi^2-b_m)\|e\|^2-\frac{1}{\gamma_a}\tilde{a}\dot{a}-\frac{1}{\gamma_b}\tilde{b}\dot{b}. \quad (10.a)$$

In this case, $\dot{V}$ may take positive values because of $\tilde{a}\dot{a}/\gamma_a+\tilde{b}\dot{b}/\gamma_b$. Hence, no stability conclusion can be drawn from the Lyapunov function candidate Eq.(6). But, further assume that

(a) $\tilde{a}$ and $\tilde{b}$ are bounded and $\dot{a}, \dot{b}\in L_1$. Then, the integration of both sides of Eq.(10.a) gives

$$\int_0^\infty(a_m\pi^2-b_m)\|e\|^2dt\le V(0)-V(\infty)+\int_0^\infty\left(\frac{1}{\gamma_a}\tilde{a}\dot{a}+\frac{1}{\gamma_b}\tilde{b}\dot{b}\right)dt<\infty.$$

Therefore, the tracking error $e(t)$ converges to zero as $t\to\infty$.

(b) Now, if only the boundedness of $\tilde{a}\dot{a}$ and $\tilde{b}\dot{b}$ is assumed, then no asymptotic convergence to zero can be asserted. However, if $\tilde{a}\dot{a}/\gamma_a+\tilde{b}\dot{b}/\gamma_b$ is sufficiently small, then it is guaranteed that e is uniformly ultimately bounded within an arbitrarily small neighborhood of zero[13].

Remark 1: In order to have the boundedness of $\tilde{a}\dot{a}/\gamma_a+\tilde{b}\dot{b}/\gamma_b$, the boundedness of individual $\tilde{a}$, $\tilde{b}$, $\dot{a}$, and $\dot{b}$ is required. The boundedness of $\dot{a}$ and $\dot{b}$ depends on the plant, which can be assumed. Also, in the case that a and b are slowly varying, the boundedness of $\tilde{a}$ and $\tilde{b}$ was shown through averaging analysis[12]. A successful adaptation is possible as long as the plant parameters vary slowly. Now, more general problem would be one in which a, b, $\dot{a}$ and $\dot{b}$ vary in unknown fashion but bounded (no slow varying assumption). The aim of this paper is to show not only the boundedness of all signals in the closed loop including $\tilde{a}$ and $\tilde{b}$ but also the convergence of $\tilde{a}$ and $\tilde{b}$ to zero.

3. Robust MRAC: Stability

As discussed in section 2, the adaptation laws (8.a), (8.b) with the boundedness of $\dot{a}(t)$ and $\dot{b}(t)$ were not able to assure the boundedness of e, $\tilde{a}$ and $\tilde{b}$ due to disturbance and unknown time-varying behavior. In this section, the control and adaptation laws are modified so that the boundedness of all signals in the closed loop system and the convergence of the state error e near to zero can be assured. Assume that $\|\varphi(x, t)\|$ is uniformly bounded by μ_e, i.e., $\mu_e \geq \|\varphi(x, t)\|$, where μ_e is a positive constant that is unknown. The main idea is to consider the worst case of the uncertainties in the form of possible bounds. Based upon the worst case, the following control algorithm is proposed.

Control Law :

$$u(x, t)=(a_m-\hat{a}(t))\xi_{xx}(x, t)+(b_m-\hat{b}(t))\xi(x, t)+r(x, t)+f(x, t) \quad (11)$$

where the additional term $f(x, t)$ is regarded as a new input signal to be determined based on robust control strategy. Let the additional input $f(x, t)$ be given by

$$f(x, t)=-\frac{\hat{\mu}_e^2}{\hat{\mu}_e\|e(x, t)\|+\varepsilon_e}e(x, t) \quad (12)$$

where $\varepsilon_e>0$, and $\hat{\mu}_e$ is the estimate of μ_e.

Adaptation Laws :

$$\dot{\hat{a}}=-\delta_a\hat{a}+\gamma_a(\langle e, e_{xx}\rangle+\langle e, \xi_{mxx}\rangle+g_a) \quad (13.a)$$

$$\dot{\hat{b}}=-\delta_b\hat{b}+\gamma_b(\langle e, e\rangle+\langle e, \xi_m\rangle+g_b) \quad (13.b)$$

$$\dot{\hat{\mu}}=-\delta_e\hat{\mu}_e+\gamma_e\|e\| \quad (13.c)$$

where

$$\delta_a>0,\ g_a=-\frac{\mu_a^2}{|\hat{a}|\mu_a+\varepsilon_a}\hat{a},$$

$$\varepsilon_a>0,\ \mu_a\geq|f_a|,\ f_a\triangleq-\frac{\delta_a}{\gamma_a}\left(a+\frac{\dot{a}}{\delta_a}\right),$$

$$\delta_b>0,\ g_b=-\frac{\mu_b^2}{|\hat{b}|\mu_b+\varepsilon_b}\hat{b},$$

$$\varepsilon_b>0,\ \mu_b\geq|f_b|,\ f_b\triangleq-\frac{\delta_b}{\gamma_b}\left(b+\frac{\dot{b}}{\delta_b}\right),$$

$\delta_e>0$, and $\gamma_e>0$.

Note that the adaptation laws (13.a)-(13.c) are implementable. The terms $-\delta_a\hat{a}$, $-\delta_b\hat{b}$, and $-\delta_e\hat{\mu}_e$ in Eqs.(13.a)-(13.c) are purposely inserted to enhance the convergence of $\hat{a}$, $\hat{b}$, and $\hat{\mu}_e$, respectively; g_a and g_b are introduced to cope with the variations of a and b, respectively. Since a, $\dot{a}$, b and $\dot{b}$ are assumed to be bounded, μ_a and μ_b can be selected at reasonable values by making γ_a and γ_b sufficiently large. It is also noted that the control magnitudes μ_a, μ_b, and $\hat{\mu}_e$ are to compensate the maximum possible bounds of f_a, f_b, and φ, respectively, for both positive and negative cases.

Substituting Eq.(11) into Eq.(1) yields the following closed loop plant equation.

$$\dot{\xi}=(a_m-\tilde{a})\xi_{xx}+(b_m-\tilde{b})\xi+r+f+\varphi. \quad (14)$$

Then, the following state error equation is derived.

$$\dot{e}=(a_m-\tilde{a})e_{xx}+(b_m-\tilde{b})e-(\tilde{a}\xi_{mxx}+\tilde{b}\xi_m)+f+\varphi,$$

$$e(0, t)=e(1, t)=0,\ e(x, 0)=\xi_0(x)-\xi_{m0}(x). \quad (15)$$

Now, consider a functional V_0: $L_2(0, 1)\times R^3\rightarrow R^+$ such that

$$V_0(e, \tilde{a}, \tilde{b})=\frac{1}{2}\langle e, e\rangle+\frac{1}{2\gamma_a}\tilde{a}^2+\frac{1}{2\gamma_b}\tilde{b}^2+\frac{1}{2\gamma_e}\tilde{\mu}_e^2. \quad (16)$$

Differentiating (16) with respect to t along (15) yields:

$$\dot{V}_0(t)=a_m\langle e, e_{xx}\rangle+b_m\langle e, e\rangle+\tilde{a}\left(-\langle e, e_{xx}\rangle-\langle e, \xi_{mxx}\rangle+\frac{1}{\gamma_a}\dot{\tilde{a}}\right)+\tilde{b}\left(-\langle e, e\rangle-\langle e, \xi_m\rangle+\frac{1}{\gamma_b}\dot{\tilde{b}}\right)+\langle e, f\rangle+\langle e, \varphi\rangle+\frac{1}{\gamma_e}\tilde{\mu}_e\dot{\tilde{\mu}}_e. \quad (17)$$

From Eqs.(13.a), (13.b) and $\dot{\tilde{a}}=\dot{\hat{a}}-\dot{a}$ and $\dot{\tilde{b}}=\dot{\hat{b}}-\dot{b}$, the differential equations of the adaptive estimates errors are given as

$$\dot{\tilde{a}}=-\delta_a\hat{a}+\gamma_a(\langle e, e_{xx}\rangle+\langle e, \xi_{mxx}\rangle+g_a)-\dot{a}$$

$$\dot{\tilde{b}}=-\delta_b\hat{b}+\gamma_b(\langle e, e\rangle+\langle e, \xi_m\rangle+g_b)-\dot{b}.$$

Therefore, Eq.(17) yields:

$$\begin{aligned}\dot{V}_0(t)=&-a_m\langle e_x, e_x\rangle+b_m\langle e, e\rangle+\langle f, e\rangle+\langle\varphi, e\rangle\\&+\frac{1}{\gamma_e}\tilde{\mu}_e\dot{\tilde{\mu}}_e+\frac{1}{\gamma_a}\tilde{a}(-\delta_a\hat{a}+\gamma_a g_a)-\frac{1}{\gamma_a}\tilde{a}\dot{a}\\&+\frac{1}{\gamma_a}\tilde{b}(-\delta_b\hat{b}+\gamma_b g_b)-\frac{1}{\gamma_b}\tilde{b}\dot{b}\\=&-a_m\langle e_x, e_x\rangle+b_m\langle e, e\rangle+\langle f, e\rangle+\langle\varphi, e\rangle\\&+\tilde{\mu}_e\dot{\tilde{\mu}}/\gamma_e-\lambda_a\tilde{a}^2+\tilde{a}f_a+\tilde{a}g_a-\lambda_b\tilde{b}^2+\tilde{b}f_b+\tilde{b}g_b\end{aligned} \quad (18)$$

where $\lambda_a=\delta_a/\gamma_a$ and $\lambda_b=\delta_b/\gamma_b$. From Eq.(18), the following inequality is derived.

$$\begin{aligned}\dot{V}_0(t)\leq&-(a_m\pi^2-b_m)\|e\|^2+\langle f, e\rangle+\|\varphi\|\|e\|\\&+\tilde{\mu}_e(\gamma_e\|e\|-\delta_e\hat{\mu}_e)/\gamma_e-\lambda_a\tilde{a}^2+\tilde{a}f_a+\tilde{a}g_a-\lambda_b\tilde{b}^2\\&+\tilde{b}f_b+\tilde{b}g_b\\\leq&-(a_m\pi^2-b_m)\|e\|^2+\langle f, e\rangle+\mu_e\|e\|\\&+\tilde{\mu}_e(\gamma_e\|e\|-\delta_e\hat{\mu}_e)/\gamma_e-\lambda_a\tilde{a}^2+|\hat{a}|\mu_a+\hat{a}g_a-af_a\\&-ag_a-\lambda_b\tilde{b}^2+|\hat{b}|\mu_b+\hat{b}g_b-bf_b-bg_b.\\\leq&-(a_m\pi^2-b_m)\|e\|^2-\frac{\hat{\mu}_e^2\|e\|^2}{\hat{\mu}_e\|e\|+\varepsilon_e}+\hat{\mu}_e\|e\|\\&-\frac{\delta_e}{\gamma_e}\tilde{\mu}_e\hat{\mu}_e-\lambda_a\tilde{a}^2+\|\hat{a}\|\mu_a-\frac{\mu_a^2|\hat{a}|^2}{|\hat{a}|\mu_a+\varepsilon_a}-af_a\\&+\frac{\mu_a^2}{|\hat{a}|\mu_a+\varepsilon_a}\hat{a}a-\lambda_b\tilde{b}^2+|\hat{b}|\mu_b-\frac{\mu_b^2|\hat{b}|^2}{|\hat{b}|\mu_b+\varepsilon_b}-bf_b\\&+\frac{\mu_b^2}{|\hat{b}|\mu_b+\varepsilon_b}\hat{b}b\\\leq&-(a_m\pi^2-b_m)\|e\|^2-\lambda_a\tilde{a}^2-\lambda_b\tilde{b}^2\\&-\lambda_e\tilde{\mu}_e^2+\left\{\varepsilon_a+\frac{\delta_a}{\gamma_a}a^2+\frac{1}{\gamma_a}a\dot{a}+|a|\mu_a+\varepsilon_b+\frac{\delta_b}{\gamma_b}b^2\right.\\&\left.+\frac{1}{\gamma_b}b\dot{b}+|b|\mu_b+\varepsilon_e+\frac{\delta_e}{2\gamma_e}\mu_e\right\},\end{aligned} \quad (19)$$

where $\lambda_e=\delta_e/\gamma_e$. Therefore, the derivative of the Lyapunov function candidate is bounded as follows:

$$\dot{V}_0\leq-(a_m\pi^2-b_m)\|e\|^2-\lambda_a\tilde{a}^2-\lambda_b\tilde{b}^2-\lambda_e\tilde{\mu}_e^2+v(t) \quad (20)$$

where $v(t)=\varepsilon_a+\frac{\delta_a}{\gamma_a}a^2+\frac{1}{\gamma_a}a\dot{a}+|a|\mu_a+\varepsilon_b+\frac{\delta_b}{\gamma_b}b^2+\frac{1}{\gamma_b}b\dot{b}+|b|\mu_b+\varepsilon_e+\frac{\delta_e}{2\gamma_e}\mu_e$. Note that $v(t)$ is bounded because of the assumption that a, b, $\dot{a}$, $\dot{b}$, and μ_e are bounded.

Remark 2: The existence and uniqueness of the solutions for coupled nonautonomous dynamical systems (15) and (13.a)-(13.c) are addressed in

Appendix A. Since $v(t)$ is bounded, the solutions e, $\tilde{a}$, and $\tilde{b}$ are uniformly ultimately bounded[13].

Remark 3: The equations of $|f_a|$ and $|f_b|$ can be rewritten as

$$|f_a|=\left|-\frac{\delta_a}{\gamma_a}\left(a+\frac{\dot{a}}{\delta_a}\right)\right|=\frac{1}{\gamma_a}|-\delta_a a+\dot{a}|$$

$$\text{and } |f_b|=\left|-\frac{\delta_b}{\gamma_b}\left(b+\frac{\dot{b}}{\delta_b}\right)\right|=\frac{1}{\gamma_b}|-\delta_b b+\dot{b}|.$$

From $\mu_a \geq |f_a|$ and $\mu_b \geq |f_b|$, μ_a and μ_b can be chosen at reasonable values according to $|f_a|$ and $|f_b|$, respectively. Thus, $v(t)$ can be pushed in an arbitrarily small boundedness region by making sufficiently small ε_a, ε_b, ε_e, δ_a, δ_b, δ_e and sufficiently large γ_a, γ_b, γ_e.

All the above developments are now summarized as follows:

Theorem 1: Consider the coupled nonautonomous dynamical system (15) and (13.a)-(13.c). Then all signals in the system are bounded. Furthermore, the uniform ultimate boundedness region of the state error e can be made arbitrary small near to zero by a suitable choice of ε_a, ε_b, ε_e, δ_a, δ_b, δ_e, γ_a, γ_b, and γ_e.

4. Parameter Error Convergence

Theorem 1 implies that the basic control objective is now achieved, i.e., all the signals in the closed loop are bounded and the trajectory following is achieved. In addition to the state error convergence near to zero, it is also desirable to have an adaptive control scheme to provide parameter estimation error convergence near to zero as well, i.e., the parameters $\hat{a}$ and $\hat{b}$ converge near to the true parameters $a(t)$ and $b(t)$ as quickly as possible. If the parameter error convergence is established, the robustness of the entire adaptive algorithm can be improved. To assure this, the following additional persistency of excitation condition on the reference model is required.

Let $H \triangleq H^1(0, 1)$ be a Hilbert space such that ξ, $\xi_m \in H$ which is densely and continuously embedded in $L_2(0, 1)$, and H^* be the continuous dual space of H. Let $A_m \in L(H, H^*)$ be the reference model dynamic operator, i.e., $A_m \triangleq a_m \partial^2/\partial x^2 + b_m$, and $A_1(q): H \to H^*$ be a differential operator such that $A_1(q) \triangleq q_1 \partial^2/\partial x^2 + q_2$ for each $q=(q_1, q_2, q_3) \in Q$, $Q \triangleq R^3$. And, let $Dom(A_m)=\{\psi \in H: A_m\psi \in L_2\}$ and $Dom(A_1(q))=\{\psi \in H: A_1(q)\psi \in L_2\}$. The following definition is adopted.

Definition: The reference model (2), or the triple $\{A_m, r, \xi_{m0}\}$, is persistently exciting if there exist positive constants τ_0, δ_0, ε_0, and c_0, such that for each $q \in Q$ with $|q|_Q=1$ and $t \geq 0$ sufficiently large, there exists $\bar{t} \in [t, t+\tau_0]$ for which

$$\left\|\int_{\bar{t}}^{\bar{t}+\delta_0} A_1(q)\xi(\tau)d\tau\right\|_{H^*} \geq \varepsilon_0+(|\tilde{\mu}_e|+\mu_e)\frac{\delta_0}{c_0}. \quad (21)$$

Theorem 2: If $r \in L_\infty(0, \infty; H)$ and $\xi_{m0} \in H$, and if the reference model (2) is persistently exciting, then the uniform ultimate boundedness region of the parameter estimation error vector $\theta=(\tilde{a}, \tilde{b}, \tilde{\mu}_e)$ can be made arbitrarily small near to zero by a suitable choice of ε_a, ε_b, ε_e, δ_a, δ_b, δ_e, γ_a, γ_b, and γ_e.

Proof: In this proof, the following notation is used $|\cdot|_2=\|\cdot\|_{L_2}$, $\|\cdot\|=\|\cdot\|_H$, and $\|\cdot\|_*=\|\cdot\|_{H^*}$. For the operators defined above, there exist α_0, $\alpha_1>0$ and $K_0>0$ such that for ψ_1, $\psi_2 \in H$ and $q \in Q$

$$|\langle A_m\psi_1, \psi_2\rangle| \leq \alpha_0\|\psi_1\|\|\psi_2\|, \quad (22)$$

$$|\langle A_1(q)\psi_1, \psi_2\rangle| \leq \alpha_1|q|_Q\|\psi_1\|\|\psi_2\|, \quad (23)$$

$$\|\psi_1\|_* \leq K_0|\psi_1|_2. \quad (24)$$

From Eqs. (13.a)-(13.c), the parameter estimation error system is rewritten as

$$\langle\dot{\theta}, q\rangle_Q=\langle A_1(\gamma q)\{e+\xi_m\}, e\rangle+\langle-\delta\hat{\theta}+\gamma_0|e|_2 + \gamma g-\dot{\theta}^*, q\rangle_Q \quad (25)$$

where $\gamma q=(\gamma_a q_1, \gamma_b q_2, \gamma_e q_3)$, $\delta\hat{\theta}=(\delta_a\hat{a}, \delta_b\hat{b}, \delta_e\hat{\mu}_e)$, $\gamma_0=(0, 0, \gamma_e)$, $\gamma g=(\gamma_a g_a, \gamma_b g_b, 0)$, and $\dot{\theta}^*=(\dot{a}, \dot{b}, \dot{\mu}_e)$.

Now assume that $r \in L_\infty(0, \infty; H)$ and $\xi_{m0} \in H$. Then, $\xi_m \in L_\infty(0, \infty; H)$, see Theorem 2.2 of Bohm et al.[2]. Assume that $|\theta(t)/\gamma|_Q$ is uniformly bounded by ρ where $\theta/\gamma=(\tilde{a}/\gamma_a, \tilde{b}/\gamma_b, \tilde{\mu}_e/\gamma_e)$. Then, from Eqs. (23) and (25) it follows that for $q=\frac{1}{\rho}\left(\frac{\theta(t)}{\gamma}\right)$

$$|\theta(t_2)-\theta(t_1)|_Q=\sup_{|q|_Q=1}|\langle\theta(t_2)-\theta(t_1), q\rangle_Q|$$

$$=\sup_{|q|_Q=1}\left|\left\langle\int_{t_1}^{t_2}\dot{\theta}(t)dt, q\right\rangle_Q\right|$$

$$\leq\int_{t_1}^{t_2}\sup_{|q|_Q=1}\langle A_1(\gamma q)\{e(t)+\xi_m(t)\}, e(t)\rangle|dt$$

$$+\int_{t_1}^{t_2}\sup_{|q|_Q=1}|\langle-\delta\hat{\theta}+\gamma_0|e|_2+\gamma g-\dot{\theta}^*, q\rangle_Q|dt$$

$$\leq\alpha_1\int_{t_1}^{t_2}\|e(t)\|^2dt$$

$$+\alpha_1\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\int_{t_1}^{t_2}\|e(t)\|dt+\frac{1}{\rho}\int_{t_1}^{t_2}|\langle\delta\hat{\theta}$$

$$-\gamma_0|e|_2-\gamma g+\dot{\theta}^*, \theta/\gamma\rangle_Q|dt$$

$$\leq\alpha_1\int_{t_1}^{t_2}\|e(t)\|^2dt+\alpha_1\|\xi_m(t)\|_{L_\infty(0,\infty;H)}(t_2-t_1)^{1/2}$$

$$\times\left\{\int_{t_1}^{t_2}\|e(t)\|^2dt\right\}^{1/2}+\frac{1}{\rho}\int_{t_1}^{t_2}|\langle\delta\hat{\theta}-\gamma_0|e|_2-\gamma g$$

$$+\dot{\theta}^*, \theta/\gamma\rangle_Q|dt. \quad (26)$$

For $t_2>t_1$, Eqs. (15), (22), (23), and (24) imply that

$$\left\|\int_{t_1}^{t_2}A_1(\theta(t))\xi(t)dt\right\|_*$$

$$=\left\|\int_{t_1}^{t_2}A_1(\theta(t))\{e(t)+\xi_m(t)\}dt\right\|_*$$

$$\leq\|e(t_2)_*+\|e(t_1)\|_*+\int_{t_1}^{t_2}\|A_m e(t)\|_*dt+\int_{t_1}^{t_2}\|f(t)$$

$$+\varphi(t)\|_*dt\leq K_0|e(t_2)|_2+K_0|e(t_1)|_2$$

$$+\alpha_0(t_2-t_1)^{1/2}\left\{\int_{t_1}^{t_2}\|e(t)\|^2dt\right\}^{1/2}$$

$$+\int_{t_1}^{t_2}\|f(t)+\varphi(t)\|_*dt$$

$$\leq K_0|e(t_2)|_2+K_0|e(t_1)|_2$$

$$+\alpha_0(t_2-t_1)^{1/2}\left\{\int_{t_1}^{t_2}\|e(t)\|^2dt\right\}^{1/2}+(|\tilde{\mu}_e|+\mu_e)(t_2-t_1). \quad (27)$$

Once again assume that $\lim_{t\to\infty}|\theta(t)|_Q>0$, and let $\{t_k\}_{k=1}^{\infty}$ be an increasing sequence of positive numbers for which $\lim_{k\to\infty} t_k=\infty$ and

$$|\theta(t_k)|_Q\geq c_0,\ k=1,2,\cdots, \tag{28}$$

for some $c_0>0$. Assume further that the reference model (2) is persistently exciting, and for each $k=1, 2, \cdots$, let $\bar{t}_k\in[t_k, t_k+\tau_0]$ be such that

$$\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}A_1\left(\frac{\theta(t_k)}{|\theta(t_k)|_Q}\right)\xi(t)dt\right\|_*\geq\varepsilon_0+(|\bar{\mu}_e|+\mu_e)\frac{\delta_0}{c_0}. \tag{29}$$

Then, using Eqs.(22), (23), (26) and (27), we obtain the estimate

$$\begin{aligned}0&<c_0\varepsilon_0+(|\bar{\mu}_e|+\mu_e)\delta_0\\&=c_0\left(\varepsilon_0+(|\bar{\mu}_e|+\mu_e)\frac{\delta_0}{c_0}\right)\\&\leq|\theta(t_k)|_Q\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}A_1\left(\frac{\theta(t_k)}{|\theta(t_k)|_Q}\right)\xi(t)dt\right\|_*\\&=\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}A_1(\theta(t_k))\xi_k(t)dt\right\|_*\\&\leq\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}A_1(\theta(t))\xi(t)dt\right\|_*\\&\quad+\left\|\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}A_1(\theta(t_k)-\theta(t)\{e(t)+\xi_m(t)\}dt\right\|_*\\&\leq K_0|e(\bar{t}_k+\delta_0)|_2+K_0|e(\bar{t}_k)|_2\\&\quad+\alpha_0\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}+(|\bar{\mu}_e|+\mu_e)\delta_0\\&\quad+\alpha_1|\theta(\bar{t}_k+\tau_0+\delta_0)\\&\quad-\theta(\bar{t}_k)|_Q\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\{\|e(t)\|+\|\xi_m(t)\|\}dt\\&\leq K_0|e(\bar{t}_k+\delta_0)|_2+K_0|e(\bar{t}_k)|_2\\&\quad+\alpha_0\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\\&\quad+(|\bar{\mu}_e|+\mu_e)\delta_0+\alpha_1\times\left[\alpha_1\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right.\\&\quad+\alpha_1\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\sqrt{\tau_0+\delta_0}\\&\quad\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|e(t)\|^2dt\right\}^{1/2}+\frac{1}{\rho}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}|\langle\delta\hat{\theta}-\gamma_0|e|_2\\&\quad\left.-\gamma g+\dot{\theta}^*,\ \theta/\gamma\rangle_Q|dt\right]\left[\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\right.\\&\quad\left.+\delta_0\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\right].\end{aligned} \tag{30}$$

Note that Eq.(30) is rewritten as

$$\begin{aligned}0&<c_0\varepsilon_0\\&\leq K_0|e(\bar{t}_k+\delta_0)|_2+K_0|e(\bar{t}_k)|_2+\alpha_0\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\right.\\&\quad\left.\times\|e(t)\|^2dt\right\}^{1/2}+\alpha_1\times\left[\alpha_1\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|e(t)\|^2dt\right.\\&\quad+\alpha_1\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\sqrt{\tau_0+\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\\&\quad\left.+\frac{1}{\rho}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}|\langle\delta\hat{\theta}-\gamma_0|e|_2-\gamma g+\dot{\theta}^*,\ \theta/\gamma\rangle_Q|dt\right]\\&\quad\times\left[\sqrt{\delta_0}\left\{\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}+\delta_0\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\right].\end{aligned} \tag{31}$$

From the adaptive laws (13.a)-(13.c), we can have

$$\langle\delta\hat{\theta}-\gamma g+\dot{\theta}^*,\ \theta/\gamma\rangle_Q\leq v'(t)$$

where $v'(t)=\varepsilon_a+\frac{\delta_a}{\gamma_a}a^2+\frac{1}{\gamma_a}a\dot{a}+|\dot{a}|\mu_a+\varepsilon_b+\frac{\delta_b}{\gamma_b}b^2+\frac{1}{\gamma_b}b\dot{b}+|b|\mu_b$ and $v'(t)$ can be made arbitrary small near to zero by making sufficiently small ε_a, ε_b, δ_a, δ_b, and sufficiently large γ_a, γ_b. Now, from Appendix B, for any $L>0$, $\lim_{t\to\infty}\int_t^{t+L}\|e(s)\|^2ds\approx0$. Therefore, letting $k\to\infty$ in Eq.(31) and sufficiently small ε_a, ε_b, ε_e, δ_a, δ_b, δ_e, and sufficiently large γ_a, γ_b, γ_e, Theorem 1 and Appendix B imply that

$$\begin{aligned}0&<c_0\varepsilon_0\\&\leq K_0\lim_{k\to\infty}|e(\bar{t}_k+\delta_0)|_2\\&\quad+K_0\lim_{k\to\infty}|e(\bar{t}_k)|_2\\&\quad+\alpha_0\sqrt{\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\\&\quad+\alpha_1\times\left[\lim_{k\to\infty}\alpha_1\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|e(t)\|^2dt+\alpha_1\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\right.\\&\quad\times\sqrt{\tau_0+\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\\&\quad+\frac{1}{\rho}\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\tau_0+\delta_0}|\langle\delta\hat{\theta}-\gamma_0|e|_2-\gamma g\\&\quad\left.+\dot{\theta}^*,\ \theta/\gamma\rangle_Q|dt\right]\left[\sqrt{\delta_0}\left\{\lim_{k\to\infty}\int_{\bar{t}_k}^{\bar{t}_k+\delta_0}\|e(t)\|^2dt\right\}^{1/2}\right.\\&\quad\left.+\delta_0\|\xi_m(t)\|_{L_\infty(0,\infty;H)}\right]\approx0\end{aligned}$$

which is a contradiction, and the theorem is proved. □

5. Simulations

To illustrate the application of the theory developed in the previous sections the heat transfer equation is chosen to have a time-varying coefficient and a spatiotemporally varying disturbance. Let the heat transfer equation be given with known homogeneous boundary conditions as

$$\begin{aligned}&\dot{\xi}=a(t)\xi_{xx}+u+\varphi,\ x\in[0,1],\ t>0\\&\xi(0,t)=\xi(1,t)=0,\ t>0\\&\xi(x,0)=\xi_0(x)=0.2\sin(2\pi x)\end{aligned} \tag{32}$$

where $a(t)$ is the time-varying conductivity and $\varphi(x,t)$ is the spatiotemporally varying latent heat of transformation which is treated as disturbance. $a(t)$ and $\varphi(x,t)$ are unknown, but for simulation purpose $a(t)=3-2.5\sin(3t)$ and $\varphi(x,t)=0.3+0.5\sin(3\pi x)\sin(5t)$ are assumed. The reference model is chosen as

$$\begin{aligned}&\dot{\xi}_m=0.5\xi_{mxx}+5,\ x\in[0,1],\ t<0\\&\xi_m(0,t)=\xi_m(1,t)=0,\ t>0\\&\xi_m(x,0)=-\sin(\pi x).\end{aligned} \tag{33}$$

In this case, the boundary conditions establishes $H\triangleq H_0^1(0,1)$. The control gains in Eqs.(12), (13.a), and (13.c) are chosen as $\varepsilon_e=0.1$, $\delta_a=0.01$, $\gamma_a=50$, $\mu_a=0.2$, $\varepsilon_a=0.01$, $\delta_e=0.1$, and $\gamma_e=30$, respectively. Figure 1 shows the convergence of the state error $e(x,t)$ to zero. Figure 2 shows the convergence of the

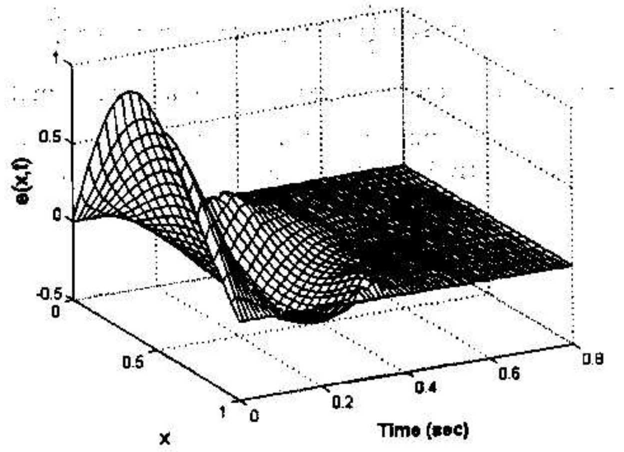

Fig. 1 Convergence of state error $e(x, t)$ to zero

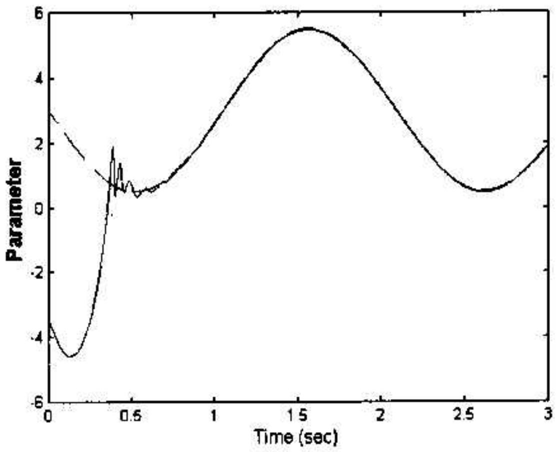

Fig. 2 Convergence of an estimated parameter $\hat{a}(t)$ (solid line) to its true value $a(t)$ (dashed line)

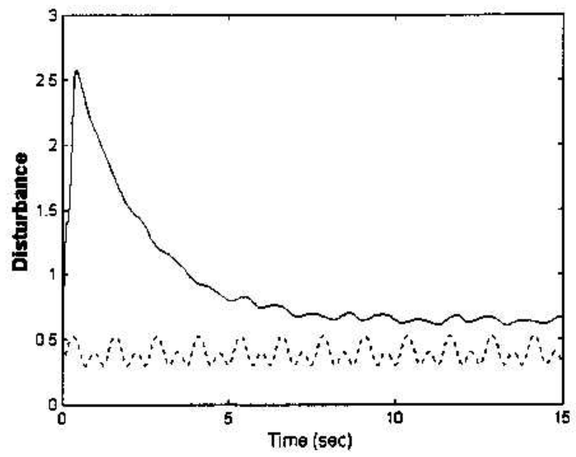

Fig. 3 Convergence of an estimated parameter $\hat{\mu}_e(t)$ (solid line) to a bounded value of disturbance norm $\|\varphi(x, t)\|$ (dashed line)

estimated parameter $\hat{a}(t)$ to the plant parameter $a(t)$. Figure 3 shows the convergence of the estimated parameter $\hat{\mu}_e(t)$ to a bounded value of disturbance norm $\|\varphi(x, t)\|$.

6. Conclusions

A robust MRAC algorithm for a radiative heat equation with unknown time-varying coefficients and spatiotemporally varying disturbance was developed. The time-varying coefficients were assumed to be uniformly bounded with uniformly bounded derivatives, but they were allowed to vary arbitrarily fast. The disturbance was also assumed to be uniformly bounded. Under the unknown plant parameters and external disturbances, the robust MRAC law developed in this paper assures the closed loop system to track a desired signal that comes from the reference model. Because the derivative of a Lyapunov function candidate was not negative semidefinite, only uniform ultimate boundedness would have been concluded. However, further analysis in this paper has shown that the state error, which remains in the derivative of the Lyapunov function candidate, converges near to zero. Also, with the additional persistence of excitation condition, the algorithm guaranteed the convergence of the adjustable controller parameters near to their nominal values. The feasibility of using a finite number of sensors and actuators is under investigation.

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Appendix A : Existence and Uniqueness

Rewrite nonlinear error equations (15) and (13.a)-(13.c) in the following form

$$\dot{z}=A(t)z+F(t, z),\ z(0)=z_0 \tag{A1}$$

where $z=(e, \tilde{a}, \tilde{b}, \tilde{\mu}_e)^T$, $\hat{a}=\tilde{a}+a$, $\hat{b}=\tilde{b}+b$, $\hat{\mu}_e=\tilde{\mu}_e+\mu_e$, and

$$A(t)=\begin{bmatrix} (a_m+a)\dfrac{\partial^2}{\partial x^2}+(b_m+b) & 0 & 0 & 0 \\ 0 & -\delta_a & 0 & 0 \\ 0 & 0 & -\delta_b & 0 \\ 0 & 0 & 0 & -\delta_e \end{bmatrix}$$

$$=\begin{bmatrix} A_0(t) & 0 & 0 & 0 \\ 0 & -\delta_a & 0 & 0 \\ 0 & 0 & -\delta_b & 0 \\ 0 & 0 & 0 & -\delta_e \end{bmatrix},$$

$$F(t,z) = \begin{bmatrix} -\hat{a}e_{xx}-\hat{b}e+(a-\hat{a})\xi_{mxx}+(b-\hat{b})\xi_m+f+\varphi \\ \gamma_a\langle e, e_{xx}\rangle+\gamma_a\langle e, \xi_{mxx}\rangle+\gamma_a g_a \\ \gamma_b\langle e, e\rangle+\gamma_b\langle e, \xi_m\rangle+\gamma_b g_b \\ \gamma_e\|e\| \end{bmatrix},$$

where $A_0(t) \triangleq (a_m+a)\frac{\partial^2}{\partial x^2}+(b_m+b)$. Define a state space as $W \triangleq L_2(0,1)\times R^3$, and

$$D(A)=\{(e, \hat{a}, \hat{b}, \hat{\mu})\in W:\ e\in H^2(0,1)\cap H_0^1(0,1)$$
$$\text{with } e(0)=0=e(1), \text{ and } \hat{a}, \hat{b}, \hat{\mu}_e\in R\}. \quad \text{(A2)}$$

Note that the boundary conditions of Eq. (15) have been incorporated in the space $H^2(0,1)\cap H_0^1(0,1)$, which is the domain of the differential operator A_0. $D(A)$ is dense, and A is a closed operator[20].

For $z\in D(A)$

$$\langle z, Az\rangle_W=\int_0^1 e(x)\left[(a_m+a)\frac{\partial^2 e(x)}{\partial x^2}+(b_m+b)e(x)\right]dx$$
$$-\delta_a\hat{a}^2-\delta_b\hat{b}^2-\delta_e\hat{\mu}_e^2$$
$$\le[-(a_m+a)\pi^2+(b_m+b)]\int_0^1 e^2(x)dx$$
$$-\delta_a\hat{a}^2-\delta_b\hat{b}^2-\delta_e\hat{\mu}_e^2$$
$$\le -C_1\langle z, z\rangle_W, \quad \text{(A3)}$$

where $C_1=\min\{(a_m+a)\pi^2-(b_m+b), \delta_a, \delta_b, \delta_e\}>0$, and $(a_m+a)\pi^2>(b_m+b)$ is assumed. Now by the linearity of A, we see that $\omega I-A$ is monotone (accretive) for every $\omega\le C_1$. Hence $A: D(A)\subset W\to W$ is the infinitesimal generator of a linear process $\{S(t)\}_{t\ge 0}=\{(\Phi(t,0), \hat{A}(t), \hat{B}(t), \hat{E}(t)\}_{t\ge 0}$ on W, see Theorem 3.2, p. 92, of Walker[20]. Note that the first component $\Phi(t,0)$ is generated by A_0. Note also that $\Phi(t,0)e_0$ is the strong solution of the evolution equation $\dot{e}(t)=A_0 e(t)$ for every $e_0\in D(A_0)$.

Now set $z=(e, \hat{a}, \hat{b}, \hat{\mu}_e)$ and $z'=(e', \hat{a}', \hat{b}', \hat{\mu}')$. Then

$$\|F(t,z)-F(t,z')\|_W^2=\|(a-\hat{a})\xi_{mxx}+(b-\hat{b})\xi_m$$
$$-(a-\hat{a}')\xi_{mxx}-(b-\hat{b}')\xi_m+f-f'-\hat{a}e_{xx}+\hat{a}'e'_{xx}$$
$$-\hat{b}e+\hat{b}'e'\|^2+\gamma_a^2|\langle e, e_{xx}\rangle+\langle e, \xi_{mxx}\rangle-\langle e', e_{xx}\rangle$$
$$+\langle e', e_{xx}\rangle-\langle e', e'_{xx}\rangle-\langle e', \xi_{mxx}\rangle+g_a-g'_a|^2$$
$$+\gamma_b^2|\langle e, e\rangle+\langle e, \xi_m\rangle-\langle e', e\rangle+\langle e', e\rangle$$
$$-\langle e', e'\rangle-\langle e', \xi_m\rangle+g_b-g'_b|^2+\gamma_e^2|\|e\|-\|e'\||^2$$
$$\le\Big(\beta_a^2K_a^2|\hat{a}'|^2+|\hat{b}'|^2+\frac{2|\hat{\mu}'_e|^6\|e'\|^2+\varepsilon_e^2|\hat{\mu}'_e|^4}{|(\hat{\mu}'_e\|e\|+\varepsilon_e)(\hat{\mu}'_e\|e'\|+\varepsilon_e)|^2}$$
$$+\gamma_a^2K_a^2(\|e_x\|^2+\|e'_x\|^2+\|\xi_{mx}\|^2)$$
$$+\gamma_b^2(\|e\|^2+\|e'\|^2+\|\xi_m\|^2)+\gamma_e^2K_d^2\Big)\|e-e'\|^2$$
$$+\Big(\beta_a^2(\|e_x\|^2+\|\xi_{mx}\|^2)$$
$$+\frac{\gamma_a^2(2\mu_a^6|\hat{a}'|^2+\varepsilon_a^2\mu_a^4)}{(|\hat{a}|\mu_a+\varepsilon_a)(|\hat{a}'|\mu_a+\varepsilon_a)}\Big)|\hat{a}-\hat{a}'|^2$$
$$+\Big(\|e\|^2+\|\xi_m\|^2+\frac{\gamma_b^2(2\mu_b^6|\hat{b}'|^2+\varepsilon_b^2\mu_b^4)}{(|\hat{b}|\mu_b+\varepsilon_b)(|\hat{b}'|\mu_b+\varepsilon_b)}\Big)|\hat{b}-\hat{b}'|^2$$
$$+\left|\frac{\hat{\mu}_e\hat{\mu}'_e\|e\|+\varepsilon_e(\hat{\mu}_e+\hat{\mu}'_e)}{(\hat{\mu}_e\|e\|+\varepsilon_e)(\hat{\mu}'_e\|e\|+\varepsilon_e)}\right|^2\|e\|^2|\hat{\mu}_e-\hat{\mu}'_e|^2,$$

where $f'=-\frac{\hat{\mu}_e'^2}{\hat{\mu}'_e\|e'\|+\varepsilon_e}e'$, $g'_a=-\frac{\mu_a^2}{|\hat{a}'|\mu_a+\varepsilon_a}\hat{a}'$, $g'_b=-\frac{\mu_b^2}{|\hat{b}'|\mu_b+\varepsilon_b}\hat{b}'$, $\|e_{xx}\|\le\beta_a\|e_x\|$, $\|e_x-e'_x\|\le K_a\|e-e'\|$, and $|\|e\|-\|e'\||\le K_d\|e-e'\|$.

Hence

$$\|F(t,z)-F(t,z')\|_W\le C_2\|z-z'\|_W, \quad \text{(A4)}$$

where C_2 is a constant. Therefore $F: W\to W$ is locally Lipschitz continuous in W. Thus a unique solution exists. Finally the solution of Eq. (15) can be written in the following variation of constant formula[16]

$$e(t)=\Phi(t,0)e(0)+\int_0^1\Phi(t,\tau)(-\hat{a}(\tau)e_{xx}(\tau)$$
$$-\hat{b}(\tau)e(\tau)-\tilde{a}(\tau)\xi_{mxx}(\tau)$$
$$-\tilde{b}(\tau)\xi_m(\tau)+f(\tau)+\varphi(\tau))d\tau, \quad \text{(A5)}$$

where $\Phi(t,s)$ is the evolution operator associated with A_0 in the space $L_2(0,1)$.

Appendix B: Tracking Error Convergence

From (17) and (20), we have

$$\dot{V}_0\le-\langle a_m e_x, e_x\rangle+\langle b_m e, e\rangle+v(t)$$
$$\le -a_m\langle e_x, e_x\rangle+\frac{b_m}{\pi^2}\langle e_x, e_x\rangle+v(t)$$
$$=-\left(a_m-\frac{b_m}{\pi^2}\right)\langle e_x, e_x\rangle+v(t)$$
$$\le-\beta_0\|e\|^2+v(t),$$

where $\beta_0>0$ and

$$v(t)=\varepsilon_e+\frac{\delta_e}{2\gamma_e}\mu_e+\varepsilon_a+\mu_a|a|+\frac{\delta_a}{\gamma_a}a^2+\frac{1}{\gamma_a}a\dot{a}$$
$$+\varepsilon_b+\mu_b|b|+\frac{\delta_b}{\gamma_b}b^2+\frac{1}{\gamma_b}b\dot{b}.$$

$v(t)$ can be made arbitrary small near to zero by making sufficiently small ε_e, ε_a, ε_b, δ_a, δ_b, δ_e and sufficiently large γ_a, γ_b, γ_e, and be assumed to be uniformly bounded by v_0. Thus, the functional $V_0(t)$ is nonincreasing and satisfies

$$V_0(t)+\beta_0\int_0^t\|e(s)\|^2ds\le\rho_0, \quad t\ge 0,$$

where $\rho_0=v_0+V_0(0)$. □

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