Adaptive Neural-Network-Based Fault-Tolerant Control for a Flexible String With Composite Disturbance Observer and Input Constraints

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***Abstract*—We propose an adaptive neural-network-based fault-tolerant control scheme for a flexible string considering the input constraint, actuator gain fault, and external disturbances. First, we utilize a radial basis function neural network to compensate for the actuator gain fault. In addition, an observer is used to handle composite disturbances, including unknown approximation errors and boundary disturbances. Then, an auxiliary system eliminates the effect of the input constraint. By integrating the composite disturbance observer and auxiliary system, adaptive fault-tolerant boundary control is achieved for an uncertain flexible string. Under rigorous Lyapunov stability analysis, the vibration scope of the flexible string is guaranteed to remain within a small compact set. Numerical simulations verify the high control performance of the proposed control scheme.**

***Index Terms*—Actuator gain fault, adaptive fault-tolerant boundary control, auxiliary system, flexible string (FS), neural-network composite disturbance observer (NNCDO).**

I. Introduction

In recent decades, flexible structure systems have attracted considerable research interest because of their excellent features, such as low energy consumption, high flexibility, and low weight. Such systems are being widely used in various fields, including aeronautics and astronautics, machine manufacturing, and medicine. However, controlling the vibrations of flexible structures remains a challenging task, for which many excellent solutions have been developed [1]–[11]. However, their solutions did not consider actuator faults, which may severely degrade the performance of the closed-loop system and even lead to instability. Thus, we propose an effective method for handling actuator faults in flexible structure systems.

Research on the actuator fault has notably advanced over the past years. The global stability of a linear, cascaded ordinary-differential-equation-beam system with actuator faults is addressed in [12], where a robust adaptive control law is employed to compensate for unknown actuator faults. An adaptive fault-tolerant boundary controller was derived for a flexible aircraft wing in [13]. A projection algorithm avoids control performance degradation due to actuator faults in the closed-loop system of the flexible aircraft wing. In [14], an adaptive fault-tolerant control scheme was introduced to estimate the unknown gain of actuator faults for a single-link flexible manipulator. In [15], a flexible satellite was described using several partial and ordinary differential equations. For the obtained model, an adaptive control scheme was derived to regulate the vibration of the satellite under actuator faults. However, these studies on flexible systems have considered the fault control-rate coefficient as a constant, which is unsuitable for a time-varying coefficient. In practice, actuator faults generally interact with the system states; however, research on adaptive fault-tolerant control for flexible systems with a time-varying fault rate coefficient is lacking, thus motivating our study.

Input constraints represent a challenging and urgent problem, and many excellent solutions have been proposed in the past few years [16]–[22]. In [23], an adaptive tracking control method was developed to prove the uniform, ultimate boundedness of uncertain MIMO nonlinear systems. The integration of an auxiliary system with an adaptive control algorithm allows handling input constraints and external disturbances, based on which boundary control was developed for a flexible aerial refueling hose [1]. A boundary controller compensates for the input constraints and mitigates external disturbances by using a Nussbaum function. In [6], boundary

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antidisturbance control was applied to a spatially nonlinear flexible string (FS) system, in which second-order disturbance observers were included to compensate for unknown external disturbances. In [24], a fault-tolerant control based on linear matrix inequality optimization was used for a class of flexible air-breathing hypersonic vehicles. Although some methods to handle actuator faults or input constraints for flexible structure systems have been developed, to the best of our knowledge, few studies have addressed unknown actuator fault, input constraint, and external disturbances in an FS system. Moreover, FS systems are widely applied in a variety of industries, such as the flexible crane system [25], suspension cable system [26], and refueling hose system [27]. Thus, the study of the FS systems is a meaningful topic. This fact motivates our proposal.

In this study, we investigate the boundary control of an FS considering constrained control input, actuator gain fault, and external disturbances. First, the unknown actuator gain fault is approximated by employing a radial basis function neural network (RBFNN). Then, a neural-network composite disturbance observer (NNCDO) is introduced to estimate unknown composite disturbances. Unlike the conventional NNCDO, the proposed observer can quickly estimate unknown composite disturbances because it adds derivative estimation information of the composite disturbances. By using a feedforward approach, an auxiliary system was established to compensate for the input constraint. Then, by combining the proposed NNCDO and auxiliary system, an adaptive neural-network-based fault-tolerant control (ANNBFTC) strategy is introduced to mitigate the vibrations in the FS. By using the proposed control scheme, the corresponding closed-loop system for the FS was proved to be uniformly bounded. Moreover, the FS vibration remains within a small compact set. The main contributions of the proposed control scheme are summarized as follows.

1) Different from the conventional NNCDO-based control scheme in [28] and [29], we introduce an NNCDO-based ANNBFTC for FS systems to handle the composite disturbances. As an unknown composite disturbance and its derivative are estimated simultaneously, a better control performance can be achieved by using NNCDO-based ANNBFTC than the conventional NNCDO-based control scheme. The advantage of the developed NNCDO-based ANNBFTC is verified in Section IV.
2) Regarding the existing auxiliary system [2], [23], if input saturation occurs, the initial value of the system must be reset. In contrast, the proposed auxiliary system not only avoids this weakness but also prevents the effect of saturation.
3) Unlike the existing methods, we provide NNCDO-based ANNBFTC aiming to suppress vibrations in an FS considering the input constraint, actuator gain fault, and external disturbances.

The remainder of this article is organized as follows. In Section II, we introduce the current problem and preliminaries. Section III details the proposed NNCDO-based ANNBFTC, including the NNCDO design, controller design, and stability analysis. The effectiveness of the proposed control scheme was verified through numerical simulations in Section IV. In Section V, we draw the conclusion and provide directions for future work.

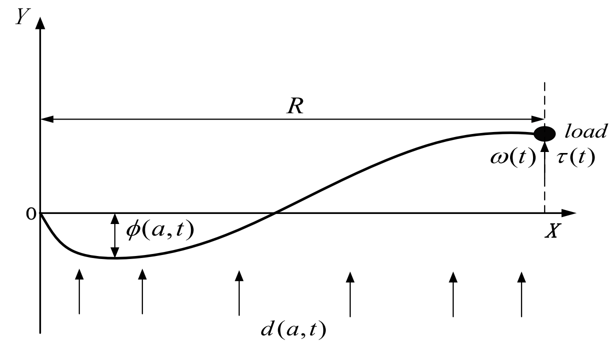

Fig. 1. Schematic of an FS system.

Notations: $\mathcal{R}$ represents the set of real numbers. $\varphi(t) \in \mathcal{L}_\infty$ indicates that the infinite norm of function $\varphi(t)$ is bounded. $Q(t) > 0$ indicates that function $Q(t)$ is a positive definite. $\mathcal{C}^1$ represents that the function is continuously differentiable. $\lambda_{\min}\{A\}$ denotes the minimum eigenvalue of matrix A. $\min\{b_1, b_2, \ldots, b_n\}$ denotes the minimum value between $b_1, b_2, \ldots, b_n$, $n \geq 2$. Partial differential variables are denoted as follows: $\phi_t(a,t) = ([\partial\phi(a,t)]/\partial t)$, $\phi_{tt}(a,t) = ([\partial^2\phi(a,t)]/\partial t^2)$, $\phi_a(a,t) = ([\partial\phi(a,t)]/\partial a)$, $\phi_{aa}(a,t) = ([\partial^2\phi(a,t)]/\partial a^2)$, and $\phi_{at}(a,t) = ([\partial^2\phi(a,t)]/\partial a\partial t)$ $\forall a \in [0, R]$, $t \in [0, \infty)$.

II. Problem Formulation and Preliminaries

The diagram of an FS is shown in Fig. 1, where XOY is the body-fixed coordinate system and $\phi(a,t)$ denotes the displacement of the FS with respect to the coordinate system XOY. As one endpoint of the FS is fixed, $\phi(0,t) = 0$ in the XOY coordinate system. In addition, $d(a,t)$ represents the distributed load exerted on the FS along the vertical direction, and $\tau(t)$ and $\omega(t)$ denote the boundary control and boundary disturbance, respectively. The FS parameters are ϖ, T, and R, which represent the uniform mass per unit length, tension, and length of the FS, respectively, and m_l denotes the mass of the endpoint load.

The governing equation of an FS is described as [2]

$$\varpi\phi_{tt}(a,t) = T\phi_{aa}(a,t) + d(a,t) \quad \forall a \in (0, R) \tag{1}$$

under boundary equations

$$\phi(0,t) = 0 \tag{2}$$

$$m_l\phi_{tt}(R,t) = -T\phi_a(R,t) + \tau(t) + \omega(t). \tag{3}$$

Nonlinear actuator faults are likely to occur in the FS. Hence, we consider the nonlinear actuator gain fault described in [19]

$$\tau_f(Z) = \o(\Pi)\tau(t), \;\; t \geq T_f \tag{4}$$

where Z and Π denote vectors composed of the FS system state variables [Z and Π are, respectively, given by (14) and (37)], T_f is the unknown fault moment, and $ø(\Pi)$ is the nonlinear fault control rate coefficient given by

$$ø(\Pi) = 1 - \frac{1}{1+\alpha e^{-\iota(\Pi)}} \tag{5}$$

where $\alpha \geq 0$ and $\iota(\Pi)$ are an unknown constant and an unknown function related to the FS system state variables, respectively.

When an actuator gain fault occurs, $\tau(t)$ may lose effect, and this loss is denoted by $\tau_f(Z)$. Thus, the actual operation of the actuator is described by $\tau(t) - \tau_f(Z)$ [19]. Then, boundary equation (3) can be rewritten as

$$m_l\phi_{tt}(R,t) = -T\phi_a(R,t) + \tau(t) - \tau_f(Z) + \omega(t). \tag{6}$$

Moreover, the actuator can be saturated due to physical limitations, and its corresponding output $\tau(t)$ can be described as

$$\tau(t) = \mathrm{Sat}(u(t)) = \begin{cases} \tau_u, & \text{if } u(t) \geq \tau_u \\ u(t), & \text{if } \tau_l < u(t) < \tau_u \\ \tau_l, & \text{if } u(t) \leq \tau_l \end{cases}$$

where $\tau_u > 0$ and $\tau_l < 0$ are the known upper and lower saturation levels of actuator input $u(t)$, respectively.

By defining saturation error $\Delta u(t) = u(t) - \tau(t)$, (6) can be rewritten as

$$m_l\phi_{tt}(R,t) = -T\phi_a(R,t) + u(t) - \Delta u(t) - \tau_f(Z) + \omega(t). \tag{7}$$

The control objective in the proposed control scheme involves the mitigation of the vibration in an FS in the presence of an input constraint, an actuator gain fault, and external disturbances. The following assumptions and lemmas are conducive to determine the stability of the closed-loop FS system.

Assumption 1: For the ideal plant (i.e., no input saturation, no actuator fault, and no disturbance), the existence of a control scheme is referred to the literature. For the considered plant in (1)–(3), the existence of a control scheme is initially assumed. However, by assuring the negative semidefiniteness of the time derivative of the Lyapunov function candidate to be developed, the blow up of the state upon actuator fault and disturbance will be eliminated.

Assumption 2 [30], [31]: Distributed disturbance $d(a,t)$ $\forall a \in (0,R)$ satisfies $|d(a,t)| \leq \bar{F}$ for constant $\bar{F} > 0$.

Assumption 3 [26]: Boundary disturbance $\omega(t)$ satisfies

$$\omega(t), \dot{\omega}(t), \ddot{\omega}(t) \in \mathcal{L}_\infty.$$

Assumption 4 [23]: Input constraint error $\Delta u(t)$ is bounded and, hence, there exists a positive constant μ such that $|\Delta u(t)| \leq \mu$.

Lemma 1 [32]–[34]: Let $\kappa(0,t) = 0$. There exists constant $\pi_o > 0$ such that

$$\kappa^2(a,t) \leq R\int_0^R \kappa_a^2(a,t)\mathrm{da}$$

$$\kappa_1(a,t)\kappa_2(a,t) \leq \pi_o\kappa_1^2(a,t) + \frac{1}{\pi_o}\kappa_2^2(a,t) \quad \forall a \in [0,R]$$

with $\kappa(a,t) \in \mathcal{C}^1$ being a first-order continuous differentiable function with respect to a and $\kappa_1(a,t), \kappa_2(a,t) \in \mathcal{R}$.

Lemma 2 [28]: For a first-order continuous differentiable function $\Omega(\xi(t)) > 0$, there are constants $\delta_1 > 0$ and $\delta_2 > 0$ such that: 1) the initial value of $\Omega(\xi(t))$ is bounded; 2) $\Psi_1(\|\xi(t)\|) \leq \Omega(\xi(t)) \leq \Psi_2(\|\xi(t)\|)$ with $\Psi_1(\|\xi(t)\|)$ and $\Psi_2(\|\xi(t)\|)$ are class-$\mathfrak{K}$ functions; and 3) $\dot{\Omega}(\xi(t)) \leq -\delta_1\Omega(\xi(t)) + \delta_2$ for $\xi(t)$ to be uniformly bounded.

Remark 1: We only consider the transverse deflection of the FS. Although the transverse model can describe the dynamic characteristics of the FS to some extent, a 3-D model would more accurately describe the FS vibration. We will explore a 3-D model for the FS in future work.

Remark 2: The form of the nonlinear fault control rate coefficient $ø(\Pi) = 1 - (1/[1+\alpha e^{-\iota(\Pi)}])$ given by (5) can ensure that $ø(\Pi) \in [0,1)$. $ø(\Pi)$ belongs to $[0,1)$ and $ø(\Pi) = 0$ when $\alpha = 0$. Hence, there exists no unknown nonlinear actuator gain fault in this case. In addition, the actuator is almost at a complete failure when $\alpha e^{-\iota(\Pi)} \to +\infty$. Moreover, as actuator fault interacts with the system states, the form of the nonlinear gain fault given by (4) is more reasonable [19].

Remark 3: Controllability is defined as the precondition of controller design for an FS justifying Assumption 1. If the external disturbance, its derivative, and its second-order derivative are unbounded, the system may become uncontrollable, conflicting with Assumption 1. Thus, Assumption 3 is also feasible. Similar assumptions have been made in [35]–[37]. In Assumption 2, the energy of the distributed disturbance exerted on the FS is limited because the FS is controllable. In addition, the energy provided by the actuator is finite given its physical limitations, indicating the suitability of Assumption 4.

III. NNCDO-Based ANNBFTC Design and Stability Analysis

The proposed RBFNN approximates the unknown function, $\tau_f(Z)$, in (7), which is related to the system state variables. Then, the NNCDO handles a composite disturbance, which includes an unknown boundary disturbance and the approximation error of neural networks. Furthermore, an auxiliary system compensates for the input constraint error $\Delta u(t)$. Finally, by considering the NNCDO and auxiliary system, we derive an ANNBFTC mechanism by using the Lyapunov stability analysis. Under the proposed control scheme, the closed-loop FS system will be shown to be uniformly ultimately bounded.

A. NNCDO Design

To achieve high FS control performance, we first define $\tau_{nf}(Z) = \lambda\tau_f(Z)$ for a constant, $\lambda > 0$. Then, to approximate unknown function $\tau_{nf}(Z)$, the RBFNN is applied [11]

$$\tau_{nf}(Z) = \Delta^{*T}\Upsilon(Z) + \iota \tag{8}$$

where $\Delta^* \in \mathcal{R}^n$ denotes the optimal weight vector, and $\Upsilon(Z) = [\Upsilon_1(Z), \Upsilon_2(Z), \ldots, \Upsilon_n(Z)]^T \in \mathcal{R}^n$ is the basis function with $\Upsilon_j(Z) = \exp([-(Z-\kappa_{1j})^T(Z-\kappa_{1j})]/\kappa_{2j}^2)$, κ_{1j} and κ_{2j}, respectively, express the center and width of the neural cell for the jth hidden layer, $j = 1, 2, \ldots, n$. $Z =$

$[Z_1, Z_2, \ldots, Z_m]^T \in \mathcal{R}^m$ is the RBFNN input, and ι is the optimal approximation error satisfying $|\iota| \le \epsilon$ for constant $\epsilon > 0$.

Given an unknown actuator gain fault, the form of a disturbance observer cannot be employed in [26]. Let $D(t) = \omega(t) - (\iota/\lambda)$ be a composite disturbance. From Assumption 3, the second-order derivative of $D(t)$ is bounded, that is, $|\ddot{D}(t)| \le \delta$ for constant $\delta > 0$. To compensate for the effects of the composite disturbance $D(t)$ on the FS, we formulate the NNCDO as

$$\begin{aligned}\hat{D}(t) &= \zeta_1(t) + \varsigma_1 m_l \phi_t(R,t)\\ \dot{\zeta}_1(t) &= -\varsigma_1\left\{-T\phi_a(R,t) + \tau(t) - \frac{\hat{\Delta}^T(t)\Upsilon(Z)}{\lambda} + \hat{D}(t)\right\}\\ &\quad + \hat{\dot{D}}(t)\\ \hat{\dot{D}}(t) &= \zeta_2(t) + \varsigma_2 m_l \phi_t(R,t)\\ \dot{\zeta}_2(t) &= -\varsigma_2\left\{-T\phi_a(R,t) + \tau(t) - \frac{\hat{\Delta}^T(t)\Upsilon(Z)}{\lambda} + \hat{D}(t)\right\}\end{aligned} \quad (9)$$

where $\zeta_1(t)$ and $\zeta_2(t)$ are intermediate variables, $\hat{\Delta}^T(t)\Upsilon(Z)$ denotes the approximation function of $\tau_{nf}(Z)$, $\hat{D}(t)$ and $\hat{\dot{D}}(t)$ represent the estimates of $D(t)$ and $\dot{D}(t)$, respectively, and $\varsigma_1 > 0$ and $\varsigma_2 > 0$ are constants.

According to $\Upsilon_j(Z) = \exp([-(Z-\kappa_{1j})^T(Z-\kappa_{1j})]/\kappa_{2j}^2)$, we find that $\|\Upsilon(Z)\|$ is bounded, that is, $\|\Upsilon(Z)\| \le \vartheta$ for constant $\vartheta > 0$.

By defining $\tilde{D}(t) = D(t) - \hat{D}(t)$ and $\tilde{\dot{D}}(t) = \dot{D}(t) - \hat{\dot{D}}(t)$, letting $\tilde{Q}(t) = [\tilde{D}(t), \tilde{\dot{D}}(t)]^T$, and invoking (7)–(9), Lemma 1, $|\ddot{D}(t)| \le \delta$, and $\|\Upsilon(Z)\| \le \vartheta$, we obtain

$$\begin{aligned}\tilde{Q}^T(t)\dot{\tilde{Q}}(t) \le \tilde{Q}^T(t)&\left\{K_1 + \left(\frac{(\varsigma_1\varrho_1 + \varsigma_2\varrho_2)}{\lambda^2}\vartheta^2 + \varrho_3\right)I_2\right\}\\ &\times \tilde{Q}(t) + \left(\frac{\varsigma_1}{\varrho_1} + \frac{\varsigma_2}{\varrho_2}\right)\tilde{\Delta}^T(t)\tilde{\Delta}(t) + \frac{1}{\varrho_3}\delta^2\end{aligned} \quad (10)$$

where $K_1 = \begin{bmatrix} -\varsigma_1 & 1 \\ -\varsigma_2 & 0 \end{bmatrix}$, $I_2 = \mathrm{diag}\{1, 1\}$, $\varrho_1 > 0$, $\varrho_2 > 0$, and $\varrho_3 > 0$ are constants.

Remark 4: As $\Upsilon_j(Z) = \exp([-(Z-\kappa_{1j})^T(Z-\kappa_{1j})]/\kappa_{2j}^2)$, $|\Upsilon_j(Z)| \le 1$, $j = 1, 2, \ldots, n$. Hence, upper bound $\vartheta > 0$ of $\|\Upsilon(Z)\|$ related to n can be achieved. Moreover, ϑ can be chosen as $\sqrt{n}$.

Remark 5: We approximate unknown function $\tau_{nf}(Z) = \lambda\tau_f(Z)$ instead of $\tau_f(Z)$ to add an adjusting parameter that easily guarantees the stability of the closed-loop FS system.

B. Controller Design and Stability Analysis

Based on the proposed NNCDO described by (9), the design principle of ANNBFTC is illustrated in Fig. 2. The controller design is detailed as follows.

To handle actuator saturation, we used an auxiliary system given by

$$\dot{\eta}(t) = \frac{-l_1\eta(t) + \Delta u(t) + T[\phi_a(R,t) + \phi_t(R,t)]}{m_l} \quad (11)$$

for constant $l_1 > 0$.

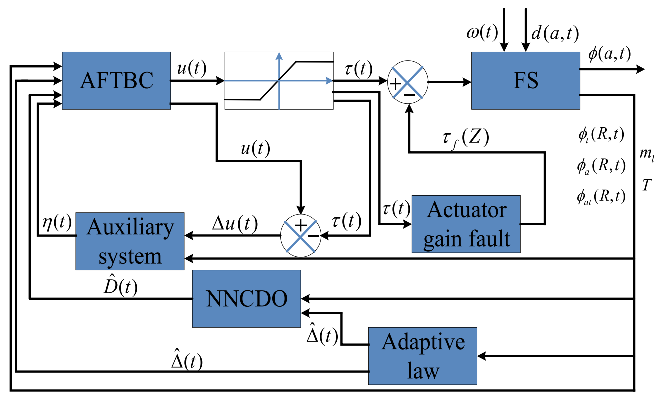

Fig. 2. Block diagram of the proposed ANNBFTC.

Based on the proposed NNCDO and auxiliary system, we design the following ANNBFTC scheme to mitigate the FS vibration:

$$\dot{\hat{\Delta}}(t) = -g_1\left\{g_2\hat{\Delta}(t) + [\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\frac{\Upsilon(Z)}{\lambda}\right\} \quad (12)$$

$$\begin{aligned}u(t) &= -T\phi_t(R,t) - m_l\phi_{at}(R,t) + l_1\eta(t)\\ &\quad - l_2[\eta(t) + \phi_t(R,t) + \phi_a(R,t)] - \hat{D}(t)\\ &\quad + \frac{\hat{\Delta}^T(t)\Upsilon(Z)}{\lambda}\end{aligned} \quad (13)$$

where g_1, g_2, and l_2 are positive constants and

$$Z = [\phi_t(R,t), \phi_a(R,t), \phi_{at}(R,t), \iota(\Pi)]^T. \quad (14)$$

To investigate the stability of the closed-loop system, we choose the following Lyapunov-candidate-function:

$$\Gamma(t) = \Gamma_1(t) + \Gamma_2(t) + \Gamma_3(t) \quad (15)$$

where

$$\begin{aligned}\Gamma_1(t) &= \frac{\gamma_1 m_l}{2}\eta^2(t) + \frac{\gamma_1 m_l}{2}[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]^2\\ &\quad + \frac{\gamma_1}{2g_1}\tilde{\Delta}^T(t)\tilde{\Delta}(t) + \frac{\gamma_1}{2}\tilde{Q}^T(t)\tilde{Q}(t)\end{aligned} \quad (16)$$

$$\Gamma_2(t) = \frac{\gamma_1\varpi}{2}\int_0^R \phi_t^2(a,t)da + \frac{\gamma_1 T}{2}\int_0^R \phi_a^2(a,t)da \quad (17)$$

$$\Gamma_3(t) = \gamma_2\varpi\int_0^R a\phi_t(a,t)\phi_a(a,t)da \quad (18)$$

with constants $\gamma_1, \gamma_2 > 0$ satisfying $([\min\{\gamma_1\varpi, \gamma_1 T\}]/\varpi R) - \gamma_2 > 0$ and $\tilde{\Delta}(t) = \Delta^* - \hat{\Delta}(t)$.

According to Lemma 2, the positive definiteness of $\Gamma(t)$ should be verified. According to Lemma 1, the cross-term $\Gamma_3(t)$ satisfies the following inequality:

$$|\Gamma_3(t)| \le \frac{\gamma_2\varpi R}{2}\int_0^R\left[\phi_t^2(a,t) + \phi_a^2(a,t)\right]da. \quad (19)$$

Then, considering $([\min\{\gamma_1\varpi, \gamma_1 T\}]/\varpi R) - \gamma_2 > 0$, (17), and (19), we obtain $\Gamma_2(t) + \Gamma_3(t) \ge (1 - [\gamma_2\varpi R]/[\min\{\gamma_1\varpi, \gamma_1 T\}])\Gamma_1(t) > 0$. Thus, $\Gamma(t)$ is a positive-definite function that can be chosen as a Lyapunov function. In the sequel, the stability of the closed-loop system is analyzed.

By invoking (11), Assumption 4, and Lemma 1, we have

$$\begin{aligned}\gamma_1 m_l \eta(t)\dot{\eta}(t) &= -l_1\gamma_1\eta^2(t) + \gamma_1\eta(t)\Delta u(t)\\ &\quad + \gamma_1 T\eta(t)[\phi_a(R,t) + \phi_t(R,t)]\\ &\le -(l_1 - \varrho_4)\gamma_1\eta^2(t) + \frac{\gamma_1}{\varrho_4}\mu^2\\ &\quad + \gamma_1 T\eta(t)[\phi_a(R,t) + \phi_t(R,t)] \qquad (20)\end{aligned}$$

where ϱ_4 is a positive constant.

By considering (7), (8), (11), (13), and Lemma 1, we obtain

$$\begin{aligned}&\gamma_1 m_l[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\\ &\quad\times [\dot{\eta}(t) + \phi_{tt}(R,t) + \phi_{at}(R,t)]\\ &= \gamma_1[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\\ &\quad\times\Big\{-l_1\eta(t) + \Delta u(t)\\ &\qquad + T[\phi_a(R,t) + \phi_t(R,t)] - T\phi_a(R,t) + u(t)\\ &\qquad - \Delta u(t) - \tau_f(Z) + \omega(t) + m_l\phi_{at}(R,t)\Big\}\\ &\le -\gamma_1(l_2 - \varrho_5)[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]^2\\ &\quad - \frac{\gamma_1}{\lambda}[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\tilde{\Delta}^T(t)\Upsilon(Z)\\ &\quad + \frac{\gamma_1}{\varrho_5}\tilde{Q}^T(t)\tilde{Q}(t) \qquad (21)\end{aligned}$$

where ϱ_5 is a positive constant.

From (10) and (12), we obtain

$$\begin{aligned}&\frac{\gamma_1}{g_1}\tilde{\Delta}^T(t)\dot{\tilde{\Delta}}(t) + \gamma_1\tilde{Q}^T(t)\dot{\tilde{Q}}(t)\\ &\le \gamma_1\tilde{\Delta}^T(t)\Big\{g_2\hat{\Delta}(t) + [\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\frac{\Upsilon(Z)}{\lambda}\Big\}\\ &\quad + \gamma_1\tilde{Q}^T(t)\Big\{K_1 + \Big(\frac{\big(\varsigma_1\varrho_1 + \varsigma_2\varrho_2\big)}{\lambda^2}\vartheta^2 + \varrho_3\Big)I_2\Big\}\tilde{Q}(t)\\ &\quad + \Big(\frac{\varsigma_1}{\varrho_1} + \frac{\varsigma_2}{\varrho_2}\Big)\tilde{\Delta}^T(t)\tilde{\Delta}(t) + \frac{1}{\varrho_3}\delta^2\\ &\le -\gamma_1\Big\{g_2\Big(1 - \frac{1}{\varrho_6}\Big) - \frac{\varsigma_1}{\varrho_1} - \frac{\varsigma_2}{\varrho_2}\Big\}\tilde{\Delta}^T(t)\tilde{\Delta}(t)\\ &\quad + \gamma_1\tilde{Q}^T(t)\Big\{K_1 + \Big(\frac{(\varsigma_1\varrho_1 + \varsigma_2\varrho_2)}{\lambda^2}\vartheta^2 + \varrho_3\Big)I_2\Big\}\tilde{Q}(t)\\ &\quad + \frac{\gamma_1}{\lambda}[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]\tilde{\Delta}^T(t)\Upsilon(Z)\\ &\quad + \gamma_1 g_2\varrho_6\Delta^{*T}\Delta^* + \frac{\gamma_1}{\varrho_3}\delta^2 \qquad (22)\end{aligned}$$

where ϱ_6 is a positive constant.

By considering (16) and (20)–(22), the derivative of $\Gamma_1(t)$ is given by

$$\begin{aligned}\dot{\Gamma}_1(t) &\le -(l_1 - \varrho_4)\gamma_1\eta^2(t) - \gamma_1(l_2 - \varrho_5)\\ &\quad\times [\eta(t) + \phi_t(R,t) + \phi_a(R,t)]^2\\ &\quad - \gamma_1\Big\{g_2\Big(1 - \frac{1}{\varrho_6}\Big) - \frac{\varsigma_1}{\varrho_1} - \frac{\varsigma_2}{\varrho_2}\Big\}\tilde{\Delta}^T(t)\tilde{\Delta}(t)\\ &\quad + \gamma_1\tilde{Q}^T(t)\Big\{K_1 + \Big(\frac{(\varsigma_1\varrho_1 + \varsigma_2\varrho_2)}{\lambda^2}\vartheta^2 + \varrho_3\\ &\qquad + \frac{1}{\varrho_5}\Big)I_2\Big\}\tilde{Q}(t)\\ &\quad + \frac{\gamma_1}{\varrho_4}\mu^2 + \gamma_1 g_2\varrho_6\Delta^{*T}\Delta^* + \frac{\gamma_1}{\varrho_3}\delta^2\\ &\quad + \gamma_1 T\eta(t)[\phi_a(R,t) + \phi_t(R,t)]. \qquad (23)\end{aligned}$$

Considering (1), (2), (17), Assumption 2, and Lemma 1, we obtain

$$\begin{aligned}\dot{\Gamma}_2(t) &= \gamma_1\int_0^R \phi_t(a,t)\Big\{T\phi_{aa}(a,t) + d(a,t)\Big\}da\\ &\quad + \gamma_1 T\int_0^R \phi_a(a,t)\phi_{at}(a,t)da\\ &= \gamma_1 T\phi_t(R,t)\phi_a(R,t) + \gamma_1\int_0^R \phi_t(a,t)d(a,t)da\\ &\le \gamma_1 T\phi_t(R,t)\phi_a(R,t) + \frac{\gamma_1}{\varrho_7}\int_0^R \phi_t^2(a,t)da\\ &\quad + \gamma_1\varrho_7 R\bar{F}^2 \qquad (24)\end{aligned}$$

with $\varrho_7 > 0$ being a constant.

By using $\phi_t(R,t)\phi_a(R,t) = [([\eta(t) + \phi_t(R,t) + \phi_a(R,t)]^2)/2] - ([\eta^2(t)]/2) - ([\phi_t^2(R,t)]/2) - ([\phi_a^2(R,t)]/2) - \eta(t)\phi_t(R,t) - \eta(t)\phi_a(R,t)$, we obtain

$$\begin{aligned}\dot{\Gamma}_2(t) &\le \frac{\gamma_1 T}{2}[\eta(t) + \phi_t(R,t) + \phi_a(R,t)]^2 - \frac{\gamma_1 T}{2}\eta^2(t)\\ &\quad - \frac{\gamma_1 T}{2}\phi_t^2(R,t) - \frac{\gamma_1 T}{2}\phi_a^2(R,t) - \gamma_1 T\eta(t)\phi_t(R,t)\\ &\quad - \gamma_1 T\eta(t)\phi_a(R,t) + \frac{\gamma_1}{\varrho_7}\int_0^R \phi_t^2(a,t)da\\ &\quad + \gamma_1\varrho_7 R\bar{F}^2. \qquad (25)\end{aligned}$$

According to (1), (18), Assumption 2, and Lemma 1, we obtain

$$\begin{aligned}\dot{\Gamma}_3(t) &= \gamma_2\int_0^R a\phi_a(a,t)\Big\{T\phi_{aa}(a,t) + d(a,t)\Big\}da\\ &\quad + \gamma_2\varpi\int_0^R a\phi_t(a,t)\phi_{at}(a,t)da\\ &\le -\frac{\gamma_2\varpi}{2}\int_0^R \phi_t^2(a,t)da + \frac{\gamma_2\varpi R}{2}\phi_t^2(R,t)\\ &\quad - \Big(\frac{\gamma_2 T}{2} - \frac{\gamma_2 R}{\varrho_8}\Big)\int_0^R \phi_a^2(a,t)da\\ &\quad + \frac{\gamma_2 TR}{2}\phi_a^2(R,t) + \gamma_2 R^2\varrho_8\bar{F}^2 \qquad (26)\end{aligned}$$

with $\varrho_8 > 0$ being a constant.

By combining (15), (23), (25), and (26), we obtain

$$\begin{aligned}\dot{\Gamma}(t) &\le -(l_1 - \varrho_4)\gamma_1\eta^2(t) - \gamma_1\Big(l_2 - \varrho_5 - \frac{T}{2}\Big)\\ &\quad\times [\phi_t(R,t) + \eta(t) + \phi_a(R,t)]^2\\ &\quad - \gamma_1\Big\{g_2\Big(1 - \frac{1}{\varrho_6}\Big) - \frac{\varsigma_1}{\varrho_1} - \frac{\varsigma_2}{\varrho_2}\Big\}\tilde{\Delta}^T(t)\tilde{\Delta}(t)\\ &\quad + \gamma_1\tilde{Q}^T(t)\Big\{K_1 + \Big(\varrho_3 + \frac{1}{\varrho_5} + \frac{(\varsigma_1\varrho_1 + \varsigma_2\varrho_2)}{\lambda^2}\vartheta^2\Big)I_2\Big\}\\ &\quad\times \tilde{Q}(t) - \Big\{\frac{\gamma_2\varpi}{2} - \frac{\gamma_1}{\varrho_7}\Big\}\\ &\quad\times \int_0^R \phi_t^2(a,t)da - \Big(\frac{\gamma_2 T}{2} - \frac{\gamma_2 R}{\varrho_8}\Big)\int_0^R \phi_a^2(a,t)da\\ &\quad + \frac{\gamma_1}{\varrho_4}\mu^2 + \gamma_1 g_2\varrho_6\Delta^{*T}\Delta^* + (\gamma_1\varrho_7 R + \gamma_2 R^2\varrho_8)\bar{F}^2\end{aligned}$$

$$+\frac{\gamma_1}{\varrho_3}\delta^2-\left\{\frac{\gamma_1 T}{2}-\frac{\gamma_2\varpi R}{2}\right\}\phi_t^2(R,t)-\left\{\frac{\gamma_1 T}{2}-\frac{\gamma_2 TR}{2}\right\}\phi_a^2(R,t). \tag{27}$$

When $(\gamma_1 T/2)-(\gamma_2\varpi R/2)\geq 0$, $(\gamma_1 T/2)-(\gamma_2 TR/2)\geq 0$, $l_1-\varrho_4>0$, $l_2-\varrho_5-(T/2)>0$, $g_2(1-[1/\varrho_6])-(\varsigma_1/\varrho_1)-(\varsigma_2/\varrho_2)>0$, $\varsigma_1-\varrho_3-(1/\varrho_5)-([(\varsigma_1\varrho_1+\varsigma_2\varrho_2)]/\lambda^2)\vartheta^2>0$, $-\{\varsigma_1-\varrho_3-(1/\varrho_5)-([(\varsigma_1\varrho_1+\varsigma_2\varrho_2)]/\lambda^2)\vartheta^2\}\{\varrho_3+(1/\varrho_5)+([(\varsigma_1\varrho_1+\varsigma_2\varrho_2)]/\lambda^2)\vartheta^2\}+\varsigma_2>0$, $(\gamma_2\varpi/2)-(\gamma_1/\varrho_7)>0$, and $(\gamma_2 T/2)-(\gamma_2 R/\varrho_8)>0$, we obtain

$$\begin{aligned}\dot{\Gamma}(t)\leq&-(l_1-\varrho_4)\gamma_1\eta^2(t)-\gamma_1\left(l_2-\varrho_5-\frac{T}{2}\right)\\&\times[\phi_t(R,t)+\eta(t)+\phi_a(R,t)]^2\\&-\gamma_1\left\{g_2\left(1-\frac{1}{\varrho_6}\right)-\frac{\varsigma_1}{\varrho_1}-\frac{\varsigma_2}{\varrho_2}\right\}\tilde{\Delta}^T(t)\tilde{\Delta}(t)\\&+\gamma_1\tilde{Q}^T(t)\left\{K_1+\left(\varrho_3+\frac{1}{\varrho_5}+\frac{(\varsigma_1\varrho_1+\varsigma_2\varrho_2)}{\lambda^2}\vartheta^2\right)I_2\right\}\\&\times\tilde{Q}(t)-\left\{\frac{\gamma_2\varpi}{2}-\frac{\gamma_1}{\varrho_7}\right\}\\&\times\int_0^R\phi_t^2(a,t)da-\left(\frac{\gamma_2 T}{2}-\frac{\gamma_2 R}{\varrho_8}\right)\int_0^R\phi_a^2(a,t)da\\&+\frac{\gamma_1}{\varrho_4}\mu^2+\gamma_1 g_2\varrho_6\Delta^{*T}\Delta^*+(\gamma_1\varrho_7 R+\gamma_2 R^2\varrho_8)\bar{F}^2\\&+\frac{\gamma_1}{\varrho_3}\delta^2\\\leq&-\frac{\chi_1}{1+\frac{\gamma_2\varpi R}{\min\{\gamma_1\varpi,\gamma_1 T\}}}\Gamma(t)+\chi_2\end{aligned} \tag{28}$$

with

$$\begin{aligned}\chi_1=\min\Bigg\{&\frac{2(l_1-\varrho_4)}{m_l},\ \frac{2l_2-2\varrho_5-T}{m_l}\\&\times 2g_1\left\{g_2\left(1-\frac{1}{\varrho_6}\right)-\frac{\varsigma_1}{\varrho_1}-\frac{\varsigma_2}{\varrho_2}\right\}\\&\times 2\lambda_{\min}\left\{-K_1-\left(\frac{(\varsigma_1\varrho_1+\varsigma_2\varrho_2)}{\lambda^2}\vartheta^2+\varrho_3+\frac{1}{\varrho_5}\right)I_2\right\}\\&\times\frac{\gamma_2}{\gamma_1}-\frac{2}{\varpi\varrho_7},\ \frac{\gamma_2}{\gamma_1}-\frac{2\gamma_2 R}{\gamma_1\varrho_8 T}\Bigg\}\\\chi_2=&\frac{\gamma_1}{\varrho_4}\mu^2+\gamma_1 g_2\varrho_6\Delta^{*T}\Delta^*+\left(\gamma_1\varrho_7 R+\gamma_2 R^2\varrho_8\right)\bar{F}^2\\&+\frac{\gamma_1}{\varrho_3}\delta^2.\end{aligned}$$

Theorem 1: Let the FS given by (1), (2), and (7) satisfy Assumptions 2–4. In addition, the NNCDO and auxiliary system are designed as described in (9) and (11), respectively. Based on the developed NNCDO and auxiliary system, we derived the ANNBFTC scheme described in (12) and (13). Let $\chi_3=(\chi_1/[1+(\gamma_2\varpi R/[\min\{\gamma_1\varpi,\gamma_1 T\}])])$ and $\chi_4=1-(\gamma_2\varpi R/[\min\{\gamma_1\varpi,\gamma_1 T\}])$. Under the introduced ANNBFTC scheme, if the initial conditions of the FS are bounded, vibration amplitude $\phi(a,t)$ satisfies the following conditions.

1) $\phi(a,t)$ uniformly converges to a compact set $\Xi_1=\{\phi(a,t)||\phi(a,t)|\leq\sqrt{[2R\Gamma(0)/\gamma_1 T\chi_4]e^{-\chi_3 t}+(2R\chi_2/[\gamma_1 T\chi_3\chi_4])}\}$.
2) $\phi(a,t)$ ultimately remains in the compact set $\Xi_2=\{\phi(a,t)||\phi(a,t)|\leq\sqrt{(2R\chi_2/[\gamma_1 T\chi_3\chi_4])}\}$.

Proof: By using $\chi_3=(\chi_1/[1+(\gamma_2\varpi R/[\min\{\gamma_1\varpi,\gamma_1 T\}])])$, (28) can be rewritten as

$$\dot{\Gamma}(t)\leq-\chi_3\Gamma(t)+\chi_2. \tag{29}$$

Considering (29), we obtain

$$\dot{\Gamma}(t)e^{\chi_3 t}+\chi_3\Gamma(t)e^{\chi_3 t}\leq\chi_2 e^{\chi_3 t}. \tag{30}$$

Then, (30) can be represented as

$$\frac{d\left(\Gamma(t)e^{\chi_3 t}\right)}{dt}\leq\chi_2 e^{\chi_3 t}. \tag{31}$$

From (31), we obtain

$$\Gamma(t)\leq\left(\Gamma(0)-\frac{\chi_2}{\chi_3}\right)e^{-\chi_3 t}+\frac{\chi_2}{\chi_3}. \tag{32}$$

As $\chi_2>0$, $\chi_3>0$, and $\Gamma(0)\in\mathcal{L}_\infty$

$$\Gamma(t)\leq\Gamma(0)e^{-\chi_3 t}+\frac{\chi_2}{\chi_3}\in\mathcal{L}_\infty. \tag{33}$$

From (17) and Lemma 1, we obtain

$$\frac{\gamma_1 T}{2R}\phi^2(a,t)\leq\frac{\gamma_1 T}{2}\int_0^R\phi_a^2(a,t)da\leq\frac{\Gamma(t)}{\chi_4}. \tag{34}$$

By substituting (33) into (34), we obtain

$$|\phi(a,t)|\leq\sqrt{\frac{2R\Gamma(0)}{\gamma_1 T\chi_4}e^{-\chi_3 t}+\frac{2R\chi_2}{\gamma_1 T\chi_3\chi_4}} \tag{35}$$

that is, $\phi(a,t)$ uniformly converges to a compact set $\Xi_1=\{\phi(a,t)||\phi(a,t)|\leq\sqrt{(2R\Gamma(0)/\gamma_1 T\chi_4)e^{-\chi_3 t}+(2R\chi_2/\gamma_1 T\chi_3\chi_4)}\}$.

When $t\to\infty$, $\phi(a,t)$ ultimately remains in the compact set $\Xi_2=\{\phi(a,t)||\phi(a,t)|\leq\sqrt{(2R\chi_2/[\gamma_1 T\chi_3\chi_4])}\}$.

Remark 6: The auxiliary system described in (11) aims to provide a feedforward approach to compensate for input saturation error $\Delta u(t)$. Although the dynamics of $\eta(t)$ include $\Delta u(t)$, we only use auxiliary variable $\eta(t)$ in controller $u(t)$ instead of $\dot{\eta}(t)$. Thus, the forms of the auxiliary system and controller $u(t)$ do not result in an algebraic loop problem.

Remark 7: In the ANNBFTC scheme, $\phi(R,t)$ and $\phi_t(R,t)$ can be obtained using a laser displacement sensor and velocity sensor, respectively, while $\phi_a(R,t)$ is obtained using an inclinometer. Variable $\phi_{at}(R,t)$ is generally derived using a backward difference with the information of $\phi_a(R,t)$ [17].

Remark 8: The design parameters of the control law and disturbance observer in Section III can be chosen according to the following regulation: ς_1, ς_2, l_1, l_2, λ, and g_2 are related to the system model parameters and the inequalities $l_1-\varrho_4>0$, $l_2-\varrho_5-(T/2)>0$, $g_2(1-[1/\varrho_6])-[\varsigma_1/\varrho_1]-[\varsigma_2/\varrho_2]>0$, $\varsigma_1-\varrho_3-(1/\varrho_5)-([(\varsigma_1\varrho_1+\varsigma_2\varrho_2)]/\lambda^2)\vartheta^2>0$, and $-\{\varsigma_1-\varrho_3-(1/\varrho_5)-([(\varsigma_1\varrho_1+\varsigma_2\varrho_2)]/\lambda^2)\vartheta^2\}\{\varrho_3+(1/\varrho_5)+[(\varsigma_1\varrho_1+\varsigma_2\varrho_2)/\lambda^2]\vartheta^2\}+\varsigma_2>0$, that is, the designs for ς_1, ς_2, l_1, l_2, λ, and g_2 are based on the completeness of the stability conditions. Moreover, design parameter g_1 is selected based on experience.

Remark 9: $\Gamma_1(t)$ contains the total kinetic energy of load and auxiliary variables. The objective of selecting this format is to ensure that system variables $\phi_t(R,t)$ and $\phi_a(R,t)$, and auxiliary variables $\eta(t)$, $\tilde{\Delta}(t)$, and $\tilde{Q}(t)$ can converge to a small compact set. $\Gamma_2(t)$ contains the total kinetic energy and total potential energy of the string, which also constrain system variables $\phi_t(a,t)$ and $\phi_a(a,t)$ from converging to a small compact set. $\Gamma_3(t)$ is an auxiliary function, which guarantees that the closed-loop system of the FS is uniformly ultimately bounded.

IV. Simulation Results

In this section, we consider the FS system described by (1), (2), and (7). By using the proposed NNCDO described in (9) and the auxiliary system (11), we verify the control performance of the proposed ANNBFTC scheme described in (12) and (13) through a numerical simulation of the FS. We set the initial conditions of the FS to $\phi(a,0) = (a/25R)$ and $\dot{\phi}(a,0) = 0$, and its parameters to $R = 1.0$ m, $\varpi = 0.15$ kg/m, $T = 12$ N, and $m_l = 1.0$ kg. In addition, disturbance terms were given by $d(a,t) = (1.0 + 0.4\sin(0.1\pi t) + 0.2\sin(0.2\pi t) + 0.1\sin(0.4\pi t))a/100$ and $\omega(t) = 1.0 + 0.1\sin(0.1t) + 0.3\sin(0.3t) + 0.5\sin(0.5t)$, and the upper and lower control constraint levels were defined as $\tau_u = 1.0$ and $\tau_l = -1.8$, respectively. Moreover, a nonlinear actuator gain fault was applied at $T_f = 4s$, with the following nonlinear fault control rate coefficient [19]:

$$\o(\Pi) = 1 - 1/\left(1 + e^{-\left(2+1.4\sqrt{|\cos(\phi(R,t)\dot{\phi}(R,t))|}\right)}\right) \quad (36)$$

where

$$\Pi = \left[\phi(R,t), \dot{\phi}(R,t)\right]^T. \quad (37)$$

To compensate for composite disturbance $D(t)$, we set the NNCDO parameters to $\varsigma_1 = 10$, $\varsigma_2 = 10$, and $\lambda = 50$. The input constraint was handled by setting $l_1 = 10$ in the auxiliary system. To mitigate the FS vibration, we set the ANNBFTC parameters to $g_1 = 10$, $g_2 = 10$, and $l_2 = 20$.

The simulated vibration amplitude $\phi(a,t)$ of the FS without control is shown in Fig. 3. Considering the ANNBFTC scheme given by (12) and (13), $\phi(a,t)$ is obtained as shown in Fig. 4. The comparison of Figs. 3 and 4 shows that the proposed ANNBFTC scheme enhances the robustness against vibration of the closed-loop FS system. Amplitudes $\phi(a,t)$ at $a = R$ and $a = R/2$ with ANNBFTC are shown in Fig. 5. A small overshoot observed after $t = 4$ s was caused by the nonlinear actuator gain fault. Nevertheless, the FS vibration returns close to 0 within a short period. Thus, the proposed control method effectively handles actuator gain faults. Moreover, the residual vibration is very small, as shown by the smooth curves in Fig. 5. Fig. 6 shows disturbance $\omega(t)$ and estimated composite disturbance $\hat{D}(t)$ obtained from the proposed NNCDO. The NNCDO quickly estimates the unknown external disturbance before the fault at $T_f = 4$ s. The effect of the unknown actuator gain fault is then compensated using the NNCDO. In addition, as the composite disturbance is described as $D(t) = (\iota/\lambda) + \omega(t)$, the value of the neural-network approximation error ι is sufficiently small, as shown in $\omega(t)$ and $\hat{D}(t)$ of Fig. 6. Thus, a proper response against the unknown actuator gain fault is achieved by using the proposed control scheme. Fig. 7 shows the simulated boundary control $\tau(t)$, considering the actuator gain fault and input constraint, which are clearly visualized in the graph.

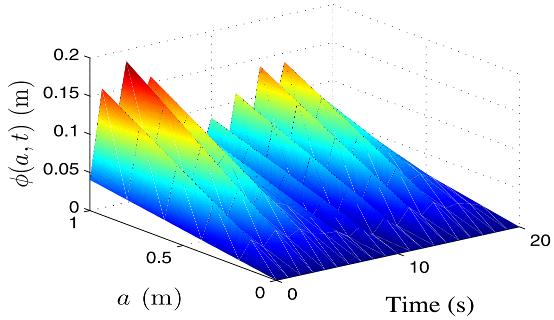

Fig. 3. Vibration response $\phi(a,t)$ of FS without control.

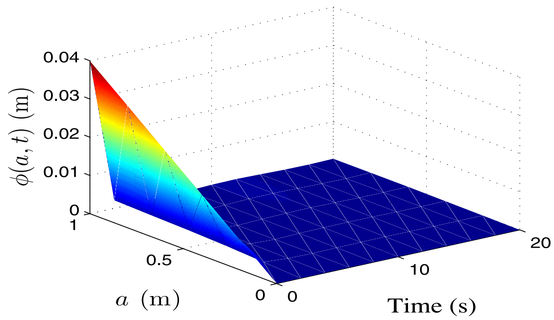

Fig. 4. Vibration response $\phi(a,t)$ of FS with the NNCDO-based ANNBFTC.

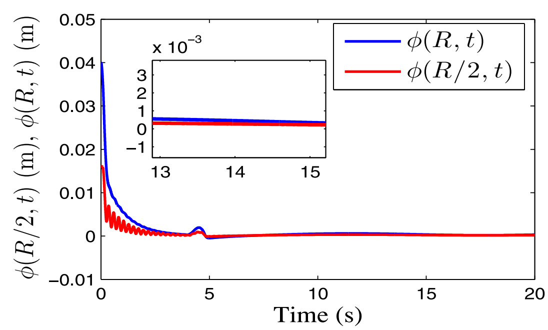

Fig. 5. Vibration response $\phi(a,t)$ at $a = R$ and $a = R/2$ with the NNCDO-based ANNBFTC.

To illustrate the superior performance of the proposed control scheme, we evaluated the following proportional–derivative (PD) controller:

$$u(t) = -l_3\phi(R,t) - l_4\dot{\phi}(R,t) \quad (38)$$

for constants $l_3 > 0$ and $l_4 > 0$.

The PD control parameters were set to $l_3 = 150$ and $l_4 = 200$, resulting in the responses for the closed-loop system shown in Figs. 8 and 9, respectively.

Figs. 4, 5, 8, and 9 show that the stability domain and chattering of vibration amplitude $\phi(a,t)$ are smaller by employing the proposed NNCDO-based ANNBFTC scheme. Thus, the proposed scheme effectively mitigates the FS vibration. Unlike

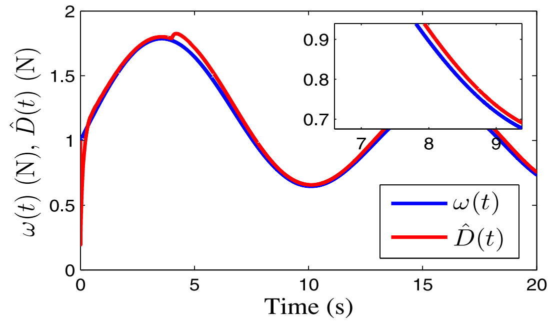

Fig. 6. Disturbance $\omega(t)$ and estimated composite disturbance $\hat{D}(t)$ under NNCDO (9).

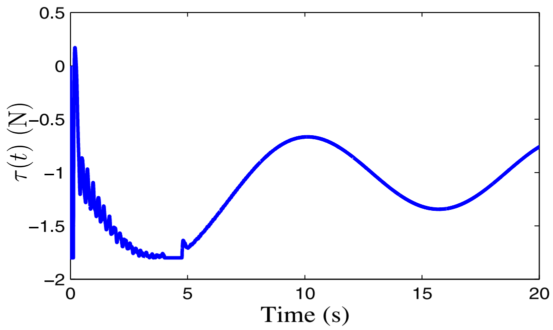

Fig. 7. Boundary control response $\tau(t)$.

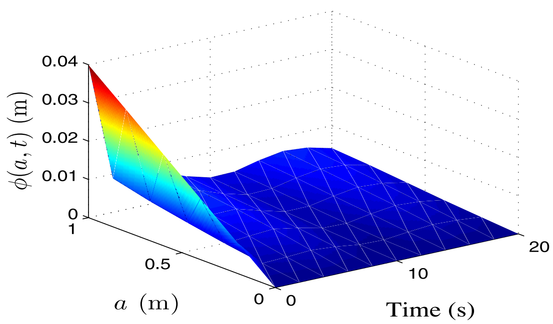

Fig. 8. Vibration response $\phi(a, t)$ of FS with PD control.

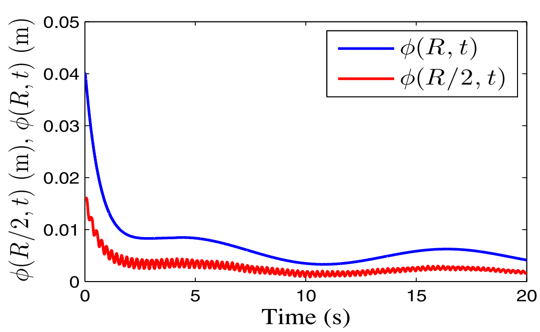

Fig. 9. Vibration response $\phi(a, t)$ at $a = R$ and $a = R/2$ with PD control.

the PD controller, higher control performance is achieved using the proposed NNCDO-based ANNBFTC.

Moreover, the conventional NNCDO-based control scheme was constructed to confirm the advantages of the proposed ANNBFTC through simulations. According to [28] and [29], the conventional NNCDO can be described as

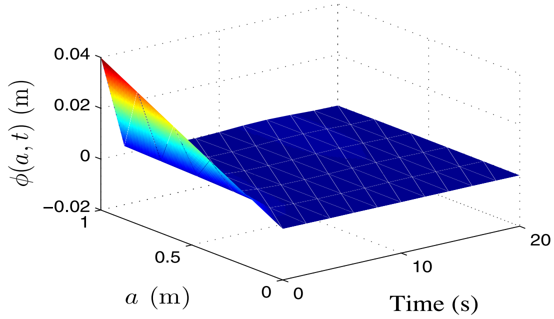

Fig. 10. Vibration response $\phi(a, t)$ of FS with the conventional NNCDO-based control scheme.

$$\hat{D}(t) = \zeta_3(t) + \varsigma_3 m_l \phi_t(R, t)$$
$$\dot{\zeta}_3(t) = -\varsigma_3\left\{-T\phi_a(R, t) + \tau(t) - \frac{\hat{\Delta}^T(t)\Upsilon(Z)}{\lambda} + \hat{D}(t)\right\} \tag{39}$$

for constant $\varsigma_3 > 0$.

Based on the designed conventional NNCDO (39) and auxiliary system (11), the conventional NNCDO-based control scheme can also be described by (12) and (13).

We set the design parameters in (39) to $\varsigma_3 = 10$ and $\lambda = 50$. The input constraint was handled using the auxiliary system with $l_1 = 10$. The design parameters of the conventional NNCDO-based control scheme were set to $g_1 = 10$, $g_2 = 10$, and $l_2 = 20$. Based on the derived NNCDO (39) and auxiliary system (11), the simulated dynamic responses of the FS are shown in Figs. 10−12 under the conventional NNCDO-based control scheme. Fig. 10 shows vibration $\phi(a, t)$ of the FS. Dynamic responses $\phi(R/2, t)$ and $\phi(R, t)$ are shown in Fig. 11. Fig. 12 shows the dynamic responses of disturbance $\omega(t)$ and estimated composite disturbance $\hat{D}(t)$ under the conventional NNCDO (39).

According to Figs. 4, 5, 10, and 11, nice control performance can be obtained using either the proposed NNCDO-based ANNBFTC or the conventional NNCDO-based control scheme before 4 s. However, the actuator gain fault occurs at 4 s, and the control performance using the proposed NNCDO-based ANNBFTC outperforms that using the conventional NNCDO scheme. Moreover, Figs. 6 and 12 show a suitable estimation using either of the observers before 4 s. However, after 4 s, when the actuator gain fault occurs, the estimation using the proposed NNCDO also outperforms that using the conventional NNCDO. Two aspects are noted regarding this estimation.

1) The estimation of the proposed NNCDO is faster.
2) The estimation accuracy of the proposed NNCDO is higher.

These results confirm that the poor disturbance rejection undermines the control performance when employing the conventional NNCDO.

Overall, the simulation results show that the proposed NNCDO-based control scheme effectively mitigates the vibration in the closed-loop FS system. In addition, it can enhance the robustness against input constraint, actuator gain fault, and

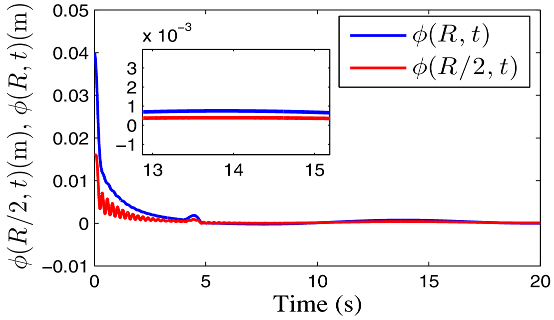

Fig. 11. Vibration response $\phi(a, t)$ at $a = R$ and $a = R/2$ with the conventional NNCDO-based control scheme.

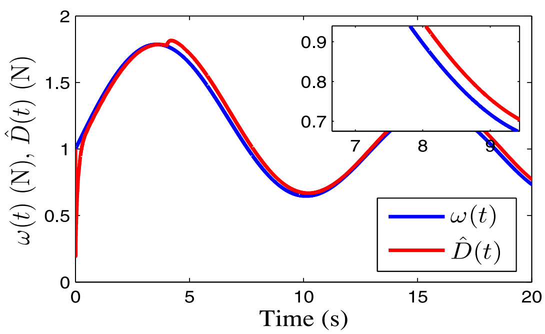

Fig. 12. Disturbance $\omega(t)$ and estimated composite disturbance $\hat{D}(t)$ under conventional NNCDO (39).

unknown external disturbances affecting the FS. Moreover, the proposed NNCDO outperforms the conventional NNCDO in terms of disturbance rejection.

V. Conclusion

We proposed a scheme to mitigate the vibration in FS systems, subject to actuator gain fault, input saturation, and external disturbances. To this end, we adopted an RBFNN and an auxiliary system to handle unknown actuator gain faults and input saturation, respectively. In addition, the proposed NNCDO estimated an unknown composite disturbance. Then, we derived an ANNBFTC scheme to determine the FS stability through the Lyapunov direct method. By using the proposed control scheme, we proved that the closed-loop system is uniformly bounded. In addition, the vibration of the FS is guaranteed to ultimately converge to a small compact set. Numerical simulations were performed to validate the high performance of the designed control scheme. Future studies will focus on flexible manipulators considering fuzzy control [38]–[41], state constraints [42]–[45], and learning-based approaches [46]–[51].

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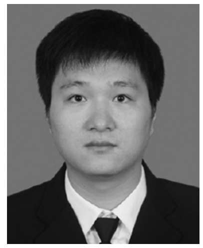

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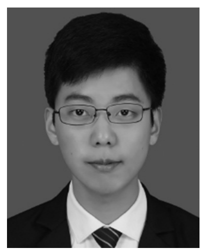

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