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An event-triggered extended dissipative control for Takagi-Sugeno fuzzy systems with time-varying delay via free-matrix-based integral inequality

Saravanan Shanmugam, Keum-Shik Hong*

School of Mechanical Engineering, Pusan National University, Busan 46241, Korea

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Abstract

This study focuses on an event-triggered control of Takagi-Sugeno (T–S) fuzzy systems. An extended dissipativity performance index is introduced, and several kinds of event-triggered control issues, such as passivity, H_∞, $L_2 - L_\infty$, and $(\mathcal{Q},\ \mathcal{S},\ \mathcal{R})$-dissipativity, are solved in a unified framework. By utilizing the free-matrix-based double integral inequality in the derivation of a Lyapunov–Krasovskii functional, sufficient conditions are provided to ensure that the considered closed-loop system is asymptotically stable. These criteria are formulated in terms of linear matrix inequalities. Two numerical examples are given to exhibit the improved conservatism and the adequacy of the obtained results.

1. Introduction

Lately, the outstanding Takagi-Sugeno (T–S) fuzzy model has been perceived as a prominent and amazing asset in approximating and depicting linear systems. The T–S fuzzy model [1] is basically a multi-model methodology in which some linear models are mixed into a general single model utilizing nonlinear participation capacities. As an outcome, incredible exertion has been made to get new results for T–S fuzzy systems [2–5]. Particularly for the

* Corresponding author.
E-mail address: kshong@pusan.ac.kr (K.-S. Hong).

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T-S fuzzy systems with time-delay, finding the most extreme delay limits has pulled an impressive consideration. As of late, the T-S fuzzy model methodology has been reached out to manage linear frameworks with time-delays. Different philosophies have been proposed for the investigation of the T-S fuzzy systems with time-delay in the wide research point, for example H_∞ control [6], $L_2 - L_\infty$ control [7], dissipative analysis [8], feedback control for nonlinear T–S fuzzy Systems [9], and H_∞ filter design [10]. Based on the T-S fuzzy model, the mixing of nearby linear frameworks can be exhibited to inexact smooth nonlinear frameworks [11–14]. For instance, in [15], the stability and stabilization with asynchronous output feedback control for the T-S fuzzy model were investigated. In [16], the stability problem for the T-S fuzzy switched system with switched signals was studied.

The Lyapunov-Krasovskii functional (LKF) approach is a well-examined technique to be utilized for dependability investigation of the T-S fuzzy frameworks with time-delay [17]. Some fundamental inequalities, for example, Jensen's inequality [18], generalized integral inequality [19], and extended reciprocally convex matrix inequality [20] were regularly presented in the estimation of the derivative of the LKF. The different types of LKF with new inequalities were refined to diminish the conservatism of delay-dependent stability criteria as much as possible. Up to now, the expanded LKF approach has been affirmed to be a viable and focused technique and the standard Bessel-Legendre disparity can cover Jensen's inequality and Wirtinger's inequality as unique cases [21,22]. Be that as it may, these strategies in [21,22] were not exhibited for the T-S fuzzy framework with time-delay. All the more critically, the increased LKF in [21,22] requires all Lyapunov matrix variables to be certain and the delay dependent LKF was not adequately utilized, which is to an improve the stability criterion of the T-S fuzzy frameworks with time-delay.

An event-triggering mechanism has been proposed in the linear control frameworks [23], in which the control task is executed if a state-dependent triggering condition is violated. The significance of the event-triggering mechanism is that it can essentially diminish the quantity of control task executions while holding an acceptable closed-loop performance, and the details can be referred to [24–29]. So as to stretch out the event-triggering mechanism to the case that the conditions of the plant cannot be quantifiable, in all respects as of late, output-based event-triggered control has been proposed to explore the dynamic output control [30]. The problem of event-triggering based on sliding mode control and dissipative control frameworks were contemplated in [31,32], respectively. In [33], an event-triggered consensus method was proposed for the linear multiagent systems with time-varying communication delays. In [34], the problem of an event-triggered dynamic output feedback controller for T-S fuzzy systems under asynchronous network communications was concentrated. In any case, on the one hand, since the characterized event-triggering conditions in [31,32] depend closely on the instant system output, the system output should be observed constantly.

As of lately, many works based on stability analysis, controller synthesis, and channel configuration have been engaged in the investigation of event-triggered control frameworks. In [35,36], the authors studied event-triggered control for networked control systems and network-based event-triggered H_∞ filtering for time delay systems. The event-triggered scheme is an effective way to keep the balance between the system control performance and the network communication was bandwidth burden. A dissipative approach for semi-Markov jump systems with the event-triggered communication is delivered in [37]. The authors in [38] explored an output feedback control of event-triggered Markovian jump systems with measured output quantizations. Network-based output tracking control for T-S fuzzy systems using an event-triggered communication scheme was studied in [39]. The proposed event-triggered scheme in networked-based T-S fuzzy systems are important in both theoretical and practical view points. Compared with the steady-state behavior of networked T-S fuzzy systems over an interval time delay, in practical systems such as communication network systems, it is very imperative to study the transient performance of the closed-loop system. In [40], a dual-side event-triggered output feedback H_∞ control for network-based systems was explored. However, only a few studies have been explored concerning the T-S fuzzy framework with an event-triggered plan. To the best of authors' knowledge, up to now, event-triggered control of extended dissipativity for T-S fuzzy systems have not been satisfactorily treated yet, which still remains an unsolved research subject.

In this brief, extended dissipative criteria and maximum allowable upper bounds on time derivatives of fuzzy premise functions are derived to guarantee the asymptotic stability of the fuzzy systems with variable sampling. A case study of the inverted pendulum is provided to show the applicability of the accomplished criteria. The contributions are stated as follows: (i) A triggering scheme is introduced to the T-S fuzzy systems with time-varying delay by choosing suitable design parameters. (ii) The T-S fuzzy system under an event-triggered scheme is modeled as a T-S fuzzy time-varying delay system. (iii) Extended dissipativity for the co-design of both the controller gain and the trigger parameters are obtained in terms of linear matrix inequalities. Based on the above, extended dissipativity conditions are derived in Theorem 1 and we produce controller gain synthesis in Theorem 2. Lastly, numerical models are given to demonstrate the adequacy of the proposed conditions. Different from the existing results [41–44], a free-matrix-based double integral inequality involved in the LKF, which plays a key role in linear matrix inequality and gives a less conservative result, is derived.

The remainder of this paper is organized as follows. The portrayal of the system model and the problem to be solved are discussed in Section 2. Our main results are presented in Section 3, where sufficient conditions for the extended dissipativity performances are given in terms of LMIs. Numerical examples demonstrate the effectiveness and superiorities of the achieved methodology in Section 4. Finally, concluding discussions and future work are given in Section 5.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For a matrix P, $P>0$ means that P is a symmetric positive definite matrix. The superscript T stands for matrix transposition. I represents the identity matrix with appropriate dimension. e_ν ($\nu = 1, 2, ..., 14$) are elementary matrices. "sym" stands for "symmetric." diag$\{\cdot\}$ denotes the diagonal matrix with diagonal elements. An asterisk (*) in a matrix is used to denote the term that is induced by symmetry.

2. System Description and Preliminaries

Consider the ith principle of the system with fuzzy IF-THEN rules as follows. Plant rule: IF $\theta_1(t)$ is $\mathbb{M}_{i1}$,..., and $\theta_p(t)$ is $\mathbb{M}_{ip}$, THEN

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + B_i w(t) + C_i u(t),\\ y(t) &= D_i x(t),\end{aligned} \tag{1}$$

where $i = 1, 2, ..., r$, r is the number of IF-THEN rules of the system. $\theta_p(t)$ ($p = 1, 2, ..., s$) are the premise variables vector, where s is the number of premise variables, and $\mathbb{M}_{ip}$ ($i = 1, 2, ..., r$; $p = 1, 2, ..., s$) present the fuzzy sets. $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^m$ is the disturbance, $u(t) \in \mathbb{R}^l$ is the control input, and $y(t) \in \mathbb{R}^p$ is the output of the system. A_i is a positive matrix, B_i, C_i, and D_i are the parameter matrices with appropriate dimensions.

The T-S fuzzy system (1) can be rewritten as follows.

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\Big[A_i x(t) + B_i w(t) + C_i u(t)\Big],$$
$$y(t) = \sum_{i=1}^{r} h_i(\theta(t))\Big[D_i x(t)\Big]. \tag{2}$$

The fuzzy basis functions are given by

$$h_i(\theta(t)) = \frac{\prod_{p=1}^{s} M_{ip}(\theta_p(t))}{\sum_{i=1}^{r}\prod_{p=1}^{s} \mathbb{M}_{ip}(\theta_p(t))}$$

where $\mathbb{M}_{ip}(\theta_p(t))$ represents the grade of the membership of $\theta_p(t)$. It can be seen that $h_i(\theta(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

The point of acquainting the event generator is to save the limited correspondence asset. Moreover, an event-triggered correspondence plan is acquainted to determine whether the current sampled signal should be transmitted or not. This event-triggered sampling strategy is based on periodically sampled data which can be written as follows [45].

$$[x(k+j)d - x(kd)]^T \Omega[x(k+j)d) - x(kd)] \leq \sigma x^T((k+j)d)\Omega x((k+j)d), \tag{3}$$

where Ω is the symmetric positive definite event-weighting matrix to be designed, $k \in N^+$, $\sigma = [0, 1)$. $(k+j)d$ and kd denote the current sampling data and the latest transmitted data with d denoting the sampling period ($d = 1, 2, \ldots$) while σ is the given event parameter. As a special case, if $\sigma = 0$ in (3), the inequality (3) is not satisfied for almost all the sampled state $x((k+j)d)$, and the event-triggered scheme reduces to a periodic release scheme.

Under the communication scheme (3), we assume that the release times are $t_0 d$, $t_1 d$, $t_2 d$, ..., where t_0 is the initial time. $\varrho_n d = t_{n+1} - t_n d$ denotes the release period which corresponds to the sampling period given by the event generator in (3). In the following, we can deal with a time-varying networked induced delay δ_k, where δ_k is the time from the instant (kd) when a sensor samples a data packet from the plant at the instant when the fuzzy system receives the data packet. It is supposed that $0 \leq \delta_m \leq \delta_k \leq \delta_M$, where δ_m and δ_M are given scalars satisfying $0 \leq \delta_m < \delta_M$. Therefore, the states $x(t_0 d)$, $x(t_1 d)$, $x(t_2 d)$, ... will arrive at the controller side at instants $t_0 + \delta_0$, $t_1 d + \delta_1$, $t_1 d + \delta_2$,..., respectively.

If the release interval satisfies $t_{k+1}d - t_k d \leq d + \delta_M - \delta_{k+1}$, time-varying delay for interval $\delta(t)$ is defined as follows:

$$\delta(t) = t - t_k d, \qquad t \in [t_k d + \delta_k, t_{k+1} d + \delta_{k+1}).$$

If the release interval satisfies $t_{k+1}d - t_k d > d + \delta_M - \delta_{k+1}$, there always exists a scalar r_M satisfying $t_{k+1}d + t_{k+1} \in [t_k d + r_M d + \delta_M, t_k d + r_M d + d + d\delta_M)$. Define the collection of time-varying delays as follows.

$$\begin{cases} \Delta_0 = [t_k d + \delta_k, t_k d + d + \delta_M), \\ \Delta_1 = [t_k d + d + \delta_M, t_k d + 2d + \delta_M), \\ \qquad\qquad \vdots \\ \Delta_{r_M} = [t_k d + r_M d + \delta_M, t_{k+1} d + \delta_{k+1}). \end{cases} \tag{4}$$

In order to employ event-triggered condition (3), two piecewise functions $\delta(t)$ and $e_k(t)$ are defined as follows:

$$\delta(t) = \begin{cases} t - t_k d, & t \in \Delta_0 \\ t - t_k d - d, & t \in \Delta_1 \\ \vdots \\ t - t_k d - r_M d, & t \in \Delta_{r_M}. \end{cases} \tag{5}$$

$$e_k(t) = \begin{cases} x(t_k d) - x(t_k d), & t \in \Delta_0 \\ x(t_k d) - x(t_k d + d), & t \in \Delta_1 \\ \vdots \\ x(t_k d) - x(t_k d + r_M d), & t \in \Delta_{r_M}. \end{cases} \tag{6}$$

From the definition of $\delta(t)$ presented above, one can see that $\delta(t) \in [\delta_m, d + \delta_M)$. It follows from (5) that $\delta_m \leq \delta(t) \leq d + \delta_M$. We define $\delta_m = \delta_1$ and $\delta_M + d = \delta_2$. From the definition of $e_k(t)$, we can write $x(t_k d) = e_k(t) + x(t - \delta(t))$. Combining (6) and (3), we have

$$e_k^T \Omega e_k(t) \leq \sigma x^T(t - \delta(t)) \Omega x(t - \delta(t)), \qquad t \in [t_k d + \delta_k, t_{k+1} d + \delta_{k+1}).$$

Using the control $u(t) = K_i x(t)$, $t \in [t_k d + \delta_k, t_{k+1} d + \delta_{k+1})$, where K_i is the desired controller gains to be determined for each $i = 1, 2, ..., r$. We can rewrite (2) as follows.

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t)) \Big[A_i x(t) + B_i \omega(t) + C_i K_i x(t - \delta(t)) + C_i K_i e_k(t) \Big],$$
$$y(t) = \sum_{i=1}^{r} h_i(\theta(t)) \Big[D_i x(t) \Big],$$
$$x(t) = \phi(t), \quad t \in [-\delta_2, 0], \tag{7}$$

where $\phi(t)$ is the initial function of $x(t)$, and $0 \leq \delta_1 \leq \delta(t) \leq \delta_2$.

Assumption 1. Matrices Φ_1, Φ_2, Φ_3, and Φ_4 satisfy the following conditions:
(i) $\Phi_1 = \Phi_1^T \leq 0$, (ii) $\Phi_3 = \Phi_3^T > 0$, (iii) $\Phi_4 = \Phi_4^T \geq 0$, and (iv) $(||\Phi_1|| + ||\Phi_2||)\Phi_4 = 0$.

Definition 1. [46]. For given matrices Φ_1, Φ_2, Φ_3, and Φ_4 satisfying Assumption 1, system (7) is said to be extended dissipative if the following inequality holds for any $T \geq 0$ and all $\omega(t) \in L_2[0, \infty)$:

$$\int_0^T J(t) dt - \sup_{0 \leq t \leq T} z^T(t) \Phi_4 z(t) \geq 0,$$

where $J(t) = z^T(t) \Phi_1 z(t) + 2 z^T(t) \Phi_2 \omega(t) + \omega^T(t) \Phi_3 \omega(t)$.

Remark 1 ([47]). The concept of extended dissipativity introduced in Definition 1 contains a few well-known performance indices as special cases by setting the weighting matrices, Φ_n $(n = 1, ..., 4)$. For instance, (i) if $\Phi_1 = -I$, $\Phi_2 = 0$, $\Phi_3 = \gamma^2 I$, $\Phi_4 = 0$, then Definition 1 refers to the H_∞ performance; (ii) if $\Phi_1 = 0$, $\Phi_2 = 0$, $\Phi_3 = \gamma^2 I$, $\Phi_4 = I$, then Definition 1 refers to the $l_2 - l_\infty$ performance; (iii) if $\Phi_1 = 0$, $\Phi_2 = I$, $\Phi_3 = \gamma I$, $\Phi_4 = 0$, then Definition 1 refers to the passivity performance; and (iv) if $\Phi_1 = \mathcal{Q}$, $\Phi_2 = \mathcal{S}$, $\Phi_3 = \mathcal{R} - \alpha I$, $\Phi_4 = 0$, then Definition 1 refers to the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance. This fact can be also seen from Assumption 1 and Definition 1. Indeed, when $\omega(t)=0$, it follows from Definition 1 that

$$\int_0^T z^T(t)\Phi_1 z(t)dt - \sup_{0\le t\le T} z^T(t)\Phi_4 z(t) \ge 0.$$

Note from Assumption 1 that $\Phi_4 \ge 0$ and $\Phi_1 \le 0$. Thus, the above inequality implies that the performance index is not less than zero.

Lemma 1 ([48]). *For $a, b \in \mathbb{R}^n$ and $M \in \mathbb{R}^{n\times n}$, the following inequality holds*

$$2a^T b \le a^T M a + b^T M^{-1} b.$$

Lemma 2 ([21]). *Let $N \in \mathbb{N}$, $\eta \in \mathbb{R}^m$, and x be a continuous and differential function mapping $[a,b] \to \mathbb{R}^n$. For any matrices $Z \in \mathbb{R}^{n\times n} > 0$ and $M \in \mathbb{R}^{(N+1)n\times m}$, the following inequality holds:*

$$-\int_a^b \dot{x}^T(s) Z \dot{x}(s) ds \le 2\xi_N^T \Xi_N^T M\eta + (b-a)\eta^T M^T \widehat{Z} M\eta,$$

where $\Xi_N = \begin{bmatrix} \pi_N^T(0) & \pi_N^T(1) & \cdots & \pi_N^T(N) \end{bmatrix}^T$, $\widehat{Z} = \mathrm{diag}\{\frac{1}{\mathrm{Z}}, \frac{1}{3\mathrm{Z}}, \cdots, \frac{1}{(2\mathrm{N}+1)\mathrm{Z}}\}$,

$$\xi_N = \begin{cases} x^T(b) \quad x^T(a)]^T, & N=0,\\ x^T(b) \quad x^T(a) \quad \frac{1}{b-a}\vartheta_0^T \cdots, \frac{1}{b-a}\vartheta_{N-1}^T, & N>0 \end{cases}$$

Lemma 3 ([49]). *Let $x(s)$ be a continuously differentiable function:$[a,b] \to \mathbb{R}^n$. If there exist positive definite symmetric matrices Z_1, $Z_2 \in R^{3n\times 3n}$, $M \in \mathbb{R}^{n\times n}$, any matrices $Z_3 \in \mathbb{R}^{3n\times 3n}$, N_1, and $N_2 \in \mathbb{R}^{3n\times n}$ satisfying*

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & M \end{bmatrix} \ge 0,$$

then the following inequality holds:

$$\int_a^b \int_\alpha^\beta \dot{x}^T(s) M \dot{x}(s) ds d\alpha \ge \varpi^T \Psi \varpi,$$

where $\Psi = -\frac{1}{2}d^2\Big[Z_1 + \frac{1}{6}Z_3\Big] - \mathrm{sym}\{\mathrm{N}_1(\mathrm{d}\mathrm{e}_1 - \mathrm{e}_2) + \mathrm{N}_2(-\frac{1}{2}\mathrm{d}\mathrm{e}_1 - \mathrm{e}_2 + \frac{3}{\mathrm{d}}\mathrm{e}_3)\}$.

3. Main Results

In this section, the extended dissipativity analysis with event-triggered control of the system with varying delay is addressed by employing the free-matrix double and generalized integral inequalities. Define the following notations:

$$\Upsilon_1(t) = \begin{bmatrix} x^T(t) & x^T(t-\delta_1) & x^T(t-\delta_2) \end{bmatrix}^T,$$

$$\Upsilon_2(t,s) = \begin{bmatrix} \dot{x}^T(s) & x^T(s) & x^T(t-\delta_1) & \int_s^t \dot{x}^T(\theta)d\theta & \int_{t-\delta_2}^s \dot{x}^T(\theta)d\theta & \int_{t-\delta_1}^s \dot{x}^T(\theta)d\theta \end{bmatrix}^T,$$

$$\begin{aligned}\xi(t) = \Big[& x^T(t) \quad x^T(t-\delta(t)) \quad x^T(t-\delta_1) \quad x^T(t-\delta_2) \quad \dot{x}^T(t-\delta_1) \quad \dot{x}^T(t-\delta_2) \quad \tfrac{1}{\delta(t)-\delta_1}\zeta_1(t) \\ & \tfrac{1}{\delta_2-\delta(t)}\zeta_2(t) \quad \tfrac{1}{(\delta(t)-\delta_1)^2}\zeta_3(t) \quad \tfrac{1}{(\delta_2-\delta(t))^2}\zeta_4(t) \quad \int_{t-\delta_1}^t x^T(s)ds \quad \int_{t-\delta_2}^{t-\delta_1} x^T(s)ds \quad \omega(t) \quad e_k(t)\Big]^T,\end{aligned}$$

$$\zeta_a(t) = \begin{bmatrix} \zeta_1(t) & \zeta_2(t) & \zeta_3(t) & \zeta_4(t) \end{bmatrix}^T,$$

$$\zeta_b(t) = \begin{bmatrix} \zeta_5(t) & \zeta_6(t) & \zeta_7(t) \end{bmatrix}^T,$$

$$\zeta_1(t) = \int_{t-\delta(t)}^{t-\delta_1} x^T(s)ds,$$

$$\zeta_2(t) = \int_{t-\delta_2}^{t-\delta(t)} x^T(s)ds,$$

$$\zeta_3(t) = \int_{-\delta(t)}^{-\delta_1} \int_{t+\theta}^{t} x^T(s)dsd\theta,$$

$$\zeta_4(t) = \int_{-\delta_2}^{-\delta(t)} \int_{t+\theta}^{t-\delta(t)} x^T(s)dsd\theta,$$

$$\zeta_5(t) = \int_{s}^{t} \dot{x}^T(\theta)d\theta,$$

$$\zeta_6(t) = \int_{t-\delta_2}^{s} \dot{x}^T(\theta)d\theta,$$

$$\zeta_7(t) = \int_{t-\delta_1}^{s} \dot{x}^T(\theta)d\theta.$$

Theorem 1. *For given scalars $\delta_1 \geq 0$, $\delta_2 > 0$, and matrices Φ_1, Φ_2, Φ_3, and Φ_4 satisfying Assumption 1, system (7) is extended dissipative and satisfies the performance index in Definition 1 for any admissible time-varying delay $0 \leq \delta_1 \leq \delta(t) \leq \delta_2$, if there exist (i) symmetric matrices $P>0, Q>0, R>0, S>0, T>0$, and $U>0$ with appropriate dimensions, (ii) matrices $\mathcal{X}_m > 0$ $(m = 1, 2, ..., 6)$, $\mathcal{Y}_n > 0$ $(n = 1, 2)$, $\mathcal{Z}_l > 0$ $(l = 1, 2, ..., 6)$, $M > 0$ with appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 & \mathcal{Z}_3 \\ * & \mathcal{X}_3 & \mathcal{Z}_4 \\ * & * & U \end{bmatrix} \geq 0, \quad \begin{bmatrix} \mathcal{X}_4 & \mathcal{X}_5 & \mathcal{Z}_5 \\ * & \mathcal{X}_6 & \mathcal{Z}_6 \\ * & * & U \end{bmatrix} \geq 0, \tag{8}$$

$$P - D_i^T \Phi_4 D_i \geq 0, \tag{9}$$

$$\widehat{\Sigma}_1 = \begin{bmatrix} \Gamma(\delta(t) = \delta_1) & \sqrt{\delta_2}\mathcal{Z}_1 \\ * & -\mathcal{M} \end{bmatrix} < 0, \quad \widehat{\Sigma}_2 = \begin{bmatrix} \Gamma(\delta(t) = \delta_2) & \sqrt{\delta_2}\mathcal{Z}_1 \\ * & -\mathcal{M} \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{aligned} \Gamma = {} & \mathrm{sym}\{\Psi_0^{\mathrm{T}} \mathrm{P} \Psi_1\} + \Psi_2^{\mathrm{T}} \mathrm{Q} \Psi_2 - \Psi_3^{\mathrm{T}} \mathrm{Q} \Psi_3 + \Psi_3^{\mathrm{T}} \mathrm{R} \Psi_3 - \Psi_4^{\mathrm{T}} \mathrm{S} \Psi_4 + (\delta_2 - \delta_1) \mathrm{e}_\mathrm{d}^{\mathrm{T}} \mathrm{T} \mathrm{e}_\mathrm{d} \\ & + \frac{(\delta_2 - \delta_1)^2}{2} e_d^T U e_d + e_2^T \sigma \Omega e_2 + \mathrm{sym}\{\Psi_5^{\mathrm{T}} \mathrm{S} \Psi_6 + \Psi_7^{\mathrm{T}} \mathrm{S} \Psi_6 + \mathcal{Z}_1 \mathcal{Y}_1 + \mathcal{Z}_2 \mathcal{Y}_2\} \\ & - \Psi_8^T \Pi_2 \Psi_8 - \Psi_9^T \Pi_3 \Psi_9 + \Psi_{10}^T \Phi_1 \Psi_{10} - 2\Psi_{10}^T \Phi_2 \Psi_{11} - 2\Psi_{11}^T \Phi_3 \Psi_{11}, \end{aligned}$$

$$\Psi_0(\delta_1) = \Big[e_1^T \quad e_2^T \quad e_3^T]^T,$$

$$\Psi_1 = \Big[e_d^T \quad e_5^T \quad e_6^T]^T,$$

$$\Psi_2 = \Big[e_d^T \quad e_1^T \quad e_3^T \quad 0 \quad e_1^T - e_4^T \quad e_1^T - e_3^T]^T,$$

$$\Psi_3 = \Big[e_5^T \quad e_3^T \quad e_3^T \quad e_1^T - e_3^T \quad e_3^T - e_4^T \quad 0]^T,$$

$$\Psi_4 = \Big[e_6^T \quad e_4^T \quad e_3^T \quad e_1^T - e_4^T \quad 0 \quad e_4^T - e_3^T \Big]^T,$$

$$\Psi_5 = \Big[e_1^T - e_3^T \quad \delta_1 e_7^T \quad \delta_1 e_3^T \quad \delta_1 (e_1^T - e_{11}^T) \quad \delta_1 (e_{11}^T - e_4^T) \quad \delta_1 (e_{11}^T - e_3^T) \Big]^T,$$

$$\Psi_6 = \Big[0 \quad 0 \quad 0 \quad e_d^T \quad -e_6^T \quad -e_5^T\Big]^T,$$
$$\Psi_7 = \Big[e_3^T - e_4^T \quad (\delta_2-\delta_1)e_{12}^T \quad (\delta_2-\delta_1)e_3^T \quad (\delta_2-\delta_1)(e_1^T - e_{12}^T)$$
$$(\delta_2-\delta_1)(e_{12}^T - e_4^T) \quad (\delta_2-\delta_1)(e_{12}^T - e_3^T)\Big]^T,$$
$$\Psi_8 = \Big[e_1^T \quad e_7^T \quad e_9^T]^T,$$
$$\Psi_9 = \Big[e_1^T \quad e_8^T \quad e_{10}^T]^T,$$
$$\Psi_{10} = \Big[D_i^T e_1^T]^T, \quad \Psi_{11} = \Big[e_{13}^T]^T,$$
$$\Pi_2 = -\frac{1}{2}\delta^2(t)\Big[\mathcal{X}_1 + \frac{1}{6}\mathcal{X}_3\Big] - [\delta_1 e_1 - e_6]\mathcal{Z}_3[\delta_1 e_1 - e_9]^T$$
$$-[-\frac{1}{2}\delta_1 e_1 - e_7 + 3e_9]\mathcal{Z}_4[-\frac{1}{2}\delta_1 e_1 - e_7 + 3e_9]^T,$$
$$\Pi_3 = -\frac{1}{2}(\delta_2-\delta_1)^2(t)\Big[\mathcal{X}_4 + \frac{1}{6}\mathcal{X}_6\Big] - [(\delta_2-\delta_1)e_1 - e_8]\mathcal{Z}_5[(\delta_2-\delta_1)e_1 - e_8]^T$$
$$-[-\frac{1}{2}(\delta_2-\delta_1)e_1 - e_8 + 3e_{10}]\mathcal{Z}_6[-\frac{1}{2}(\delta_2-\delta_1)e_1 - e_8 + 3e_{10}]^T,$$

$$\mathcal{Y}_1 = \begin{bmatrix} e_1 - e_3 \\ e_1 + e_3 - 2e_7 \\ e_1 - e_3 + 6e_7 - 12e_9 \end{bmatrix}, \quad \mathcal{Y}_2 = \begin{bmatrix} e_3 - e_4 \\ e_3 + e_4 - 2e_8 \\ e_3 - e_4 + 6e_8 - 12e_{10} \end{bmatrix},$$
$$e_d = A_i e_1 + B_i e_{13} + C_i K_i e_2 + C_i K_i e_{14},$$
$$\mathcal{M} = \text{diag}\{\text{M}, 3\text{M}, 5\text{M}\}$$
$$e_\nu = [0_{n\times(\nu-1)n} \quad I_{n\times n} \quad 0_{n\times(14-\nu)n}]^T, \text{ where } \nu = 1, 2, \ldots 14.$$

Proof. For system (7), the following Lyapunov-Krasovskii functional is considered.

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) + V_4(x(t)) + V_5(x(t)), \tag{11}$$

where

$$V_1(x(t)) = \Upsilon_1^T(t)P\Upsilon_1(t),$$
$$V_2(x(t)) = \int_{t-\delta_1}^{t} \Upsilon_2^T(t,s)Q\Upsilon_2(t,s)ds,$$
$$V_3(x(t)) = \int_{t-\delta_2}^{t-\delta_1} \Upsilon_2^T(t,s)R\Upsilon_2(t,s)ds,$$
$$V_4(z(t)) = \int_{-\delta_2}^{-\delta_1}\int_{t+\theta}^{t} \dot{x}^T(s)S\dot{x}(s)dsd\theta,$$
$$V_5(z(t)) = \int_{t-\delta_2}^{t-\delta_1}\int_{\theta}^{t}\int_{\gamma}^{t} \dot{x}^T(s)T\dot{x}(s)dsd\theta d\gamma.$$

Calculating the derivative of (11) along system (7), we obtain

$$\dot{V}_1(t) = 2\Upsilon_1^T(t)P\dot{\Upsilon}_1(t), \tag{12}$$

$$\begin{aligned}\dot{V}_2(t) &= \Upsilon_2^T(t,t)Q\Upsilon_2^T(t,t) - \Upsilon_2^T(t,t-\delta_1)Q\Upsilon_2(t,t-\delta_1)\\ &\quad + \mathrm{sym}\Big(\int_{\mathrm{t}-\delta_1}^{\mathrm{t}} \Upsilon_2^{\mathrm{T}}(\mathrm{t,s})\mathrm{dsQ}\Big)\frac{\partial\Upsilon_2(\mathrm{t,s})}{\partial \mathrm{t}},\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_3(t) &= \Upsilon_2^T(t,t-\delta_1)R\Upsilon_2^T(t,t-\delta_1) - \Upsilon_2^T(t,t-\delta_2)Q\Upsilon_2(t,t-\delta_2)\\ &\quad + \mathrm{sym}\Big(\int_{\mathrm{t}-\delta_2}^{\mathrm{t}-\delta_1} \Upsilon_2^{\mathrm{T}}(\mathrm{t,s})\mathrm{dsR}\Big)\frac{\partial\Upsilon_2(\mathrm{t,s})}{\partial \mathrm{t}},\end{aligned} \tag{14}$$

$$\dot{V}_4(t) = (\delta_2-\delta_1)e_d^T T e_d - \int_{t-\delta_2}^{t-\delta_1} \dot{x}^T(s)Z\dot{x}(s)ds, \tag{15}$$

$$\dot{V}_5(t) = \frac{(\delta_2-\delta_1)^2}{2}e_d^T U e_d - \int_{t-\delta_2}^{t-\delta_1}\int_{\theta}^{t} \dot{x}^T(s)U\dot{x}(s)dsd\theta, \tag{16}$$

where

$$\begin{aligned}
\Upsilon_2(t,t) &= \Big[\dot{x}^T(t) \quad x^T(t) \quad x^T(t-\delta_1) \quad \zeta_b^T(t)\Big]^T,\\
\Upsilon_2(t,t-\delta_1) &= \Big[\dot{x}^T(t-\delta_1) \quad x^T(t-\delta_1) \quad x^T(t-\delta_1) \quad \zeta_b^T(t)\Big]^T,\\
\Upsilon_2(t,t-\delta_2) &= \Big[\dot{x}^T(t-\delta_2) \quad x^T(t-\delta_2) \quad x^T(t-\delta_1) \quad \zeta_b^T(t)\Big]^T,\\
\int_{t-\delta_1}^{t}\Upsilon_2(t,s)ds &= \Big[x^T(t)-x^T(t-\delta_1) \quad \delta_1\zeta_1^T(t) \quad \delta_1 x^T(t-\delta_1) \quad \delta_1\zeta_b^T(t)\Big]^T,\\
\int_{t-\delta_2}^{t-\delta_1}\Upsilon_2(t,s)ds &= \Big[x^T(t-\delta_1)-x^T(t-\delta_2) \quad (\delta_2-\delta_1)\zeta_2^T(t) \quad (\delta_2-\delta_1)x^T(t-\delta_1)\\
&\qquad \times (\delta_2-\delta_1)\zeta_b^T(t)\Big]^T,\\
\frac{\partial\Upsilon_2(t,s)}{\partial t} &= \Big[0 \quad 0 \quad 0 \quad \dot{\zeta}_b^T(t)\Big]^T,\\
\dot{\zeta}_b(t) &= \Big[\dot{x}^T(t) \quad -\dot{x}^T(t-\delta_2) \quad -\dot{x}^T(t-\delta_1)\Big]^T,
\end{aligned}$$

and

$$\dot{V}(t) = \xi^T(t)\Pi_0\xi(t) - \int_{t-\delta_2}^{t-\delta_1}\dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-\delta_2}^{t-\delta_1}\int_{\theta}^{t}\dot{x}^T(s)U\dot{x}(s)dsd\theta, \tag{17}$$

where

$$\Pi_0 = 2\begin{bmatrix} x(t) \\ x(t-\delta_1) \\ x(t-\delta_2) \end{bmatrix}^T P \begin{bmatrix} x(t) \\ x(t-\delta_1) \\ x(t-\delta_2) \end{bmatrix} + \begin{bmatrix} \dot{x}(t) \\ x(t) \\ x(t-\delta_1) \\ 0 \\ x(t)-x(t-\delta_2) \\ x(t)-x(t-\delta_1) \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(t) \\ x(t) \\ x(t-\delta_1) \\ 0 \\ x(t)-x(t-\delta_2) \\ x(t)-x(t-\delta_1) \end{bmatrix}$$

$$- \begin{bmatrix} \dot{x}(t-\delta_1) \\ x(t-\delta_1) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_1) \\ x(t-\delta_1)-x(t-\delta_2) \\ 0 \end{bmatrix}^T Q \begin{bmatrix} \dot{x}(t-\delta_1) \\ x(t-\delta_1) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_1) \\ x(t-\delta_1)-x(t-\delta_2) \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \dot{x}(t-\delta_1) \\ x(t-\delta_1) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_1) \\ x(t-\delta_1)-x(t-\delta_2) \\ 0 \end{bmatrix}^T R \begin{bmatrix} \dot{x}(t-\delta_1) \\ x(t-\delta_1) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_1) \\ x(t-\delta_1)-x(t-\delta_2) \\ 0 \end{bmatrix}$$

$$- \begin{bmatrix} \dot{x}(t-\delta_2) \\ x(t-\delta_2) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_2) \\ 0 \\ x(t-\delta_2)-x(t-\delta_1) \end{bmatrix}^T R \begin{bmatrix} \dot{x}(t-\delta_2) \\ x(t-\delta_2) \\ x(t-\delta_1) \\ x(t)-x(t-\delta_2) \\ 0 \\ x(t-\delta_2)-x(t-\delta_1) \end{bmatrix}$$

$$+(\delta_2-\delta_1)e_d^T T e_d + \tfrac{(\delta_2-\delta_1)^2}{2} e_d^T U e_d + \mathrm{sym}\{\Psi_5^{\mathrm{T}} \mathrm{S} \Psi_6 + \Psi_7^{\mathrm{T}} \mathrm{S} \Psi_6\},$$

$$\Pi_0 = \mathrm{sym}\{\Psi_0^{\mathrm{T}} \mathrm{P} \Psi_1\} + \Psi_2^{\mathrm{T}} \mathrm{Q} \Psi_2 - \Psi_3^{\mathrm{T}} \mathrm{Q} \Psi_3 + \Psi_3^{\mathrm{T}} \mathrm{R} \Psi_3 - \Psi_4^{\mathrm{T}} \mathrm{S} \Psi_4 + (\delta_2-\delta_1)\mathrm{e}_{\mathrm{d}}^{\mathrm{T}} \mathrm{T} \mathrm{e}_{\mathrm{d}}$$
$$+ \tfrac{(\delta_2-\delta_1)^2}{2} e_d^T U e_d + \mathrm{sym}\{\Psi_5^{\mathrm{T}} \mathrm{S} \Psi_6 + \Psi_7^{\mathrm{T}} \mathrm{S} \Psi_6\}.$$

Lemma 2 and (15) yield

$$-\int_{t-\delta_2}^{t-\delta_1} \dot{x}^T(s) T \dot{x}(s) ds = -\int_{t-\delta(t)}^{t-\delta_1} \dot{x}^T(s) T \dot{x}(s) ds - \int_{t-\delta_2}^{t-\delta(t)} \dot{x}^T(s) T \dot{x}(s) ds$$
$$\leq \xi^T(t) \Pi_1(\delta(t)) \xi(t), \tag{18}$$

where $\Pi_1(\delta(t)) = \mathrm{sym}\{\mathcal{Z}_1 \mathcal{Y}_1 + \mathcal{Z}_2 \mathcal{Y}_2\} - (\delta(\mathrm{t}) - \delta_1) \mathcal{Z}_1 \mathcal{M}^{-1} \mathcal{Z}_1^{\mathrm{T}} - (\delta_2 - \delta(\mathrm{t})) \mathcal{Z}_2 \mathcal{M}^{-1} \mathcal{Z}_2^{\mathrm{T}}$. By Lemma 3, the following inequality is obtained.

$$-\int_{t-\delta_2}^{t-\delta_1} \int_{\theta}^{t} \dot{x}^T(s) U \dot{x}(s) ds d\theta = -\int_{t-\delta(t)}^{t-\delta_1} \int_{\theta}^{t} \dot{x}^T(s) U \dot{x}(s) ds d\theta - \int_{t-\delta_2}^{t-\delta_1} \int_{\theta}^{t} \dot{x}^T(s) U \dot{x}(s) ds d\theta$$
$$\leq -\vartheta_1^T(t) \Pi_2 \vartheta_1(t) - \vartheta_2^T(t) \Pi_3 \vartheta_2(t)$$
$$\leq \xi^T(t) \Big(-\vartheta_1^T(t) \Pi_2 \vartheta_1(t) - \vartheta_2^T(t) \Pi_3 \vartheta_2(t) \Big) \xi(t)$$
$$\leq \xi^T(t) [\Pi_2 + \Pi_3] \xi(t), \tag{19}$$

where

$\vartheta_1(t) = [x^T(t),\ \frac{1}{\delta(t)-\delta_1}\int_{t-\delta(t)}^{t-\delta_1} x^T(s)ds,\ \frac{1}{(\delta(t)-\delta_1)^2}\int_{-\delta(t)}^{-\delta_1}\int_{t+\theta}^{t-\delta_1} x^T(s)dsd\theta]^T,$
$\vartheta_2(t) = [x^T(t),\ \frac{1}{\delta_2-\delta(t)}\int_{t-\delta_2}^{t-\delta(t)} x^T(s)ds,\ \frac{1}{(\delta_2-\delta(t))^2}\int_{-\delta_2}^{-\delta(t)}\int_{t+\theta}^{t-\delta(t)} x^T(s)dsd\theta]^T.$

Combining (12)–(19) with the condition on Schur complement lemma [48], the following inequality is obtained.

$$\xi^T(t)\Sigma\xi(t) < 0. \tag{20}$$

It is clear that the membership function $\sum_{i=1}^r \omega_i\theta(t)) = 1$ and $\omega_i\theta(t)) \geq 0$. Finally, we obtain

$$\dot V(x(t)) < \sum_{i=1}^{r} \omega_k(\theta(t))\xi^T(t)\widetilde{\Sigma}_{1,2}\xi(t) < 0. \tag{21}$$

By the event-triggered condition, we can get

$$\dot V(x(t)) - J(t) < \sum_{i=1}^{r} \omega_k(\theta(t))\xi^T(t)\widehat{\Sigma}_{1,2}\xi(t) < 0, \tag{22}$$

where $J(t) = y^T(t)\Phi_1 y(t) + 2y^T(t)\Phi_2\omega(t) + \omega^T(t)\Phi_3\omega(t)$. □

According to Definition 1, we need to prove that the following inequality holds for matrices Φ_1, Φ_2, Φ_3, and Φ_4 with the zero initial condition $V(0) = 0$.

$$\int_0^t J(s)ds \geq V(t). \tag{23}$$

Meanwhile,

$$V(t) \geq x^T(t)Px(t) > 0. \tag{24}$$

We also have

$$\int_0^t J(s)ds \geq x^T(t)Px(t). \tag{25}$$

To prove that system (7) is extended dissipative, Definition 1 should be satisfied. To satisfy (22), the following inequality should hold:

$$\int_0^{T_f} J(t)dt - \sup_{0\leq t\leq T_f} y^T(t)\Phi_4 y(t) \geq 0. \tag{26}$$

According to, the extended dissipativity criteria can be described: (i) $\Phi_4 = 0$ if the H_∞ performance, the passivity, and the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipativity conditions are satisfied. (ii) $\Phi_4 > 0$ if the $L_2 = L_\infty$ performance conditions are provided.

Case 1. When $\Phi_4 = 0$, we have

$$\int_0^{T_f} J(t)dt = \sup_{0\leq t\leq T_f} y^T(s)\Phi_4 y(s), \qquad T_f \geq 0.$$

Then system (7) can be deduced to perform H_∞, $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity and the passivity.

Case 2. When $\Phi_4 > 0$, we have, from Assumption 1, that $\Phi_1 = 0$, $\Phi_2 = 0$, and $\Phi_3 > 0$, whereupon we obtain

$$\int_0^t J(s)ds = \int_0^t w^T(s)\Phi_3 w(s)ds. \tag{27}$$

Then for any $t \geq 0$, $T_f \geq 0$, $T_f \geq t$, $0 \leq t \leq T_f$, for all $t \in [0, T_f]$, we have

$$\int_0^{T_f} J(s)ds > \int_0^{t} J(s)ds \geq x^T(t)Px(t) > 0.$$

From (9) we obtain

$$\int_0^{T_f} J(s)ds \geq x^T(t)Px(t) \geq y^T(t)\Phi_4 y(t). \tag{28}$$

Finally, we obtain

$$\int_0^{T_f} J(t)dt - \sup_{0 \leq t \leq T} y^T(t)\Phi_4 y(t) \geq 0. \tag{29}$$

As discussed above, system (7) is said to be extended dissipative in the sense of Definition 1. This completes the proof.

In the following, a criterion is proposed to design the feedback gain K_i under the event-triggered condition (3) from Theorem 1.

Theorem 2. *For given scalars $\delta_1 \geq 0$, $\delta_2 > 0$, and matrices Φ_1, Φ_2, Φ_3, and Φ_4 satisfying Assumption 1, system (1) under the event-triggered condition (3) and gain matrix $K_i = YX^{-1}$ is extended dissipative and satisfies the performance index in Definition 1 for any admissible time-varying delay $0 \leq \delta_1 \leq \delta_2$, if there exist (i) symmetric matrices $X > 0$, $\widehat{Q} > 0$, $\widehat{R} > 0$, $\widehat{S} > 0$, $\widehat{T} > 0$, and $\widehat{U} > 0$ with appropriate dimensions, (ii) matrices $\widehat{\mathcal{X}}_m > 0$ $(m = 1, 2, ..., 6)$, $\mathcal{Y}_n > 0$ $(n = 1, 2)$, $\widehat{\mathcal{Z}}_l > 0$ $(l = 1, 2, ..., 6)$, $M > 0$ with appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \widehat{\mathcal{X}}_1 & \widehat{\mathcal{X}}_2 & \widehat{\mathcal{Z}}_3 \\ * & \widehat{\mathcal{X}}_3 & \widehat{\mathcal{Z}}_4 \\ * & * & \widehat{U} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \widehat{\mathcal{X}}_4 & \widehat{\mathcal{X}}_5 & \widehat{\mathcal{Z}}_5 \\ * & \widehat{\mathcal{X}}_6 & \widehat{\mathcal{Z}}_6 \\ * & * & \widehat{U} \end{bmatrix} \geq 0, \tag{30}$$

$$X - D_i^T(t)\Phi_4 D_i \geq 0, \tag{31}$$

$$\overline{\Sigma}_1 = \begin{bmatrix} \Gamma(\delta(t) = \delta_1) & \sqrt{\delta_2}\widehat{\mathcal{Z}}_1 \\ * & -\widehat{\mathcal{M}} \end{bmatrix} < 0, \ \overline{\Sigma}_2 = \begin{bmatrix} \Gamma(\delta(t) = \delta_2) & \sqrt{\delta_2}\widehat{\mathcal{Z}}_1 \\ * & -\widehat{\mathcal{M}} \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \Gamma = {} & \mathrm{sym}\{\Psi_0^T P \Psi_1\} + \Psi_2^T Q \Psi_2 - \Psi_3^T Q \Psi_3 + \Psi_3^T R \Psi_3 - \Psi_4^T S \Psi_4 + (\delta_2 - \delta_1) e_d^T T e_d \\ & + \frac{(\delta_2 - \delta_1)^2}{2} e_d^T U e_d + e_2^T \sigma \Omega e_2 + \mathrm{sym}\{\Psi_5^{\mathrm{T}} \mathrm{S} \Psi_6 + \Psi_7^{\mathrm{T}} \mathrm{S} \Psi_6 + \mathcal{Z}_1 \mathcal{Y}_1 + \mathcal{Z}_2 \mathcal{Y}_2\} \\ & - \Psi_8^T \Pi_2 \Psi_8 - \Psi_9^T \Pi_3 \Psi_9 + \Psi_{10}^T \Phi_1 \Psi_{10} - 2\Psi_{10}^T \Phi_2 \Psi_{11} - 2\Psi_{11}^T \Phi_3 \Psi_{11}, \\ \Psi_0 = {} & [e_1^T \quad e_2^T \quad e_3^T]^T, \\ \Psi_1 = {} & [e_d^T \quad e_5^T \quad e_6^T]^T, \\ \Psi_2 = {} & [e_d^T \quad e_1^T \quad e_3^T \quad 0 \quad e_1^T - e_4^T \quad e_1^T - e_3^T]^T, \\ \Psi_3 = {} & [e_5^T \quad e_3^T \quad e_3^T \quad e_1^T - e_3^T \quad e_3^T - e_4^T \quad 0]^T, \end{aligned}$$

$$\Psi_4 = \begin{bmatrix} e_6^T & e_4^T & e_3^T & e_1^T - e_4^T & 0 & e_4^T - e_3^T \end{bmatrix}^T,$$

$$\Psi_5 = \begin{bmatrix} e_1^T - e_3^T & \delta_1 e_7^T & \delta_1 e_3^T & \delta_1(e_1^T - e_{11}^T) & \delta_1(e_{11}^T - e_4^T) & \delta_1(e_{11}^T - e_3^T) \end{bmatrix}^T,$$

$$\Psi_6 = \begin{bmatrix} 0 & 0 & 0 & e_d^T & -e_6^T & -e_5^T \end{bmatrix}^T,$$

$$\Psi_7 = \Big[e_3^T - e_4^T \quad (\delta_2-\delta_1)e_{12}^T \quad (\delta_2-\delta_1)e_3^T \quad (\delta_2-\delta_1)(e_1^T - e_{12}^T)$$
$$(\delta_2-\delta_1)(e_{12}^T - e_4^T) \quad (\delta_2-\delta_1)(e_{12}^T - e_3^T) \Big]^T,$$

$$\Psi_8 = \Big[e_1^T \quad e_7^T \quad e_9^T]^T,$$

$$\Psi_9 = \Big[e_1^T \quad e_8^T \quad e_{10}^T]^T,$$

$$\Psi_{10} = \Big[D_{il}^T e_1^T]^T, \quad \Psi_{11} = \Big[e_{13}^T]^T,$$

$$\Pi_2 = -\frac{1}{2}\delta^2(t)\Big[\widehat{\mathcal{X}}_1 + \frac{1}{6}\widehat{\mathcal{X}}_3\Big] - [\delta_1 e_1 - e_5]\widehat{\mathcal{Z}}_3[\delta_1 e_1 - e_8]^T$$
$$- [-\frac{1}{2}\delta_1 e_1 - e_6 + 3e_8]\widehat{\mathcal{Z}}_4[-\frac{1}{2}\delta_1 e_1 - e_6 + 3e_8]^T,$$

$$\Pi_3 = -\frac{1}{2}(\delta_2-\delta_1)^2(t)\Big[\widehat{\mathcal{X}}_4 + \frac{1}{6}\widehat{\mathcal{X}}_6\Big] - [(\delta_2-\delta_1)e_1 - e_7]\widehat{\mathcal{Z}}_5[(\delta_2-\delta_1)e_1 - e_7]^T$$
$$- [-\frac{1}{2}(\delta_2-\delta_1)e_1 - e_7 + 3e_9]\widehat{\mathcal{Z}}_6[-\frac{1}{2}(\delta_2-\delta_1)e_1 - e_7 + 3e_9]^T,$$

$$\mathcal{Y}_1 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \\ e_1 - e_2 + 6e_6 - 12e_8 \end{bmatrix}, \quad \mathcal{Y}_2 = \begin{bmatrix} e_2 - e_3 \\ e_2 + e_3 - 2e_7 \\ e_2 - e_3 + 6e_7 - 12e_9 \end{bmatrix},$$

$$e_d = A_i e_1 + B_i e_{10} + C_{il} K e_2 + C_i K e_{14},$$
$$\widehat{\mathcal{M}} = \text{diag}\{\text{M, 3M, 5M}\}, \text{ and}$$
$$e_\nu = [0_{n\times(\nu-1)n} \quad I_{n\times n} \quad 0_{n\times(14-\nu)n}]^T, \text{ where } \nu = 1, 2, ..., 14.$$

Proof. The proof follows the same line as Theorem 1. Define $X = P^{-1}$, then, pre and post multiplying (10) with diag{X, X, X, X, X, X, X, X, X, X, X, X, I, I}, respectively, and defining new matrix variables $\widehat{Q} = XQX$, $\widehat{R} = XRX$, $\widehat{S} = XSX$, $\widehat{T} = XTX$, $\widehat{U} = XUX$, $\widehat{\mathcal{X}}_n = X\mathcal{X}_n X$ $(n = 1, 2, ..., 6)$, $\widehat{\mathcal{Z}}_n = X\mathcal{Z}_n X$ $(n = 1, 2, ..., 6)$, $\widehat{\mathcal{M}} = X\mathcal{M}X$ in (32), we can readily reach (32). That is, if (30)-(32) holds, the closed loop system (1) is extended dissipative under the event triggered condition (2). □

Remark 2. In Theorem 1, the extended dissipativity investigation model, which unifies the $L_2 - L_\infty$ performance, passivity, dissipativity, and H_∞ performance, is proposed for T-S fuzzy system system (7). The investigation method in Theorem 1 demonstrates a technique for the comparing execution issues.

Remark 3. In Theorem 1 and Theorem 2, a large number of variables in our LMIs were used, which is more than that in [41–44]. These variables increase the computational complexity. However, the process to gain the feasible solution just needs a few seconds by using the MATLAB toolbox [50]. Besides, according to Table 2, it is easy to see that the stability criteria obtained in this study are less conservative than the existing ones. Therefore, it is worthy to reduce the conservation of stability criteria for a few more seconds. Our target of further study is to get better stability conditions for less variables.

Remark 4. Theorem 1 and 2 give a new condition for the extended dissipativity of the systems as the event-triggered scheme is implemented. As can be seen from derivation, free-matrix-based integral inequalities were implemented in its deduction, which may bring some conservatism to some extent. It should be mentioned that the LMI technique has been widely applied to tackle various time-delay problems in control theory and applications. As the size of LMI gets bigger, e.g., the LMIs (8)-(10) and (30)-(32), there may be a key issue of its computational complexity. Fortunately, due to the fast speed of current computer processing and the case that controllers may not require online solving, its application is still very convenient. In general, the research on complexity and solution of a high dimensional LMI is an issue of concern and also an active topic in the field.

4. Numerical Examples

To show the advantage of our method, two examples for the effectiveness and reduced conservatism are provided in this section.

Example 1. Consider the well-examined case of balancing an inverted pendulum on a cart, see [51]. The pendulum angle (x_1) and the angular velocity (x_2) satisfy

$$\dot{x}_1(t) = x_2,$$
$$\dot{x}_2(t) = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + w$$

where $g = 9.8$ is the gravity constant, u is the control force applied to the cart, w is the external disturbance which is assumed to be $w = \cos(2\pi t)$, $2l$ is the length of the pendulum, $a = 1/(m + M)$, where m is the mass of the pendulum and M is the mass of the cart. This nonlinear system can be depicted by a fuzzy model as in [51] with two IF-THEN rules: Let $i = 1, 2$. Consider the accompanying plant principles for the invented pendulum:

Plant Rule 1: IF $\theta_1(t)$ is $\mathbb{M}_{i1}$, $\theta_2(t)$ is $\mathbb{M}_{i2}$,..., and $\theta_p(t)$ is $\mathbb{M}_{ip}$ (i.e., IF x_1 is about 0), THEN

$$\dot{x}(t) = h_1(\theta(t))\Big[A_1x(t) + B_1\omega(t) + C_1u(t)\Big],$$
$$y(t) = h_1(\theta(t))\Big[D_1x(t)\Big].$$

Plant Rule 2: IF $\theta_1(t)$ is $\mathbb{M}_{i1}$, $\theta_2(t)$ is $\mathbb{M}_{i2}$,..., and $\theta_p(t)$ is $\mathbb{M}_{ip}$ (i.e., IF x_1 is about $\pm\pi/2$), THEN

$$\dot{x}(t) = h_2(\theta(t))\Big[A_2x(t) + B_2\omega(t) + C_2u(t)\Big],$$
$$y(t) = h_2(\theta(t))\Big[D_2x(t)\Big].$$

The membership functions are $h_1(\theta(t)) = (1 - 1/1 + e^{-7(x_1-\pi/4)})(1/1 + e^{-7(x_1+\pi/4)})$ and $h_2(\theta(t)) = 1 - h_1(\theta(t))$. In order to illustrate the use of Theorem 2, we assume the fuzzy

Table 1
Minimum γ for fixed $\delta_1 = 0.06$ and different values of δ_2 in Example 1.

	δ_2				
Method	0.1	0.2	0.3	0.4	0.5
H_∞ Performance	0.1134	0.1071	0.0844	0.0140	0.0022
$l_2 - l_\infty$ Performance	0.2286	0.1751	0.1307	0.0433	0.0164
Passivity	0.1501	0.1281	0.1136	0.0440	0.0075
Dissipativity	0.0121	0.0116	0.0088	0.0013	0.0009

time-delay model considered here as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.29 & 0 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ D_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 12.63 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ D_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

(1) ***H_∞* Performance**: Let $\Phi_1 = -I_2$, $\Phi_2 = 0$, $\Phi_3 = \gamma^2 I_2$, and $\Phi_4 = 0$. The extended dissipativity reduces to the standard H_∞ performance. Utilizing Theorem 2, the permissible least H_∞ performance γ can be obtained for fixed $\sigma = 0.2$, $\delta_1 = 0.06$ and $\delta_2 = 0.5$. The relation between γ and δ_2 (for fixed δ_1) is demonstrated in Table 1. Then, we can get the unknown matrices with gain matrices K_1 and K_2, and the trigger matrix Ω as follows.

$$P = 10^5 \times \begin{bmatrix} 2.2673 & -1.9147 \\ -1.9147 & 1.6956 \end{bmatrix}, Q = 10^5 \times \begin{bmatrix} 0.9718 & 0.2700 \\ 0.2700 & 8.5700 \end{bmatrix}, R = \begin{bmatrix} 0.3622 & 0.1423 \\ 0.1423 & 4.1175 \end{bmatrix},$$

$$S = 10^4 \times \begin{bmatrix} 1.0444 & 0.0191 \\ 0.0191 & 0.1814 \end{bmatrix}, \ T = 10^7 \times \begin{bmatrix} 1.1573 & 0.0220 \\ 0.0220 & 0.0005 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 1.03612 & 0.0274 \end{bmatrix}, \ K_2 = \begin{bmatrix} 1.0876 & 0.2347 \end{bmatrix}, \ \Omega = 10^{12} \times \begin{bmatrix} 2.4363 & -0.0000 \\ -0.0000 & 0.1704 \end{bmatrix}.$$

(2) **$L_2 - L_\infty$ Performance**: Let $\Phi_1 = 0$, $\Phi_2 = 0$, $\Phi_3 = \gamma^2 I_2$, and $\Phi_4 = I_2$. The extended dissipativity becomes the $l_2 - l_\infty$ performance. For given $\sigma = 0.2$, $\delta_1 = 0.06$ and $\delta_2 = 0.5$, the minimum estimation of γ is obtained as given in Table 1 by solving the LMIs in Theorem 2. We can get the unknown matrices with gain matrices as follows.

$$P = 10^8 \times \begin{bmatrix} 7.4866 & -6.3528 \\ -6.3528 & 5.6500 \end{bmatrix}, Q = 10^9 \times \begin{bmatrix} 0.3207 & 0.0901 \\ 0.0901 & 2.8531 \end{bmatrix}, R = \begin{bmatrix} 0.1185 & 0.0468 \\ 0.0468 & 1.3575 \end{bmatrix},$$

$$S = 10^{12} \times \begin{bmatrix} 3.4234 & 0.0658 \\ 0.0658 & 0.5960 \end{bmatrix}, \ T = 10^{12} \times \begin{bmatrix} 3.8858 & 0.0736 \\ 0.0736 & 0.0017 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -0.5563 & 0.1275 \end{bmatrix}, \ K_2 = \begin{bmatrix} -1.4396 & 0.5763 \end{bmatrix}, \ \Omega = 10^{15} \times \begin{bmatrix} 7.9471 & 0.0004 \\ 0.0004 & 0.6202 \end{bmatrix}.$$

(3) **Passivity**: When $\Phi_1 = 0$, $\Phi_2 = I_2$, $\Phi_3 = \gamma I_2$, and $\Phi_4 = 0$, the passivity performance is obtained. For $\sigma = 0.2$, $\delta_1 = 0.06$, and $\delta_2 = 0.3$, the different value of γ is obtained as listed in Table 1 by solving the LMIs in Theorem 2 with a different value of upper bound δ_2. Then, we can get the unknown matrices with gain matrices K_1 and K_2, and the trigger matrix Ω as

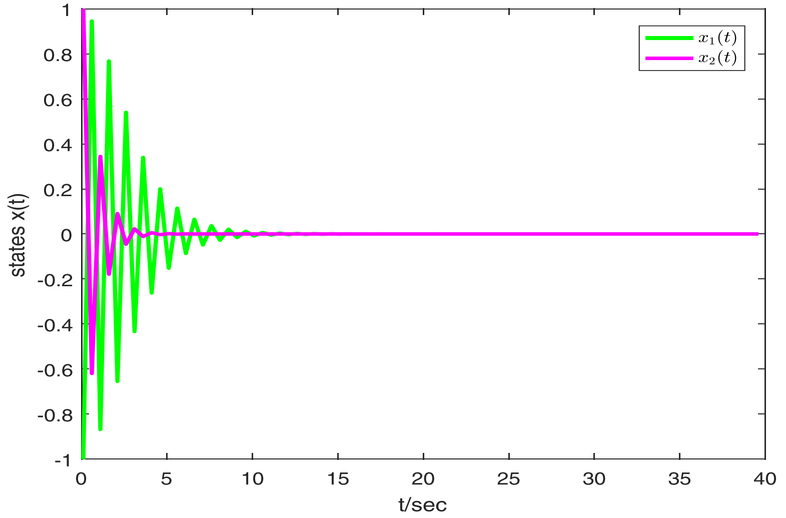

Fig. 1. State responses of system (1).

follows.

$$P = 10^8 \times \begin{bmatrix} 7.1465 & -6.4113 \\ -6.4113 & 6.0355 \end{bmatrix}, Q = 10^9 \times \begin{bmatrix} 0.3062 & 0.0741 \\ 0.0741 & 2.9729 \end{bmatrix}, R = \begin{bmatrix} 0.1137 & 0.0373 \\ 0.0373 & 1.3763 \end{bmatrix},$$

$$S = 10^7 \times \begin{bmatrix} 3.3026 & 0.0860 \\ 0.0860 & 0.6291 \end{bmatrix}, \quad T = 10^{10} \times \begin{bmatrix} 4.2897 & 0.0786 \\ 0.0786 & 0.0018 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0.0027 & 0.3467 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1.6954 & 0.7561 \end{bmatrix}, \quad \Omega = 10^{15} \times \begin{bmatrix} 9.1241 & 0.0005 \\ 0.0005 & 1.3401 \end{bmatrix}.$$

(4) **Dissipativity**: Let $\Phi_1 = -7I_2$, $\Phi_2 = I_2$, $\Phi_3 = 3I_2$, and $\Phi_4 = 0$. If the LMIs in (30)–(32) are feasible, then system (1) is (Φ_1, Φ_2, Φ_3)-dissipative. When $\Phi_3 = \mathcal{R} - \gamma$, the performance reduces to $(\Phi_1,\ \Phi_2,\ \mathcal{R}) - \gamma$-dissipative, where γ denotes the level of dissipativity. The various estimations of admissible maximum dissipativity level are shown in Table 1, from which it can be observed that the smaller the upper bound of delay δ_2 is, the larger the dissipativity level is. The simulation results are shown in Figs. 1-5. Fig. 1 shows the state responses of system (1). Fig. 2 shows the responses of the closed-loop control effort $u(t)$. Fig. 3 shows the angle displacement of the pendulum from the vertical. Fig. 4 shows the angular velocity of the pendulum, and Fig. 5 shows that the position of the cart.

$$P = 10^5 \times \begin{bmatrix} 5.0607 & -4.2527 \\ -4.2527 & 3.7438 \end{bmatrix}, Q = 10^6 \times \begin{bmatrix} 0.2166 & 0.0625 \\ 0.0625 & 1.9009 \end{bmatrix}, R = \begin{bmatrix} 0.7963 & 0.3242 \\ 0.3242 & 9.0438 \end{bmatrix},$$

$$S = 10^4 \times \begin{bmatrix} 2.3002 & 0.0418 \\ 0.0418 & 0.3935 \end{bmatrix}, \quad T = 10^7 \times \begin{bmatrix} 2.5642 & 0.0488 \\ 0.0488 & 0.0011 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 1.2049 & 0.8461 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 2.0012 & 0.9415 \end{bmatrix}, \quad \Omega = 10^{12} \times \begin{bmatrix} 5.2393 & -0.0001 \\ -0.0001 & 0.3660 \end{bmatrix}$$

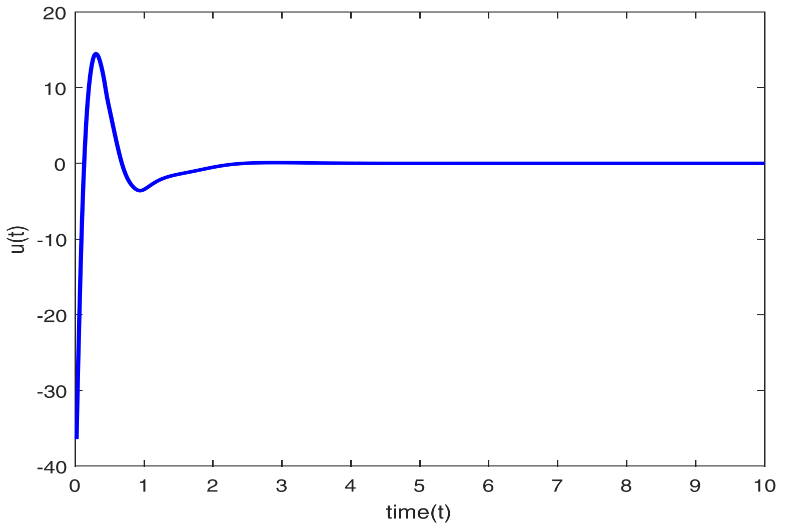

Fig. 2. The closed-loop control force $u(t)$.

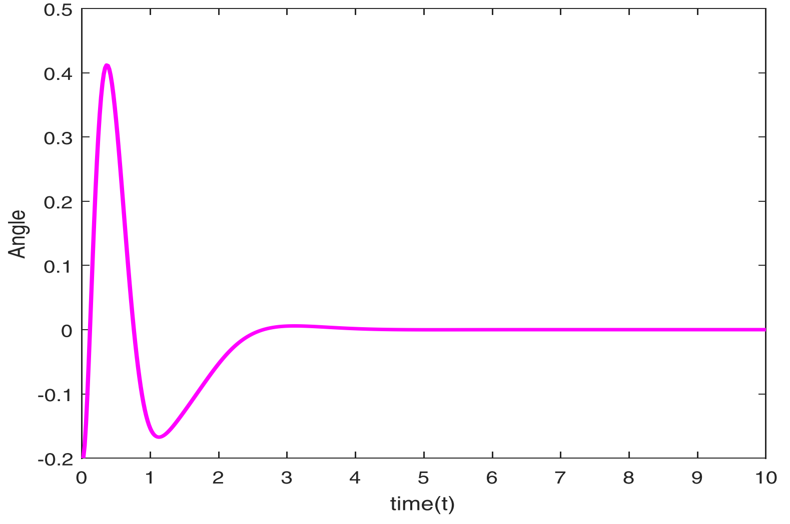

Fig. 3. The angle displacement of the pendulum from the vertical.

Example 2. Consider the T-S fuzzy system (7) with time-varying delay and the following rules [41–44]: Plant Rule 1: IF $\theta(t)$ is $\pm\pi/2$, THEN

$$\dot{x}(t) = h_1(\theta(t))\Big[A_1x(t) + B_1\omega(t) + C_1K(t-\delta(t)) + C_1Ke_k(t)\Big],$$

$$y(t) = h_1(\theta(t))\Big[D_1x(t)\Big].$$

IF $\theta(t)$ is 0, THEN

$$\dot{x}(t) = h_2(\theta(t))\Big[A_2x(t) + B_2\omega(t) + C_2K(t-\delta(t)) + C_2Ke_k(t)\Big],$$

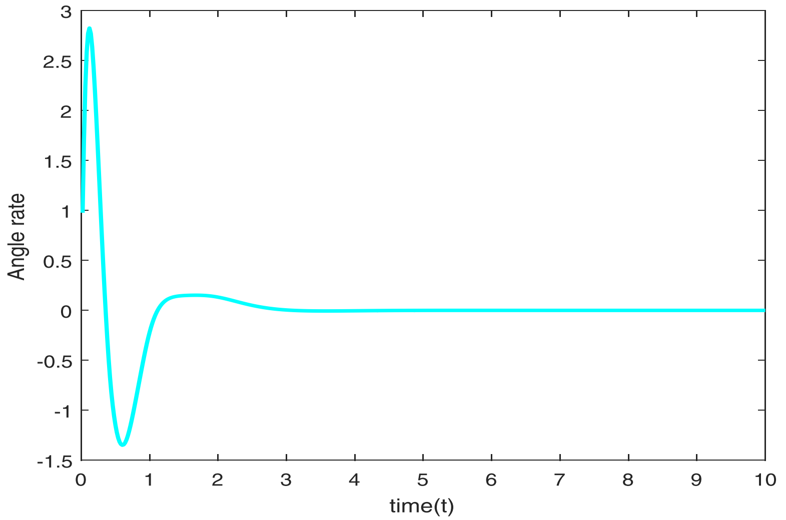

Fig. 4. The angular velocity of the pendulum.

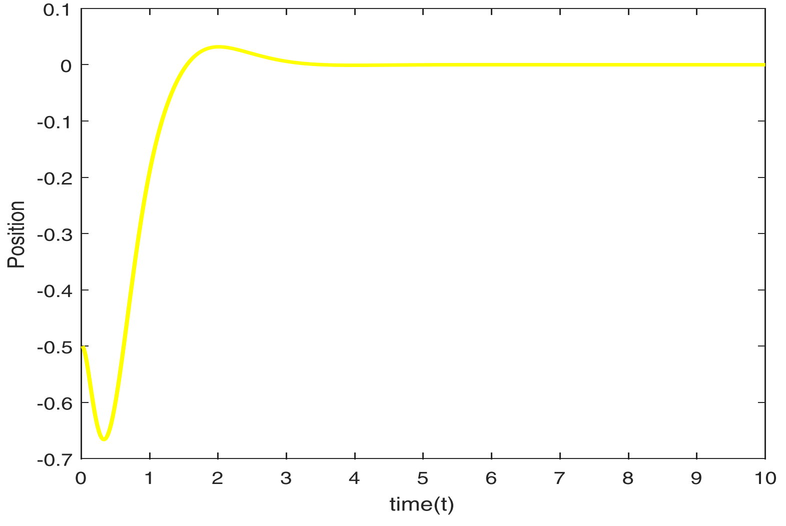

Fig. 5. The cart position response.

$$y(t) = h_2(\theta(t))\Big[D_2 x(t)\Big].$$

where

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} C_1 = \begin{bmatrix} -1.5 & 1 \\ 0 & -0.75 \end{bmatrix},\ D_1 = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ C_2 = \begin{bmatrix} -1 & 0 \\ 1 & -0.85 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

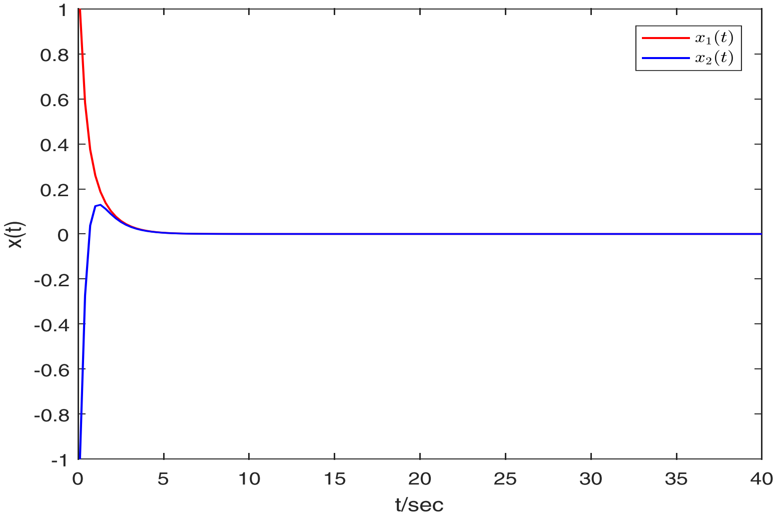

Fig. 6. State responses of initial conditions.

Table 2
Maximum allowable upper bound δ_2 for four different lower bounds of δ_1 in Example 2.

δ_1	0.2	0.4	0.6	0.8	Method
	0.7945	0.8487	0.9316	1.0325	[41]
	1.141	1.150	1.72	1.209	[42]
δ_2	1.1639	1.1734	1.1994	1.2532	[43]
	1.3101	1.3116	1.2999	1.2674	[44]
	1.7410	1.8628	2.0433	2.5410	Theorem 1

The membership functions for above Rules 1 and 2 are $h_1(\theta(t)) = \sin^2(\theta(t))$, $h_2(\theta(t)) = \cos^2(\theta(t))$, where $\theta(t) = x_1(t)$. For different values of δ_1, the admissible maximum upper bound of δ_2 obtained from [41–44] and that of Theorem 1 in this paper are compared in Table 2. As shown in Table 2, the maximum allowable upper bounds are larger than those obtained form [41–44]. With the initial state conditions $[1, -1]^T$, Fig. 7 demonstrates the reproduction aftereffects of the state responses of the T-S fuzzy system (7). It shows from the simulation results (Fig. 7) that the admissible maximum allowable upper bound of δ_2 listed in Table 2 are capable of guaranteeing asymptotical stability of (7).

5. Conclusion

In this study, an event-triggered control problem has been proposed through the extended dissipativity of a T-S fuzzy model. By picking a novel Lyapunov-Krasovskii functional, criteria for the extended dissipativity were derived based on free-matrix-based double integral inequality. The numerical examples were presented to illustrate the proposed methodology. The obtained results were less conservative than the existing ones. The control performance was demonstrated with an inverted pendulum through analyzing the stability and stabilization issues. As further works, dynamic output-feedback controller and adaptive boundary control

[52–55], event-triggered control system [56,57], and adaptive event-triggered communication scheme will be sought to obtain a better control and communication performance.

Declaration of Competing Interest

The authors declare that they have no known competing financialinterestsor personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE trans. syst. man cybern. 1 (1985) 116–132.
[2] C. Peng, L.Y. Wen, J.Q. Yang, On delay-dependent robust stability criteria for uncertain T-S fuzzy systems with interval time-varying delay, Int. J. Fuzzy Syst. 13 (2011) 35–44.
[3] O. Kwon, M.J. Park, S.M. Lee, J.H. Park, Augmented Lyapunov–Krasovskii functional approaches to robust stability criteria for uncertain Takagi–Sugeno fuzzy systems with time-varying delays, Fuzzy Sets Syst. 201 (2012) 1–19.
[4] H.B. Zeng, J.H. Park, J.W. Xia, S.P. Xiao, Improved delay-dependent stability criteria for T–S fuzzy systems with time-varying delay, Appl. Math. Comput. 235 (2014) 492–501.
[5] J. Yang, W.P. Luo, Y.H. Wang, C.S. Duan, Improved stability criteria for T-S fuzzy systems with time-varying delay by delay-partitioning approach, Int. J. Control Autom. Syst. 13 (2015) 1521–1529.
[6] J. Wu, Q. Fu, Z. Li, H_∞ control via state observer feedback for the T–S fuzzy singular system, Int. J. Mach. Learn. Cybern. 8 (2017) 619–626.
[7] X. Su, F. Xia, Y.D. Song, M.V. Basin, L. Zhao, L_2–L_∞ output feedback controller design for fuzzy systems over switching parameters, IEEE Trans. Fuzzy Syst. 26 (2018) 3755–3769.
[8] Y. Ma, M. Chen, Q. Zhang, Memory dissipative control for singular T–S fuzzy time-varying delay systems under actuator saturation, J. Frankl. Inst. 352 (2015) 3947–3970.
[9] W. Zheng, Z.M. Zhang, H.B. Wang, H.R. Wang, P.H. Yin, Stability analysis and dynamic output feedback control for nonlinear T-S fuzzy system with multiple subsystems and normalized membership functions, Int. J. Control Autom. Syst. 16 (2018) 2801–2813.
[10] J. An, G. Wen, C. Lin, R. Li, New results on a delay-derivative-dependent fuzzy H_∞ filter design for T–S fuzzy systems, IEEE Trans. Fuzzy Syst. 19 (2011) 770–779.
[11] L.Y. Hao, J.H. Park, D. Ye, Fuzzy logic systems-based integral sliding mode fault-tolerant control for a class of uncertain non-linear systems, IET Contr. Theory Appl. 10 (2016) 300–311.
[12] J. Lian, S. Li, J. Liu, T–S fuzzy control of positive Markov jump nonlinear systems, IEEE Trans. Fuzzy Syst. 26 (2018) 2374–2383.
[13] X.H. Chang, J. Xiong, J.H. Park, Fuzzy robust dynamic output feedback control of nonlinear systems with linear fractional parametric uncertainties, Appl. Math. Comput. 291 (2016) 213–225.
[14] A.T. Nguyen, T. Laurain, R. Palhares, J. Lauber, C. Sentouh, J.C. Popieul, LMI-based control synthesis of constrained Takagi–Sugeno fuzzy systems subject to L_2 or L_∞ disturbances, Neurocomputing 207 (2016) 793–804.
[15] C. Du, C. Yang, F. Li, P. Shi, Asynchronous output feedback control for fuzzy Markovian jump systems via sliding mode, J. Frankl. Inst. 356 (2019) 8952–8970.
[16] L. Liu, Y. Yin, L. Wang, R. Bai, Stability analysis for switched positive T–S fuzzy systems, Neurocomputing 173 (2016) 2009–2013.
[17] C. Lin, G. Wang, T.H. Lee, Y. He, LMI approach to analysis and control of Takagi-Sugeno fuzzy systems with time delay, Springer Science & Business Media, 2007.
[18] C.C. Ku, G.W. Chen, Robust H_∞ control of uncertain stochastic systems with time-varying interval delays, Int. J. Control Autom. Syst. 16 (2018) 243–255.

[19] B. Wu, C.L. Wang, Y.J. Hu, X.L. Ma, Stability analysis for time-delay systems with nonlinear disturbances via new generalized integral inequalities, Int. J. Control Autom. Syst. 16 (2018) 2772–2780.

[20] H.J. Yu, Y. He, M. Wu, Improved robust passive criteria of neural networks with discrete and distributed delays based on extended reciprocally convex matrix inequality, Int. J. Control Autom. Syst. 16 (2018) 2480–2488.

[21] A. Seuret, F. Gouaisbaut, Hierarchy of LMI conditions for the stability analysis of time-delay systems, Syst. Control Lett. 81 (2015) 1–7.

[22] X.M. Zhang, Q.L. Han, Z. Zeng, Hierarchical type stability criteria for delayed neural networks via canonical Bessel–Legendre inequalities, IEEE T. Cybern. 48 (2017) 1660–1671.

[23] W. Heemels, J. Sandee, P. Van Den Bosch, Analysis of event-driven controllers for linear systems, Int. J. Control 81 (2008) 571–590.

[24] C. Peng, J. Zhang, Event-triggered output-feedback H_∞ control for networked control systems with time-varying sampling, IET Contr. Theory Appl. 9 (2015) 1384–1391.

[25] C. Peng, M. Yang, J. Zhang, M. Fei, S. Hu, Network-based H_∞ control for T–S fuzzy systems with an adaptive event-triggered communication scheme, Fuzzy Sets Syst. 329 (2017) 61–76.

[26] H. Yan, S. Yan, H. Zhang, H. Shi, L_2 control design of event-triggered networked control systems with quantizations, J. Frankl. Inst. 352 (2015) 332–345.

[27] Z. Tang, Event-triggered consensus of linear discrete-time multi-agent systems with time-varying topology, Int. J. Control Autom. Syst. 16 (3) (2018) 1179–1185.

[28] D. Zhao, T. Dong, W. Hu, Event-triggered consensus of discrete time second-order multi-agent network, Int. J. Control Autom. Syst. 16 (1) (2018) 87–96.

[29] Z. Hu, P. Shi, L. Wu, C.K. Ahn, Event-based distributed filtering approach to nonlinear stochastic systems over sensor networks, Int. J. Control Autom. Syst. 17 (2019) 896–906.

[30] M. Donkers, W. Heemels, Output-based event-triggered control with guaranteed L_∞-gain and improved and decentralized event-triggering, IEEE Trans. Autom. Control 57 (2011) 1362–1376.

[31] L. Wu, Y. Gao, J. Liu, H. Li, Event-triggered sliding mode control of stochastic systems via output feedback, Automatica 82 (2017) 79–92.

[32] Y. Wen, Y. Tan, X. Su, Event-triggered dissipative filtering of fuzzy nonlinear dynamic systems, in: 2017 Eighth International Conference on Intelligent Control and Information Processing (ICICIP), IEEE, 2017, pp. 249–254.

[33] C. Deng, M.J. Er, G.H. Yang, N. Wang, Event-triggered consensus of linear multiagent systems with time-varying communication delays, IEEE T. Cybern. (2019) doi:10.1109/TCYB.2019.2922740.

[34] D. Liu, G.H. Yang, M.J. Er, Event-triggered control for T–S fuzzy systems under asynchronous network communications, IEEE Trans. Fuzzy Syst. 28 (2020) 390–399.

[35] F.L. Qu, Z.H. Guan, D.X. He, M. Chi, Event-triggered control for networked control systems with quantization and packet losses, J. Frankl. Inst. 352 (2015) 974–986.

[36] H. Ji, H. Zhang, B. Cui, Event-triggered H_∞ filtering control for a class of distributed parameter systems with Markovian switching topology, J. Frankl. Inst. 355 (2018) 5928–5956.

[37] J. Wang, M. Chen, H. Shen, Event-triggered dissipative filtering for networked semi-Markov jump systems and its applications in a mass-spring system model, Nonlinear Dyn. 87 (2017) 2741–2753.

[38] L. Zha, J.A. Fang, X. Li, J. Liu, Event-triggered output feedback H_∞ control for networked Markovian jump systems with quantizations, Nonlinear Anal.-Hybrid Syst. 24 (2017) 146–158.

[39] D. Zhang, Q.L. Han, X. Jia, Network-based output tracking control for T–S fuzzy systems using an event-triggered communication scheme, Fuzzy sets syst. 273 (2015) 26–48.

[40] F. Li, L. Gao, G. Dou, B. Zheng, Dual-side event-triggered output feedback H_∞ control for NCS with communication delays, Int. J. Control Autom. Syst. 16 (2018) 108–119.

[41] C.H. Lien, K.W. Yu, W.D. Chen, Z.L. Wan, Y.J. Chung, Stability criteria for uncertain Takagi-Sugeno fuzzy systems with interval time-varying delay, IET Contr. Theory Appl. 1 (2007) 764–769.

[42] E. Tian, D. Yue, Y. Zhang, Delay-dependent robust H_∞ control for T–S fuzzy system with interval time-varying delay, Fuzzy sets syst. 160 (2009) 1708–1719.

[43] F.O. Souza, V.C. Campos, R.M. Palhares, On delay-dependent stability conditions for Takagi–Sugeno fuzzy systems, J. Frankl. Inst. 351 (2014) 3707–3718.

[44] X. Su, H. Zhou, Y.D. Song, An optimal divisioning technique to stabilization synthesis of T–S fuzzy delayed systems, IEEE T. Cybern. 47 (2016) 1147–1156.

[45] D. Yue, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2012) 475–481.

[46] H. Gao, J. Xia, G. Zhuang, Robust finite-time extended dissipative control for a class of uncertain switched delay systems, Int. J. Control Autom. Syst. 16 (2018) 1459–1468.

[47] B. Zhang, W.X. Zheng, S. Xu, Filtering of Markovian jump delay systems based on a new performance index, IEEE Trans. Circuits Syst. I: Regul. Pap. 60 (2013) 1250–1263.
[48] O. Kwon, J.H. Park, S.M. Lee, Augmented lyapunov functional approach to stability of uncertain neutral systems with time-varying delays, Appl. Math. Comput. 207 (2009) 202–212.
[49] C. Hua, S. Wu, X. Yang, X. Guan, Stability analysis of time-delay systems via free-matrix-based double integral inequality, Int. J. Syst. Sci. 48 (2017) 257–263.
[50] G. Balas, R. Chiang, A. Packard, M. Safonov, Robust control toolbox user guide, The MathWorks Inc., Natick, MA (2007).
[51] C. Lin, Q.G. Wang, T.H. Lee, Y. He, Design of observer-based H_∞ control for fuzzy time-delay systems, IEEE Trans. Fuzzy Syst. 16 (2008) 534–543.
[52] Q.C. Nguyen, K.S. Hong, Asymptotic stabilization of a nonlinear axially moving string by adaptive boundary control, J. Sound Vibr. 329 (2010) 4588–4603.
[53] W. He, S.S. Ge, B.V.E. How, Y.S. Choo, K.S. Hong, Robust adaptive boundary control of a flexible marine riser with vessel dynamics, Automatica 47 (2011) 722–732.
[54] G.H. Kim, K.S. Hong, Adaptive sliding mode control of an offshore container crane with unknown disturbances, IEEE-ASME Trans. Mechatron. 24 (2019) 2850–2861.
[55] K.S. Hong, P.T. Pham, Control of axially moving systems: A review, Int. J. Control Autom. Syst. 17 (2019) 2983–3008.
[56] W. Lin, X. Li, D. Yao, X. Gao, Q. Zhou, Observer-based event-triggered sliding mode control for Markov jump systems with partially unknown transition probabilities, Int. J. Control Autom. Syst. 17 (2019) 1626–1633.
[57] Z. Tang, C. Li, Distributed event-triggered containment control for discrete-time multi-agent systems, Int. J. Control Autom. Syst. 16 (2018) 2727–2732.