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Non-fragile synchronization of genetic regulatory networks with randomly occurring controller gain fluctuation☆

M. Syed Ali[a,*], R. Agalya[a], Keum-Shik Hong[b]

[a] Department of Mathematics, Thiruvalluvar University, Vellore-632115, Tamil Nadu, India
[b] School of Mechanical Engineering, Pusan National University, 2 Busandaehak-ro, Geumjeong-gu, Busan 46241, Republic of Korea

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ABSTRACT

This study examines the non-fragile synchronization of genetic regulatory networks (GRNs) with time-varying delays. Genetic regulatory network is formulated and sufficient conditions are derived to guarantee its synchronization based on master-slave system approach. The non-fragile observer based feedback controller gains are assumed to have the random fluctuations, two different types of uncertainties which perturb the gains are taken into account. By constructing a suitable Lyapunov-Krasovskii stability theory together with linear matrix inequality (LMI) approach we derived the delay-dependent criteria to ensure the asymptotic stability of the error system, which guarantees the master system synchronize with the slave system. The expressions for the non-fragile controller can be obtained by solving a set of LMIs using standard softwares. Finally, some numerical examples are included to show that the proposed method is less conservative than existing ones.

1. Introduction

During the past few decades, various genetic regulatory networks (GRNs) have been created because their biological characteristics refer to data in genes that collectively affect biological propagation, inheritance, and variation. Stunning progression has been achieved with the rapid advance of gene sequencing technology. However, organism research still requires complex, difficult methodologies together with massive amounts of experimental measurements, observations, and analyses. In a living cell, there is a complete set of genes, but they are not all expressed in every tissue. GRNs are actually significant mechanisms that regulate gene expression, that is, the expression is regulated by its production. GRNs, which consist of DNA, RNA, little molecules, proteins, and the regulatory interactions among them, have received considerable attention in the fields of medical and biological technologies [1–13]. Basically, there are two kinds of GRNs. The first is the Boolean model [14], where each gene's activity is represented by one of two states (for example, ON or OFF). The second is the differential equation [15,16], where the variables represent mRNAs and proteins. Many studies on GRNs with time-delayed signals have been presented in the literature [17,18]. [19–25] discussed the stability analysis problem of GRNs with delay signals. In [25], the authors examined robust filtering problems for GRNs with stochastic effects and delay signals. The majority of the pragmatic issues [26,27] concern framework strength, so the examination of stability of delayed GRNs has drawn interest from various fields, and a lot of astounding results have been accounted (see for instance, [17,18] and the references therein).

On the other hand, the synchronization control of chaotic systems plays a very important role in applications cherish secure

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* Corresponding author.
E-mail addresses: syedgru@gmail.com (M. Syed Ali), kshong@pusan.ac.kr (K.-S. Hong).

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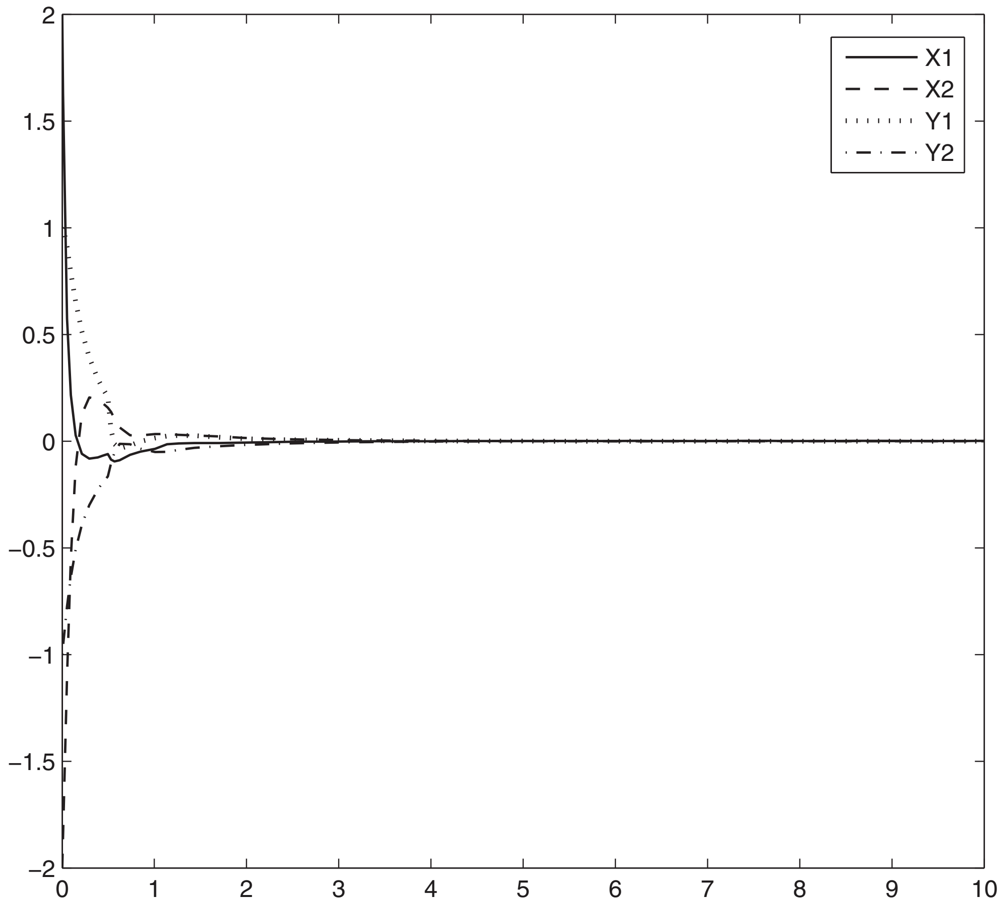

Fig. 1. State trajectory of the system in Example 4.1.

Table 1
Maximum value of σ for different values of τ in Example 4.2.

τ	0.125	0.25	0.55	1.0	1.1
[52]	0.5	-	-	-	-
[53]	-	-	1.0	-	-
[54]	2.8273	2.1661	1.1544	0.4904	0.3845
[55]	5.2268	4.8848	3.7840	2.5066	2.3018
[56]	11.9400	9.3495	6.7436	4.7079	4.3571
Corollary 3.4	12.1022	10.0027	7.2242	5.9114	4.5841

communication, image encryption, image segmentation, two-dimensional motion control and information processing [28–34]. Chaotic synchronization means that the states of the connected systems are coincide one another, that is, it's a method to synchronize two identical chaotic systems (drive-response concept) with completely different initial conditions. Chaotic signal is conjectured to be utilized in communication schemes thanks to its inherent wide band characteristic, as a result, several synchronization ways work has been dispensed for chaotic neural system with time delays [35,36]. The issue of synchronization of chaotic systems with randomly occurring uncertainties via stochastic sampled-data control has been investigated in [37].

It is insinuated that, in sensible things as a section of a close-by float framework as noted in sensible designed controller the proceed out of uncertainity in its coefficients isn't stayed aloof from, then the murkiness within the controller execution brought on by the compelled word length in any modernised structures or further turning of parameters within the final controller use is happened. on these lines, it's of nice importance to chart a non-fragile controller specified the controller isn't attentive to uncertainties. the problem of non-fragile management has became a beguiling purpose in each theory and accomplishable execution. The execution of the running strategy of framework, the controller use is susceptible to the exactitude of A/D conversion and D/A conversion, spherical off errors in numerical numbers and additionally the parameter of electronic elements area unit finished in non-fragile,etc. Within the recent years, there was AN vast examination believed being utilised of non-fragile controller (for event [38–47]).

Inspired by the above investigations, this paper examines the non-fragile synchronization for GRNs with randomly occurring controller gain fluctuation. Regardless, to the simplest of our data, the problem of non-fragile synchronization of GRNs with randomly occurring controller gain fluctuation has not been entirely inspected. This motivates our study.

The main contribution of this study is given below:

* By constructing a proper Lyapunov-Krasovskii functional (LKF) with double and triple integrals and standard integral inequality techniques, new stability criteria for GRNs with randomly occurring controller gain fluctuation are obtained in terms of LMI.
* Gain matrices for the proposed controller design are determined by solving the proposed LMI condition.
* Finally, numerical results are provided to demonstrate the adequacy of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. the notation * represents the elements below the main diagonal of a symmetric matrix. A^T means the transpose of A and A^{-1} is the inverse of A. I denotes the identity matrix with appropriate dimensions. $X > 0$ means that the matrix X is real symmetric positive definite with appropriate dimensions and $diag\{a, b, ...,z\}$ denotes the block-diagonal matrix with $a, b, ...,z$ in the diagonal entries.

2. System description and preliminaries

Consider the following GRNs with time varying delay:

$$\left.\begin{aligned}\dot{\zeta}_i(t) &= -a_i\zeta_i(t) + \sum_{j=1}^{n} b_{ij}^{(1)}\tilde{g}_j(\delta_j(t-\sigma(t))),\\ \dot{\delta}_j(t) &= -c_j\delta_j(t) + \sum_{i=1}^{n} d_{ji}^{(1)}(\zeta_i(t-\tau(t)))\end{aligned}\right\}$$
$$\zeta_i(t) = \phi_i(t),\ \ \delta_j(t) = \phi_j^*(t),\ \ \forall\ t \in [-h, 0], \tag{1}$$

where $\zeta_i(t)$ *and* $\delta_j(t)$ are concentrations of mRNA and protein at time t, respectively. The non linear function $\tilde{g}_j(\cdot)$ is activation function. The positive constants a_i, c_j are the degradation rates of the mRNA and protein, respectively. $b_{ij}^{(1)}$ is the regulative. $d_{ji}^{(1)}$ is the translation rate. $\sigma(t)$, $\tau(t)$ denotes the time varying delays satisfying

$$0 \le \sigma(t) \le \sigma,\ \dot{\sigma}(t) = \mu_1,\ 0 \le \tau(t) \le \tau,\ \dot{\tau}(t) = \mu_2,$$

where σ, τ, μ_1, and μ_2 are constants. $\phi_i(t)$ and $\phi_j^*(t)$ are initial conditions, where $h \in \sigma \vee \tau$.

Assumption (A): For $i \in \{1, 2,...,n\}$, $\forall\ x, \tilde{x} \in \mathbb{R}$, $x \ne \tilde{x}$, the genetic activation function $\tilde{g}_i(\cdot)$ is continuous, bounded and satisfies,

$$L_i^- \le \frac{\tilde{g}_i(x) - \tilde{g}_i(\tilde{x})}{x - \tilde{x}} \le L_i^+ \tag{2}$$

where l_i^- *and* l_i^+ are constants. Denote $L_1 = diag\{l_1^- l_1^+,...,l_n^- l_n^+\}$, $L_2 = diag\left\{\frac{l_1^- + l_1^+}{2}, \ ..., \frac{l_n^- + l_n^+}{2}\right\}$.

Then, the master system (1) can be written as the following compact matrix form:

$$\mathcal{M}: \begin{cases}\dot{\zeta}(t) = -\mathscr{A}\zeta(t) + \mathscr{B}\mathrm{g}(\delta(t-\sigma(t))),\\ \dot{\delta}(t) = -\mathscr{C}\delta(t) + \mathscr{D}\zeta(t-\tau(t))\end{cases} \tag{3}$$

where $\mathscr{A} = diag\{a_1, a_2,...,a_n\} > 0$, $\mathscr{C} = diag\{c_1, c_2,...,c_n\} > 0$, $\mathscr{B} = (b_{ij}^{(k)})_{(n\times n)}$, $\mathscr{D} = (d_{ij}^{(k)})_{(n\times n)}$.

Here we use the master-slave synchronization approach to derive the synchronization criteria, where the master system with the state variables denoted by $\zeta(t)$ and $\delta(t)$ and the slave system having identical dynamical equations denoted by the state variables $\hat{\zeta}(t)$ and $\hat{\delta}(t)$. However, the initial conditions on the master system are different from those of the slave system. In order to derive the synchronization behavior for the considered genetic regulatory networks, we consider the following slave system corresponding to the master system (3):

$$\mathcal{S}: \begin{cases}\dot{\hat{\zeta}}(t) = -\mathscr{A}\hat{\zeta}(t) + \mathscr{B}\mathrm{g}(\hat{\delta}(t-\sigma(t))) + u(t),\\ \dot{\hat{\delta}}(t) = -\mathscr{C}\delta(t) + \mathscr{D}\hat{\zeta}(t-\tau(t)) + v(t),\end{cases} \tag{4}$$

where u(t) and v(t) are control inputs. We consider the following non-fragile controller:

$$\left.\begin{aligned}u(t) &= (\mathscr{K}_1 + \alpha(t)\Delta\mathscr{K}_1(t))x(t),\\ v(t) &= (\mathscr{K}_2 + \beta(t)\Delta\mathscr{K}_2(t))y(t),\end{aligned}\right\} \tag{5}$$

where $\mathscr{K}_1$, $\mathscr{K}_2$ are the controller gain matrices to be determined, and the real-valued matrix $\Delta\mathscr{K}_i(t)(i = 1, 2)$ represents possible controller gains. It is assumed that $\Delta\mathscr{K}_i(t)$ has the following structure:

$$\Delta\mathscr{K}_i(t) = \mathscr{H}_i\tilde{\Delta}(t)\mathscr{E}_i,\ \ (i = 1, 2), \tag{6}$$

where $\tilde{\Delta}(t) \in \mathbb{R}^{k\times l}$ is an unknown time-varying matrix satisfying

$$\tilde{\Delta}(t)^T\tilde{\Delta}(t) \le I, \tag{7}$$

and $\mathscr{H}_i$, $\mathscr{E}_i$ are known constant matrices. The stochastic variable $\alpha(t)$, $\beta(t) \in \mathbb{R}$ is introduced to describe the phenomena of randomly occurring controller gain fluctuation, which is Bernoulli-distributed white noise sequences taking on values of zero or one with

$$Pr\{\alpha(t) = 1\} = \alpha,\ \ Pr\{\alpha(t) = 0\} = 1 - \alpha,\ Pr\{\beta(t) = 1\} = \beta,\ \ Pr\{\beta(t) = 0\} = 1 - \beta,$$

where $\alpha, \beta \in [0, 1]$ is known constant.

Let the synchronization error signals $x(t) = \hat{\zeta}(t) - \zeta(t)$, and $y(t) = \hat{\delta}(t) - \delta(t)$. Thus, error dynamics between systems (3) and (4) can be expressed as:

$$\left.\begin{aligned} \dot{x}(t) &= (-\mathscr{A} + (\mathscr{K}_1 + \alpha(t)\Delta\mathscr{K}_1(t)))x(t) + \mathscr{B}g(y(t - \sigma(t)),\\ \dot{y}(t) &= (-\mathscr{C} + (\mathscr{K}_2 + \beta(t)\Delta\mathscr{K}_2(t)))y(t) + \mathscr{D}x(t - \tau(t)) \end{aligned}\right\} \tag{8}$$

where $g(x(t)) = \tilde{g}(\hat{\delta}(t)) - \tilde{g}(\delta(t))$.

To obtain our main results we use the following Lemmas:

Lemma 2.1 ([48]). *Let $\mathscr{M}$, $\mathscr{P}$, $\mathscr{Q}$ be given matrices such that $\mathscr{Q} > 0$, then*

$$\begin{bmatrix} \mathscr{P} & \mathscr{M}^T \\ \mathscr{M} & -\mathscr{Q} \end{bmatrix} < 0 \quad \Leftrightarrow \mathscr{P} + \mathscr{M}^T\mathscr{Q}^{-1}\mathscr{M} < 0$$

.

Lemma 2.2 ([49]). *Given matrices $\mathscr{Q} = \mathscr{Q}^T$, mathscrH, mathscrE, and $\mathscr{R} = \mathscr{R}^T > 0$ with appropriate dimensions$\mathscr{Q} + \mathscr{H}\mathscr{F}(t)\mathscr{E} + \mathscr{E}^T\mathscr{F}^T(t)\mathscr{H}^T < 0$,for all $\mathscr{F}(t)$ satisfying $\mathscr{F}^T(t)\mathscr{F}(t) \le I$ if and only if there exists a scalar $\varepsilon > 0$ such that$\mathscr{Q} + \varepsilon^{-1}\mathscr{H}\mathscr{H}^T + \varepsilon\mathscr{E}^T\mathscr{R}\mathscr{E} < 0$.or, equivalently,*

$$\begin{bmatrix} \mathscr{Q} & \varepsilon\mathscr{H} & \mathscr{E}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0.$$

Lemma 2.3 ([50]). *For a given matrix $R \in S_n^+$ and a function φ: $[a, b] \to \mathbb{R}^n$ whose derivative $\dot{\varphi} \in PC([a, b], mathbbR^n)$,the following inequalities hold: $\int_a^b \dot{\varphi}^T(s)R\dot{\varphi}(s)ds \ge \frac{1}{b-a}\hat{\chi}\bar{R}\hat{\chi}$, where $\bar{R} = diag\{R, 3R, 5R\}, \hat{\chi} = [\chi_1^T\ \chi_2^T\ \chi_3^T]^T, \chi_1 = \varphi(b) - \varphi(a), \chi_2 = \varphi(b) + \varphi(a) - \frac{2}{b-a}\int_a^b \varphi(s)ds,\ \chi_3 = \varphi(b) - \varphi(a) + \frac{6}{b-a}\int_a^b \varphi(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_s^b \varphi(u)duds$.*

Lemma 2.4 ([51]). *For a given matrix $\mathcal{M} > 0$, given scalars a and b satisfying $a < b$, the following inequality holds for all continuously differentiable function in $[a, b] \to \mathbb{R}$*

$$\frac{(b-a)^2}{2!}\int_a^b\int_\theta^b \dot{x}^T(s)\mathcal{M}\dot{x}^T(s)dsd\theta \ge \left(\int_a^b\int_\theta^b \dot{x}(s)dsd\theta\right)^T\mathcal{M}\left(\int_a^b\int_\theta^b \dot{x}(s)dsd\theta\right) + 2\Theta_d\mathcal{M}\Theta_d,$$

where $\Theta_d = -\int_a^b\int_\theta^b \dot{x}(s)dsd\theta + \frac{3}{b-a}\int_a^b\int_\theta^b\int_\nu^b \dot{x}(s)dsd\theta d\nu$.

3. Main results

In this section, we will establish a criterion to implement the non-fragile synchronization of GRNs with time-varying delays in the presence of controller gain perturbations. The sufficient conditions for ensuring the stability of system (8) are derived.

Theorem 3.1. *Under assumption (A), for given positive scalars α, β, γ_i, σ, τ, μ_1, and μ_2 if there exist matrices $\mathscr{P}_i > 0$, $\mathscr{Q}_i > 0$, $\mathscr{R}_i > 0$, and $\mathscr{Z}_i > 0$, diagonal matrices $\mathscr{S} > 0$, any matrices $\mathscr{J}_i$, and $\mathscr{G}_i$, $(i = 1, 2)$ satisfying*

$$\Psi = \begin{bmatrix} \Xi & \Phi \\ * & \Upsilon \end{bmatrix} < 0, \tag{9}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \frac{3\mathscr{R}_2}{\tau} & 0 & \frac{-24\mathscr{R}_2}{\tau^2} & \frac{60\mathscr{R}_2}{\tau^3} & 3\mathscr{Z}_2 & \Xi_{17} & 0 \\ * & \frac{-9\mathscr{R}_2}{\tau} & 0 & \frac{36\mathscr{R}_2}{\tau^2} & \frac{-60\mathscr{R}_2}{\tau^3} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & \mathscr{D}^T\mathscr{G}_2 \\ * & * & * & \Xi_{44} & \frac{360\mathscr{R}_2}{\tau^4} & \frac{6\mathscr{Z}_2}{\sigma} & 0 & 0 \\ * & * & * & * & \frac{-720\mathscr{R}_2}{\tau^5} & 0 & 0 & 0 \\ * & * & * & * & * & \frac{-18\mathscr{Z}_2}{\sigma^2} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 \\ * & * & * & * & * & * & * & \Xi_{88} \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0 & 0 & 0 & \mathscr{G}_1\mathscr{B} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\mathscr{D}^T\mathscr{G}_2^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1\mathscr{G}_1\mathscr{B} & 0 & 0 & 0 & 0 \\ \frac{3\mathscr{R}_1}{\sigma} & 0 & \mathscr{S}\mathscr{L}_2 & 0 & \frac{-24\mathscr{R}_1}{\sigma^2} & \frac{60\mathscr{R}_1}{\sigma^3} & 3\mathscr{Z}_1 & \Phi_{88} \end{bmatrix},$$

$$\Upsilon = \begin{bmatrix} \frac{-9\mathscr{R}_1}{\sigma} & 0 & 0 & 0 & \frac{36\mathscr{R}_1}{\sigma^2} & \frac{-60\mathscr{R}_1}{\sigma^3} & 0 & 0 \\ * & \Upsilon_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & \frac{360\mathscr{R}_1}{\sigma^4} & \frac{6\mathscr{Z}_1}{\sigma} & 0 \\ * & * & * & * & * & \frac{-720\mathscr{R}_1}{\sigma^5} & 0 & 0 \\ * & * & * & * & * & * & \frac{-18\mathscr{Z}_1}{\sigma^2} & 0 \\ * & * & * & * & * & * & * & \Upsilon_{88} \end{bmatrix},$$

$\Xi_{11} = \mathscr{Q}_2 - \mathscr{G}_1\mathscr{A} - \mathscr{A}^T\mathscr{G}_1^T - \frac{9\mathscr{R}_2}{\tau} - \frac{3\tau^2\mathscr{Z}_2}{2} + \mathscr{J}_1 + \mathscr{J}_1^T + \alpha\mathscr{G}_1\Delta\mathscr{K}_1(t) + \alpha\Delta\mathscr{K}_1^T(t)\mathscr{G}_1^T,$
$\Xi_{17} = \mathscr{P}_1 - \mathscr{G}_1 - \gamma_1\mathscr{A}^T\mathscr{G}_1^T + \gamma_1\mathscr{J}_1 + \gamma_1\mathscr{G}_1\alpha\Delta\mathscr{K}_1(t),\ \Xi_{33} = -\mathscr{Q}_2(1-\mu_2),$
$\Xi_{44} = -\frac{192\mathscr{R}_2}{\tau^3} - 3\mathscr{Z}_2,\ \Xi_{77} = -\gamma_1\mathscr{G}_1 - \gamma_1\mathscr{G}_1^T + \tau\mathscr{R}_2 + \frac{\tau^4\mathscr{Z}_2}{4},$
$\Xi_{88} = \mathscr{Q}_1 - \frac{9\mathscr{R}_1}{\sigma} - \mathscr{S}\mathscr{L}_1 - \frac{3\sigma^2\mathscr{Z}_1}{2} - \mathscr{G}_2\mathscr{C} - \mathscr{C}^T\mathscr{G}_2^T + \mathscr{J}_2 + \mathscr{J}_2^T,$
$\Phi_{88} = \mathscr{P}_2 - \mathscr{G}_2 - \gamma_2\mathscr{C}^T\mathscr{G}_2^T + \gamma_2\mathscr{J}_2 + \gamma_2\mathscr{G}_2\beta\Delta\mathscr{K}_2(t),\ \Upsilon_{22} = -\mathscr{Q}_1(1-\mu_1),\ \Upsilon_{33} = \mathscr{Q}_3 - \mathscr{S}$
$\Upsilon_{44} = -\mathscr{Q}_3(1-\mu_1),\ \Upsilon_{55} = -\frac{192\mathscr{R}_1}{\sigma^3} - 3\mathscr{Z}_1,\ \Upsilon_{88} = -\gamma_2\mathscr{G}_2 + \sigma\mathscr{R}_1 + \frac{\sigma^4\mathscr{Z}_1}{4}.$

then the system (8) is asymptotically stable. Moreover the controller gain matrices in (5) are given by $\mathscr{K}_i = \mathscr{G}_i^{-1}\mathscr{J}_i$.

Proof. Choose the following Lyapunov-Kraovskii functional candidate as:

$$V(x_t, y_t, t) = \sum_{i=1}^{4} V_i(x_t, y_t, t), \tag{10}$$

where

$$\begin{aligned} V_1(x_t, y_t, t) &= x^T(t)\mathscr{P}_1x(t) + y^T(t)\mathscr{P}_2y(t), \\ V_2(x_t, y_t, t) &= \int_{t-\sigma(t)}^{t} y^T(s)\mathscr{Q}_1y(s)ds + \int_{t-\tau(t)}^{t} x^T(s)\mathscr{Q}_2x(s)ds + \int_{t-\sigma(t)}^{t} g^T(y(s))\mathscr{Q}_3g(y(s))ds, \\ V_3(x_t, y_t, t) &= \int_{-\sigma}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)\mathscr{R}_1\dot{y}(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)\mathscr{R}_2\dot{x}(s)dsd\theta, \\ V_4(x_t, y_t, t) &= \frac{\tau^2}{2}\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \dot{x}^T(s)\mathscr{Z}_1\dot{x}(s)dsd\nu d\theta + \frac{\sigma^2}{2}\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \dot{y}^T(s)\mathscr{Z}_2\dot{y}(s)dsd\nu d\theta. \end{aligned}$$

The infinitesimal operator $\mathcal{L}$ of $V(x_t, y_t, t)$ is

$$\mathcal{L}V(x_t, y_t, t) = \lim_{\Delta\to 0}\frac{1}{\Delta}\{\mathbb{E}\{V(x_{t+\Delta}, y_{t+\Delta}, t)|(x_t, y_t, t)\} - V(x_t, y_t, t)\}. \tag{11}$$

We calculate the stochastic derivative of $V(x_t, y_t, t)$ gives

$$\mathbb{E}\{\mathcal{L}V_1(x_t, y_t, t)\} = 2x^T(t)\mathscr{P}_1\dot{x}(t) + 2y^T(t)\mathscr{P}_2\dot{y}(t), \tag{12}$$

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_2(x_t, y_t, t)\} \le\ & y^T(t)\mathscr{Q}_1 y(t) - (1-\mu_1)y(t-\sigma(t))\mathscr{Q}_1 y(t-\sigma(t)) + x^T(t)\mathscr{Q}_2 x(t)\\ & -(1-\mu_2)x^T(t-\tau(t))\mathscr{Q}_2 x(t-\tau(t)) + g^T(y(t))\mathscr{Q}_3 g(y(t))\\ & -(1-\mu_1)g^T(y(t-\sigma(t)))\mathscr{Q}_3 g(y(t-\sigma(t))),\end{aligned} \tag{13}$$

$$\mathbb{E}\{\mathcal{L}V_3(x_t, y_t, t)\} = \sigma\dot{y}^T(t)\mathscr{R}_1\dot{y}(t) - \int_{t-\sigma}^t \dot{y}(s)\mathscr{R}_1\dot{y}(s)ds + \tau\dot{x}^T(t)\mathscr{R}_2\dot{x}(t) - \int_{t-\tau}^t \dot{x}(s)\mathscr{R}_2\dot{x}(s)ds, \tag{14}$$

By applying lemma (2.3) in above equation (13), we can obtain

$$-\int_{t-\sigma}^t \dot{y}^T(s)\mathscr{R}_1\dot{y}(s)ds \le -\frac{1}{\tau}\eta_1^T(t)\begin{bmatrix}\mathscr{R}_1 & 0 & 0\\ 0 & 3\mathscr{R}_1 & 0\\ 0 & 0 & 5\mathscr{R}_1\end{bmatrix}\eta_1(t), \tag{15}$$

$$-\int_{t-\tau}^t \dot{x}^T(s)\mathscr{R}_2\dot{x}(s)ds \le -\frac{1}{\sigma}\eta_2^T(t)\begin{bmatrix}\mathscr{R}_2 & 0 & 0\\ 0 & 3\mathscr{R}_2 & 0\\ 0 & 0 & 5\mathscr{R}_2\end{bmatrix}\eta_2(t), \tag{16}$$

where

$$\begin{aligned}\eta_1(t) =\ & \Big[y^T(t) - y^T(t-\sigma) \quad y^T(t) + y^T(t-\sigma) - \frac{2}{\sigma}\int_{t-\sigma}^t y^T(s)ds\\ & y^T(t) - y^T(t-\sigma) + \frac{6}{\sigma}\int_{t-\sigma}^t y^T(s)ds - \frac{12}{\sigma^2}\int_{t-\tau}^t\int_s^t y^T(u)duds\Big]^T,\\ \eta_2(t) =\ & \Big[x^T(t) - x^T(t-\tau) \quad x^T(t) + x^T(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^t x^T(s)ds\\ & x^T(t) - x^T(t-\tau) + \frac{6}{\tau}\int_{t-\tau}^t x^T(s)ds - \frac{12}{\tau^2}\int_{t-\tau}^t\int_s^t x^T(u)duds\Big]^T.\end{aligned}$$

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_4(x_t, y_t, t)\} =\ & \frac{\sigma^4}{4}\dot{y}^T(t)\mathscr{Z}_1\dot{y}(t) - \frac{\sigma^2}{2}\int_{-\sigma}^0\int_{t+\theta}^t \dot{y}^T(s)\mathscr{Z}_1\dot{y}(s)dsd\theta + \frac{\tau^4}{4}\dot{x}^T(t)\mathscr{Z}_2\dot{x}(t)\\ & -\frac{\tau^2}{2}\int_{-\tau}^0\int_{t+\theta}^t \dot{x}^T(s)\mathscr{Z}_2\dot{x}(s)dsd\theta.\end{aligned} \tag{17}$$

By using Lemma 2.4 in (17), we get

$$\begin{aligned}-\frac{\sigma^2}{2}\int_{-\sigma}^0\int_{t+\theta}^t \dot{y}^T(s)\mathscr{Z}_1\dot{y}(s)dsd\theta \le\ & -\left(\int_{-\sigma}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta\right)^T \mathcal{X}_1\left(\int_{-\sigma}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta\right) + 2\Sigma_d^T\mathcal{M}\Sigma_d,\\ \le\ & \begin{bmatrix}\sigma y(t)\\ \int_{t-\sigma}^t y(s)ds\end{bmatrix}^T\begin{bmatrix}-\mathscr{Z}_1 & \mathscr{Z}_1\\ * & -\mathscr{Z}_1\end{bmatrix}\begin{bmatrix}\sigma y(t)\\ \int_{t-\sigma}^t y(s)ds\end{bmatrix}\\ & +2\begin{bmatrix}\frac{\sigma}{2}y(t)\\ \int_{t-\sigma}^t y(s)ds\\ \frac{3}{\sigma}\int_{-\sigma}^0\int_{t+\theta}^t y(s)dsd\theta\end{bmatrix}^T\begin{bmatrix}-\mathscr{Z}_1 & -\mathscr{Z}_1 & \mathscr{Z}_1\\ * & -\mathscr{Z}_1 & \mathscr{Z}_1\\ * & * & -\mathscr{Z}_1\end{bmatrix}\\ & \begin{bmatrix}\frac{\sigma}{2}y(t)\\ \int_{t-\sigma}^t y(s)ds\\ \frac{3}{\sigma}\int_{-\sigma}^0\int_{t+\theta}^t y(s)dsd\theta\end{bmatrix},\end{aligned} \tag{18}$$

where $\Sigma_d = -\int_{-\sigma}^0\int_{t+\theta}^t \dot{x}(s)dsd\theta + \frac{3}{\sigma}\int_{-\sigma}^0\int_\theta^0\int_{t+\vartheta}^t \dot{x}(s)dsd\vartheta d\theta$.

Similar to (18), we have

$$-\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathscr{Z}_2\dot{x}(s)dsd\theta \le \begin{bmatrix}\tau x(t)\\ \int_{t-\tau}^{t}x(s)ds\end{bmatrix}^T\begin{bmatrix}-\mathscr{Z}_2 & \mathscr{Z}_2\\ * & -\mathscr{Z}_2\end{bmatrix}\begin{bmatrix}\tau x(t)\\ \int_{t-\tau}^{t}x(s)ds\end{bmatrix} +2\begin{bmatrix}\frac{\tau}{2}x(t)\\ \int_{t-\tau}^{t}x(s)ds\\ \frac{3}{\tau}\int_{-\tau}^{0}\int_{t+\theta}^{t}x(s)dsd\theta\end{bmatrix}^T\begin{bmatrix}-\mathscr{Z}_2 & -\mathscr{Z}_2 & \mathscr{Z}_2\\ * & -\mathscr{Z}_2 & \mathscr{Z}_2\\ * & * & -\mathscr{Z}_2\end{bmatrix}\begin{bmatrix}\frac{\tau}{2}x(t)\\ \int_{t-\tau}^{t}x(s)ds\\ \frac{3}{\tau}\int_{-\tau}^{0}\int_{t+\theta}^{t}x(s)dsd\theta\end{bmatrix}. \quad (19)$$

From assumption (A), there exists a diagonal matrix $\mathscr{S} > 0$ such that the following inequality holds:

$$0 \le \begin{bmatrix}y(t)\\ g(y(t))\end{bmatrix}^T\begin{bmatrix}-\mathscr{S}\mathscr{L}_1 & \mathscr{S}\mathscr{L}_2\\ * & -\mathscr{S}\end{bmatrix}\begin{bmatrix}y(t)\\ g(y(t))\end{bmatrix}. \quad (20)$$

For any appropriately dimensioned matrix $\mathscr{G}_1$, $\mathscr{G}_2$, and scalar γ_1, γ_2, the following equations holds:

$$0 = 2[x^T(t) + \gamma_1\dot{x}^T(t)]\mathscr{G}_1[-\dot{x}(t) + (-\mathscr{A} + (\mathscr{K}_1 + \alpha(t)\Delta\mathscr{K}_1(t)))x(t) + \mathscr{B}g(y(t-\sigma(t))], \quad (21)$$

$$0 = 2[y^T(t) + \gamma_2\dot{y}^T(t)]\mathscr{G}_2[-\dot{y}(t) + (-\mathscr{C} + (\mathscr{K}_2 + \beta(t)\Delta\mathscr{K}_2(t)))y(t) + \mathscr{D}x(t-\tau(t))]. \quad (22)$$

Now, combining (12)-(22), we have

$$\mathbb{E}\{\mathcal{L}V(x_t, y_t, t)\} \le \xi^T(t)\,\Psi\,\xi(t), \quad (23)$$

where

$$\xi^T(t) = \Big[x^T(t)\; x^T(t-\tau)\; x^T(t-\tau(t))\; \int_{t-\tau}^{t}x^T(s)ds\; \int_{t-\tau}^{t}\int_{s}^{t}x^T(u)duds\; \int_{-\tau}^{0}\int_{t+\theta}^{t}x^T(s)dsd\theta\dot{x}(t)\; y^T(t)$$
$$y^T(t-\sigma)\; y^T(t-\sigma(t))\; g^T(y(t))\; g^T(y(t-\sigma(t)))\; \int_{t-\sigma}^{t}y^T(s)ds\; \int_{t-\sigma}^{t}\int_{s}^{t}y^T(u)duds\; \int_{-\sigma}^{0}\int_{t+\theta}^{t}y^T(s)dsd\theta\; \dot{y}(t)\Big].$$

It is obvious that $\Psi < 0$, which indicates from the Lyapunov stability theory that the system (8) is asymptotically stable. This completes the proof. □

Theorem 3.2. *Under assumption (A), for given scalars* α, $\beta > 0$ *and* ϵ_i *if there exist matrices* $\mathscr{P}_i > 0$, $\mathscr{Q}_i > 0$, $\mathscr{R}_i > 0$, $\mathscr{Z}_i > 0$, *diagonal matrices* $\mathscr{S} > 0$, *matrices* $\mathscr{J}_i$, *and* $\mathscr{G}_i$, *and a scalar* $\rho_i > 0$ $(i = 1, 2)$ *satisfying*

$$\tilde{\Psi} = \begin{bmatrix}\hat{\Psi} & \Lambda_1 & \rho_1\Lambda_2 & \Omega_1 & \Omega_2\\ * & -\rho_1 I & 0 & 0 & 0\\ * & * & -\rho_1 I & 0 & 0\\ * & * & * & -\rho_2 I & 0\\ * & * & * & * & -\rho_2 I\end{bmatrix} < 0, \quad (24)$$

where

$$\hat{\Psi} = \begin{bmatrix} \hat{\Xi} & \Phi \\ * & \hat{\Upsilon} \end{bmatrix}, \hat{\Xi} = \begin{bmatrix} \hat{\Xi}_{11} & \frac{3\mathscr{R}_2}{\tau} & 0 & \frac{-24\mathscr{R}_2}{\tau^2} & \frac{60\mathscr{R}_2}{\tau^3} & 3\mathscr{Z}_2 & \hat{\Xi}_{17} & 0 \\ * & \frac{-9\mathscr{R}_2}{\tau} & 0 & \frac{36\mathscr{R}_2}{\tau^2} & \frac{-60\mathscr{R}_2}{\tau^3} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & \mathscr{D}^T\mathscr{G}_2 \\ * & * & * & \Xi_{44} & \frac{360\mathscr{R}_2}{\tau^4} & \frac{6\mathscr{Z}_2}{\sigma} & 0 & 0 \\ * & * & * & * & \frac{-720\mathscr{R}_2}{\tau^5} & 0 & 0 & 0 \\ * & * & * & * & * & \frac{-18\mathscr{Z}_2}{\sigma^2} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 \\ * & * & * & * & * & * & * & \hat{\Xi}_{88} \end{bmatrix},$$

$$\hat{\Upsilon} = \begin{bmatrix} \frac{-9\mathscr{R}_1}{\sigma} & 0 & 0 & 0 & \frac{36\mathscr{R}_1}{\sigma^2} & \frac{-60\mathscr{R}_1}{\sigma^3} & 0 & 0 \\ * & \Upsilon_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & \frac{360\mathscr{R}_1}{\sigma^4} & \frac{6\mathscr{Z}_1}{\sigma} & 0 \\ * & * & * & * & * & \frac{-720\mathscr{R}_1}{\sigma^5} & 0 & 0 \\ * & * & * & * & * & * & \frac{-18\mathscr{Z}_1}{\sigma^2} & 0 \\ * & * & * & * & * & * & * & \Upsilon_{88} \end{bmatrix},$$

$$\begin{aligned} \hat{\Xi}_{11} &= \mathscr{R}_1 - \frac{9\mathscr{Z}_1}{\tau} - \mathscr{J}_1\mathscr{A} + \mathscr{G}_1 + \mathscr{G}_1^T - \mathscr{A}^T\mathscr{J}_1^T, \hat{\Xi}_{17} = -\mathscr{J}_1 - \epsilon_1\mathscr{A}^T\mathscr{J}_1^T + \epsilon_1\mathscr{G}_1^T + \mathscr{P}_1, \\ \hat{\Lambda}_{11} &= \mathscr{R}_2 - \mathscr{S}\mathscr{L}_1 - \mathscr{J}_2\mathscr{B} + \mathscr{G}_2 - \frac{9\mathscr{Z}_2}{\sigma} - \mathscr{B}^T\mathscr{J}^T + \mathscr{G}_2^T, \hat{\Lambda}_{17} = -\mathscr{J}_2 - \epsilon_2\mathscr{B}^T\mathscr{J}_2^T + \epsilon_2\mathscr{G}_2^T + \mathscr{P}_2, \\ \Upsilon_1 &= [\alpha\mathscr{H}_1^T\mathscr{J}_1^T \underbrace{0,\dots,0}_{5elements} \alpha\epsilon_1\mathscr{H}_1^T\mathscr{J}_1^T \underbrace{0,\dots,0}_{7elements}]^T, \Upsilon_2 = [\mathscr{E}_1 \underbrace{0,\dots,0}_{13elements}]^T, \\ \Psi_1 &= [\underbrace{0,\dots,0}_{7elements} \beta\mathscr{H}_2^T\mathscr{J}_2^T \underbrace{0,\dots,0}_{5elements} \beta\epsilon_2\mathscr{H}_2^T\mathscr{J}_2^T]^T, \Psi_2 = [\underbrace{0,\dots,0}_{7elements} \rho_2\mathscr{E}_2 \underbrace{0,\dots,0}_{6elements}]^T, \end{aligned}$$

then the system (8) is asymptotically stable. Moreover the controller gain matrices in (5) are given by $\mathscr{K}_i = \mathscr{J}_i^{-1}\mathscr{G}_i$.

Proof. By using the Schur complement, Lemma (2.1) we get,

$$[\hat{\Xi}] + \Upsilon_1\Delta\mathscr{K}_1(t)\Upsilon_2^T + \Upsilon_2\Delta\mathscr{K}_1^T(t)\Upsilon_1^T + \Psi_1\Delta\mathscr{K}_2(t)\Psi_2^T + \Psi_2\Delta\mathscr{K}_2^T(t)\Psi_1^T < 0.$$

Based on Lemma 2.1, the above equation is equivalent to

$$\tilde{\Xi} = \begin{bmatrix} \hat{\Xi} & \Upsilon_1 & \rho_1\Upsilon_2 & \Psi_1 & \Psi_2 \\ * & -\rho_1 I & 0 & 0 & 0 \\ * & * & -\rho_1 I & 0 & 0 \\ * & * & * & -\rho_2 I & 0 \\ * & * & * & * & -\rho_2 I \end{bmatrix} < 0.$$

It is not difficult to see that the inequalities in (24) hold. This completes the proof. □

Remark 3.3. By constructing proper Lyapunov Krasovskii functionals and using LMI method, sufficient conditions for the GRNs are given in Theorems 3.1 to guarantee the asymptotic stable. It can be seen that these stability criteria are formulated by no model transformation. By introducing appropriate free-weighting matrices, we develop less conservative stability results than the other literatures.

The master system (3) become the network reference [52], so the result is more general than [52–56]:

$$\mathcal{M}: \begin{cases} \dot{\zeta}(t) = -\mathscr{A}\zeta(t) + \mathscr{B}\mathfrak{g}(\delta(t-\sigma(t))), \\ \dot{\delta}(t) = -\mathscr{C}\delta(t) + \mathscr{D}\zeta(t-\tau(t)) \end{cases} \tag{25}$$

Based on the result in Theorem 3.1, we can easily get the following criterion.

Corollary 3.4. *Under assumption (A), for given positive scalars γ_i, σ, τ, μ_1, and μ_2, if there exist matrices $\mathscr{P}_i > 0$, $\mathscr{Q}_i > 0$, $\mathscr{R}_i > 0$, and $\mathscr{Z}_i > 0$, diagonal matrices $\mathscr{S} > 0$, any matrices $\mathscr{G}_i$, $(i = 1, 2)$ satisfying*

$$\Omega = \begin{bmatrix} \hat{\Xi} & \hat{\Phi} \\ * & \hat{\Upsilon} \end{bmatrix} < 0, \tag{26}$$

where

$$\hat{\Xi} = \begin{bmatrix} \hat{\Xi}_{11} & \frac{3\mathscr{R}_2}{\tau} & 0 & \frac{-24\mathscr{R}_2}{\tau^2} & \frac{60\mathscr{R}_2}{\tau^3} & 3\mathscr{Z}_2 & \hat{\Xi}_{17} & 0 \\ * & \frac{-9\mathscr{R}_2}{\tau} & 0 & \frac{36\mathscr{R}_2}{\tau^2} & \frac{-60\mathscr{R}_2}{\tau^3} & 0 & 0 & 0 \\ * & * & \hat{\Xi}_{33} & 0 & 0 & 0 & 0 & \mathscr{D}^T\mathscr{G}_2 \\ * & * & * & \hat{\Xi}_{44} & \frac{360\mathscr{R}_2}{\tau^4} & \frac{6\mathscr{Z}_2}{\sigma} & 0 & 0 \\ * & * & * & * & \frac{-720\mathscr{R}_2}{\tau^5} & 0 & 0 & 0 \\ * & * & * & * & * & \frac{-18\mathscr{Z}_2}{\sigma^2} & 0 & 0 \\ * & * & * & * & * & * & \hat{\Xi}_{77} & 0 \\ * & * & * & * & * & * & * & \hat{\Xi}_{88} \end{bmatrix},$$

$$\hat{\Phi} = \begin{bmatrix} 0 & 0 & 0 & \mathscr{G}_1\mathscr{B} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\mathscr{D}^T\mathscr{G}_2^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1\mathscr{G}_1\mathscr{B} & 0 & 0 & 0 & 0 \\ \frac{3\mathscr{R}_1}{\sigma} & 0 & \mathscr{S}\mathscr{L}_2 & 0 & \frac{-24\mathscr{R}_1}{\sigma^2} & \frac{60\mathscr{R}_1}{\sigma^3} & 3\mathscr{Z}_1 & \hat{\Phi}_{88} \end{bmatrix},$$

$$\hat{\Upsilon} = \begin{bmatrix} \frac{-9\mathscr{R}_1}{\sigma} & 0 & 0 & 0 & \frac{36\mathscr{R}_1}{\sigma^2} & \frac{-60\mathscr{R}_1}{\sigma^3} & 0 & 0 \\ * & \hat{\Upsilon}_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \hat{\Upsilon}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Upsilon}_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \hat{\Upsilon}_{55} & \frac{360\mathscr{R}_1}{\sigma^4} & \frac{6\mathscr{Z}_1}{\sigma} & 0 \\ * & * & * & * & * & \frac{-720\mathscr{R}_1}{\sigma^5} & 0 & 0 \\ * & * & * & * & * & * & \frac{-18\mathscr{Z}_1}{\sigma^2} & 0 \\ * & * & * & * & * & * & * & \hat{\Upsilon}_{88} \end{bmatrix},$$

$$\begin{aligned} \hat{\Xi}_{11} &= \mathscr{Q}_2 - \mathscr{G}_1\mathscr{A} - \mathscr{A}^T\mathscr{G}_1^T - \frac{9\mathscr{R}_2}{\tau} - \frac{3\tau^2\mathscr{Z}_2}{2},\ \hat{\Xi}_{17} = \mathscr{P}_1 - \mathscr{G}_1 - \gamma_1\mathscr{A}^T\mathscr{G}_1^T,\ \hat{\Xi}_{33} = -(1-\mu_2)\mathscr{Q}_2, \\ \hat{\Xi}_{44} &= -\frac{192\mathscr{R}_2}{\tau^3} - 3\mathscr{Z}_2,\ \hat{\Xi}_{77} = -\gamma_1\mathscr{G}_1 - \gamma_1\mathscr{G}_1^T + \tau\mathscr{R}_2 + \frac{\tau^4\mathscr{Z}_2}{4}, \\ \hat{\Xi}_{88} &= \mathscr{Q}_1 - \frac{9\mathscr{R}_1}{\sigma} - \mathscr{S}\mathscr{L}_1 - \frac{3\sigma^2\mathscr{Z}_1}{2} - \mathscr{G}_2\mathscr{C} - \mathscr{C}^T\mathscr{G}_2^T,\ \hat{\Phi}_{88} = \mathscr{P}_2 - \mathscr{G}_2 - \gamma_2\mathscr{C}^T\mathscr{G}_2^T, \\ \hat{\Upsilon}_{22} &= -\mathscr{Q}_1(1-\mu_1),\ \Upsilon_{33} = \mathscr{Q}_3 - \mathscr{S},\ \hat{\Upsilon}_{44} = -(1-\mu_1)\mathscr{Q}_3,\ \hat{\Upsilon}_{55} = -\frac{192\mathscr{R}_1}{\sigma^3} - 3\mathscr{Z}_1, \\ \hat{\Upsilon}_{88} &= -\gamma_2\mathscr{G}_2 + \sigma\mathscr{R}_1 + \frac{\sigma^4\mathscr{Z}_1}{4}. \end{aligned}$$

Proof. *Choose the following Lyapunov-Kraovskii functional candidate as:*

$$V(x_t, y_t, t) = \sum_{i=1}^{4} V_i(x_t, y_t, t), \tag{27}$$

where

$$\begin{aligned} V_1(x_t, y_t, t) &= x^T(t)\mathscr{P}_1x(t) + y^T(t)\mathscr{P}_2y(t),\\ V_2(x_t, y_t, t) &= \int_{t-\sigma(t)}^{t} y^T(s)\mathscr{Q}_1y(s)ds + \int_{t-\tau(t)}^{t} x^T(s)\mathscr{Q}_2x(s)ds + \int_{t-\sigma(t)}^{t} g(y(s))\mathscr{Q}_3g(y(s))ds,\\ V_3(x_t, y_t, t) &= \int_{-\sigma}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)\mathscr{R}_1\dot{y}(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)\mathscr{R}_2\dot{x}(s)dsd\theta,\\ V_4(x_t, y_t, t) &= \frac{\tau^2}{2}\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \dot{x}^T(s)\mathscr{Z}_1\dot{x}(s)dsd\nu d\theta + \frac{\sigma^2}{2}\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \dot{y}^T(s)\mathscr{Z}_2\dot{y}(s)dsd\nu d\theta. \end{aligned}$$

we can prove that Lyapunov functional in this paper can usefulness results and other symbols are the same as those defined in Theorem 3.1. □

Remark 3.5. Kwon et.al., has introduced the Wirtinger-based double integral inequality (WDI) for double integral terms in the derivative of LKF in [51]. While, how to better the results has still attracted much attention. Constructing better LKFs and reducing the enlargement of the derivative of LKF are the main challenges of this problem. The main contribution of this paper is that by using a new analysis method based on the time-varying delays and an appropriate LKF with double and triple integral terms, together with WDI inequality techniques, sufficient conditions for the asymptotic stability is presented in terms of LMIs.

Remark 3.6. Compared with [52–55], a novel technique was introduced to deal with the double and triple integral terms in the time-derivative of the proposed LKF (27) with the Wirtinger-based double integral inequality approach to obtain the less conservative stability condition. Hence, in this paper, the novel LKF was presented to utilize the GRNs together with WDI inequality techniques and some useful integral inequalities to establish a stability condition that is less conservative than those presented in [52–55]. Due to the utilization of Writinger-based double integral inequality approach, there is no complexity in numerical computation. Its very useful to obtain the less conservative results when compare to existing results.

4. Numerical example

This section provides a numerical result to demonstrate the effectiveness of the presented control strategy.

Example 4.1. Consider the following GRNs with time varying delay:

$$\begin{aligned} \dot{x}(t) &= (-\mathscr{A} + (\mathscr{K}_1 + \alpha(t)\Delta\mathscr{K}_1(t)))x(t) + \mathscr{B}g(y(t-\sigma(t)),\\ \dot{y}(t) &= (-\mathscr{C} + (\mathscr{K}_2 + \beta(t)\Delta\mathscr{K}_2(t)))y(t) + \mathscr{D}x(t-\tau(t)) \end{aligned}$$

with the following parameters:

$$\mathscr{A}=\begin{bmatrix}2&0\\0&2\end{bmatrix},\ \mathscr{B}=\begin{bmatrix}1&2\\0.8&0.1\end{bmatrix},\ \mathscr{C}=\begin{bmatrix}2&0\\0&2\end{bmatrix},\ \mathscr{D}=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \mathscr{L}_1=I,\ \mathscr{H}_1=0.25I,\quad \mathscr{H}_2=0.15I,\ \mathscr{E}_1=0.35I,\ \mathscr{E}_2=0.15I.$$

We take $\tau(t) = \sigma(t) = 0.2|\sin(t)|$, $\tau = \sigma = 0.2$, $\mu_1 = \mu_2 = 0.5$, and $\alpha = \beta = 0.1$. By using MATLAB LMI control toolbox solving the LMIs in Theorem 3.2, we get the feasible solutions as

$$\begin{aligned} \mathscr{P}_1 &= 10^4\times\begin{bmatrix}0.2805&-0.0094\\-0.0094&0.2774\end{bmatrix},\ \mathscr{P}_2=\begin{bmatrix}747.2111&-0.0000\\-0.0000&747.2271\end{bmatrix},\ \mathscr{Q}_1=\begin{bmatrix}0.0078&-0.0002\\-0.0002&0.0078\end{bmatrix},\\ \mathscr{Q}_2 &= \begin{bmatrix}0.0027&-0.0000\\-0.0000&0.0027\end{bmatrix},\ \mathscr{Q}_3=\begin{bmatrix}0.0127&0.0005\\0.0005&0.0127\end{bmatrix},\ \mathscr{R}_1=10^3\times\begin{bmatrix}0.5376&-0.0064\\-0.0064&0.5359\end{bmatrix},\\ \mathscr{R}_2 &= \begin{bmatrix}0.0037&-0.0000\\-0.0000&0.0036\end{bmatrix},\ \mathscr{Z}_1=\begin{bmatrix}1.0744&0.0082\\0.0082&1.0734\end{bmatrix},\ \mathscr{Z}_2=\begin{bmatrix}0.8820&-0.0071\\-0.0071&0.8773\end{bmatrix},\\ \mathscr{G}_2 &= \begin{bmatrix}0.0047&0.0000\\0.0000&0.0047\end{bmatrix},\ \mathscr{J}_1=\begin{bmatrix}-0.1571&0.1097\\0.1097&-0.1133\end{bmatrix},\ \mathscr{J}_2=\begin{bmatrix}0.0228&0.0001\\0.0001&0.0228\end{bmatrix},\\ \rho_1 &= 0.0043,\ \rho_2 = 0.3473. \end{aligned}$$

We then obtain the following gain matrices of the proposed non-fragile controller:

$$\mathscr{K}_1=\begin{bmatrix}-2.1247&0.6786\\0.6803&-1.8887\end{bmatrix},\ \mathscr{K}_2=\begin{bmatrix}4.8762&-0.0037\\-0.0037&4.8720\end{bmatrix}.$$

According to Theorem 3.2, the system (8) in this example is asymptotically stable.

Example 4.2. Consider the delayed neural networks (25) with the following parameters:

$$\begin{aligned} \mathscr{A} &= \begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix},\ \mathscr{B}=\begin{bmatrix}0&0&-2.5\\-2.5&0&0\\0&-2.5&0\end{bmatrix},\ \mathscr{C}=\begin{bmatrix}2.5&0&0\\0&2.5&0\\0&0&2.5\end{bmatrix},\ \mathscr{D}=\begin{bmatrix}0.8&0&0\\0&0.8&0\\0&0&0.8\end{bmatrix},\\ \mathscr{S} &= diag\{0.65, 0.65, 0.65\}, \end{aligned}$$

and $\mu_1 = \mu_2 = 0.5$. By using Corollary 3.4, the maximum upper bound of σ are listed in Table 2, which can show that the asymptotical stability criterion for GRNs with time-varying delay (25) in this paper gives some less conservative results than ones given in [52–56].

Remark 4.1. The practical application of our result in this paper to a biological network is mentioned through the following Example. The biological network was presented in Escherichia coli [58], as a repressilator mathematical model. By solving the LMIs in Corollary 3.4 that the repressor with below mentioned set of parameters is asymptotically stable within a stability region.

Example 4.3. Taking into account the time-varying delay, we now consider the genetic regulatory network (GRN) as follows [57,59].

$$\begin{cases} \dot{x}(t) = -\mathcal{A}x(t) + \mathcal{B}f(y(t-\tau(t))) + \mathcal{J}, \\ \dot{y}(t) = -\mathcal{C}y(t) + \mathcal{D}x(t-\tau(t)). \end{cases} \tag{28}$$

where $x(t) \in \mathcal{R}^n$, $y(t) \in \mathcal{R}^n$, $\mathcal{A} = diag\{a_1, a_2, ..., a_n\}$ is a diagonal matrix with positive entries, $\mathcal{B}$, C, and D are real known constant matrices. $\mathcal{J} = [\mathcal{J}_1, mathcalJ_2, ..., \mathcal{J}_n]^T$ is a constant input vector, and $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), ..., f_n(x_n(t))]^T \in \mathcal{R}^p$ denotes the continuous activation function with $f_i(0) = 0$ satisfy

$$0 \leq \frac{f_i(x) - f_i(y)}{x - y} \leq \mathcal{L}_i^+, x \neq y \in \mathbb{R}. \tag{29}$$

Now consider a three-dimensional network (e.g., n = 3). For any i, we choose $f(k) = \frac{k^2}{(1+k^2)}$, which implies $\mathcal{L}_i^+ = 0.65 (i = 1, 2, 3)$, and

$$\mathcal{A} = C = \mathcal{I}_3, \mathcal{B} = \begin{bmatrix} 0 & 0 & -0.8 \\ -0.6 & 0 & 0 \\ 0 & -0.60 & 0 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \mathcal{J} = \begin{bmatrix} 0.8 \\ 0.6 \\ 0.6 \end{bmatrix},$$

Assume that (x^*, y^*) are the equilibrium point of (28). By applying the transformations $\hat{x}(t) = x(t) - x^*$, $\hat{y}(t) = y(t) - y^*$ and $\varphi(\hat{y}(t)) = f(\hat{y}(t) + y^*) - f(y^*)$, we can transform GRN ((28)) into the following form:

$$\begin{cases} \dot{\hat{x}}(t) = -\mathcal{A}\hat{x}(t) + \mathcal{B}\varphi(\hat{y}(t-\tau(t))), \\ \dot{\hat{y}}(t) = -\mathcal{C}\hat{y}(t) + \mathcal{D}\hat{x}(t-\tau(t)), \end{cases} \tag{30}$$

It is easy to know from Corollary 3.4 that the repressor with this set of parameters is asymptotically stable within a stability region, and stability region bound can be estimated is $\tau = \sigma \leq 0.5$. This shows that the GRNs (28) is stable.

5. Conclusion

This paper has considered the problem of non-fragile synchronization of genetic regulatory networks (GRNs) with time-varying delays. Based on the non-fragile control and randomly occurring controller gain fluctuation, less conservative delay-dependent stability conditions are derived. By constructing a L-K functional and using standard integral inequality technique sufficient conditions are obtained in terms of linear matrix inequalities (LMIs), which guarantees asymptotically stability of addressed to GRNs. Finally, numerical examples are given to illustrate effectiveness of the results presented in this paper. Hence, the proposed techniques, results and methods can be extended and applicable to many famous dynamical models, such as fuzzy systems, switched systems, and multi-agent systems. This will be our near future topics for research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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