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International Journal of Control, Automation and Systems 17(12) (2019) 2983-3008

ISSN:1598-6446 (print version)
eISSN:2005-4092 (electronic version)

To link to this article:
http://dx.doi.org/10.1007/s12555-019-0592-5

Springer

International Journal of Control, Automation and Systems 17(12) (2019) 2983-3008
http://dx.doi.org/10.1007/s12555-019-0592-5 ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Control of Axially Moving Systems: A Review

Keum-Shik Hong* and **Phuong-Tung Pham**

Abstract: This paper presents a comprehensive review of significant works on active vibration control of axially moving systems. Owing to their broad applications, vibration suppression techniques for these systems have generated active research over decades. Mathematical equations for five different models (i.e., string, beam, coupled, plate, and approximated model) are outlined. Active vibration control of axially moving systems can be performed based on a finite-dimensional model described by ordinary differential equations (ODEs) or an infinite-dimensional model described by partial differential equations (PDEs). For ODE models, the sliding mode control is most representative. For PDE models, however, there exist various methods, including wave cancellation, Lyapunov method, adaptive control, and hybrid control. Control applications (lifting systems, steel industry, flexible electronics, and roll-to-roll systems) are also illustrated. Finally, several issues for future research in vibration control of axially moving systems are discussed.

Keywords: Axially moving systems, control methods, modeling, PDE control, review, roll-to-roll systems, vibration suppression.

1. INTRODUCTION

Axially moving systems play an essential role in various engineering systems (Fig. 1) including continuous material manufacturing lines, roll-to-roll processes (e.g., zinc galvanization line, technical textile production lines, etc.), and transport processes (e.g., elevator cable systems, cable cars, etc.). In these systems, an adverse effect of the mechanical vibrations of the transported materials is a significant problem that affects the overall performance. As such, the analysis and suppression of the vibrations of axially moving systems have attracted considerable research interest for over six decades. The primary purpose of this paper is to present a detailed survey of the studies on vibration suppression of axially moving systems performed hitherto. The main focus is the detailed analyses of control methods and their applications.

Axially moving systems can be considered as a string model, a beam model, a coupled model, and a plate model depending on the flexibility, the existence of damping, and geometric parameters of the system. The moving string/beam/coupled models are a one-dimensional system, whereas the moving plate model is a two-dimensional one (i.e., the oscillation is a function of spatial variables x and y, and temporal variable t). Further, a moving string is often utilized to model a continuously moving system without considering the bend-

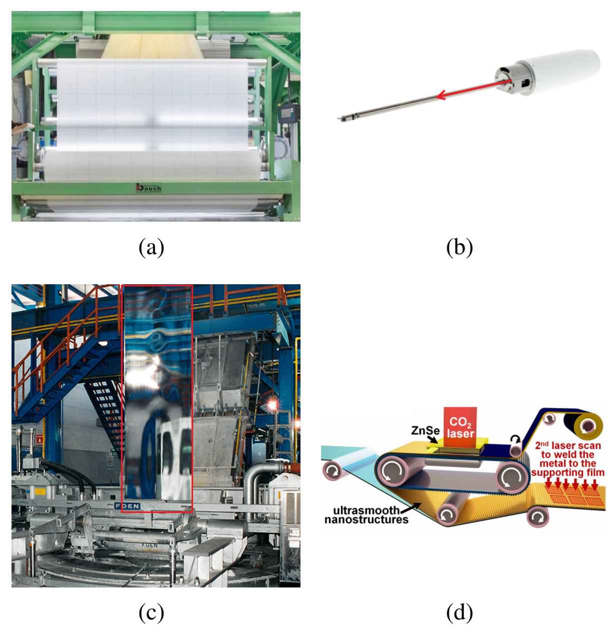

Fig. 1. Axially moving systems: (a) Technical textile-manufacturing process [1], (b) versatile instrument for minimally invasive robotic surgery [2], (c) zinc-galvanization line [3], and (d) nanoscale metal-printing process [4].

Manuscript received July 29, 2019; revised September 1, 2019; accepted September 7, 2019. Recommended by Editor Kyoung Kwan Ahn. This work was supported by the National Research Foundation (NRF) of Korea under the auspices of the Ministry of Science and ICT, Korea (grant no. NRF-2017R1A2A1A17069430).

Keum-Shik Hong and Phuong-Tung Pham are with the School of Mechanical Engineering, Pusan National University; 2 Busandaehak-ro, Geumjeong-gu, Busan 46241, Korea (e-mails: {kshong, pptung}@pusan.ac.kr).
* Corresponding author.

ing stiffness of the material [5–9] (e.g., threads in the textile-manufacturing processes (Fig. 1(a)), cables in automatic winding machines). The flexible components whose bending stiffness is significant are generally modeled using a beam model [10–14] (e.g., a cantilever beam with a prismatic joint for flexible robotic arms (Fig. 1(b)), hosting rods employed in an object carrying system). An axially moving string/beam model focuses on the influence of the lateral vibration but ignores the longitudinal vibration, whereas the coupled model accounts for both vibrations [15–20]. The coupled model is suitable for modeling materials of significant length (e.g., belts used in a power transmission system). The axially moving plate model is appropriate for the analysis of moving materials with considerable width (e.g., the metal layer in nanoscale metal printing processes, continuous roll-process technology for transferring and packaging flexible large-scale integrated circuits, see Fig. 1(d) [21–27]. Fig. 2 shows the distribution of mathematical models of axially moving systems (i.e., string, beam, coupled, and plate models), in which the string and beam models are the most commonly used models.

Fig. 3 depicts the distribution of about 40% of the total papers that developed control methods, in which the string and beam models were mostly used. Vibration suppression can be achieved based on passive control [28–36] or active control [37–46], see Fig. 4. An axially moving system is characterized by its distributed and gyroscopic properties due to the mass distribution of the material and the existence of Coriolis acceleration. An axially moving system has an infinite number of vibration modes, and it can be described as an infinite-dimensional system using partial differential equations (PDEs). Although a PDE model can precisely show the dynamic behavior of the system, the analysis and control of the vibrations of axially moving systems by directly using the PDE model is a challenge because of their infinite number of vibration modes. Therefore, the early investigations related to control were undertaken based on a finite-dimensional set of ordinary differential equations (ODEs), which was established by discretizing the PDE model to a set of ODEs. The control design based on an ODE model is convenient for its implementation using the conventional control methods, which are well developed for ODEs. Ulsoy [42] introduced a feedback control scheme for a moving string based on a modal control. Yang [47] proposed a direct velocity feedback controller for a gyroscopic system. Based on the ODE model, robust control such as variable structure control [48] and H_∞ control [49] were employed in the design of vibration controllers. Instead, a PDE model is an exact model: A direct use of this model in the control system design not only enhances the control performance but also avoids the spillover phenomenon. Due to this advantage, control strategies for vibration suppression of axially moving systems based on a PDE model have attracted significant research interest: Control methods such as wave cancellation [37,50,51], Lyapunov method [45,46,52–54], optimal control [38], computed-torque control [40], adaptive control [55–58], and intelligent control [59,60] have been utilized in the dissipation of vibrations in axially moving systems, wherein the control laws were designed in the frequency or in the time domain.

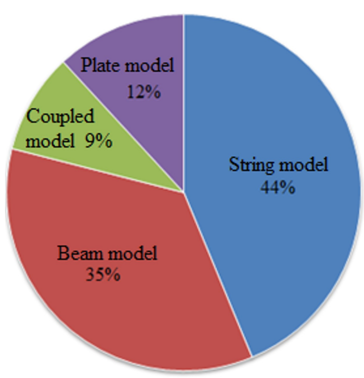

Fig. 2. Modeling (dynamics) of axially moving systems (total: 292 papers).

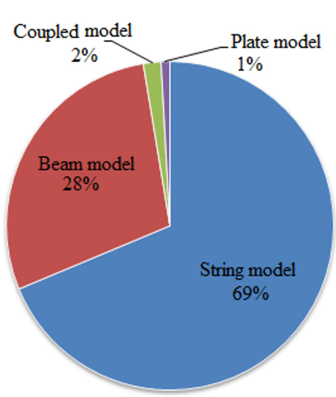

Fig. 3. Control of axially moving systems (total: 115 papers).

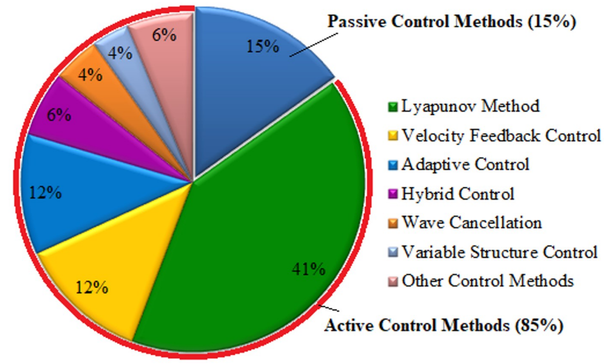

Fig. 4. Control methods applied to axially moving systems (115 papers).

Approaches for controlling PDEs can be categorized into two schemes; "in domain" control [34, 42, 50, 51, 61–64] and boundary control [43, 44, 53, 54, 65–72]. "In domain" control techniques suppress vibrations via actuators and sensors located at several points within the domain (i.e., pointwise control) or evenly distributed along the domain (i.e., distributed control), whereas boundary control techniques utilize actuators and sensors at the boundary of the considered system. Per boundary control, the actuators and sensors generally do not interfere with the operation of the system. Therefore, it is considered as a practical control solution for the control problem of axially moving systems.

Surveys on axially moving systems were conducted by Wang *et al*. [73] and Chen [74]. Wang *et al*. [73] presented insights on studies related to the suppression of linear vibrations of axially moving systems, whereas Chen [74] focused on the investigation of lateral vibration control of axially moving strings. In [74, 75], vibration control of only string models was discussed (i.e., modeling and control of the beam, coupled, and plate models were not done). This paper is a comprehensive compilation of significant studies on the control of axially moving systems. In particular, the various essential dynamic models of axially moving systems are introduced. Moreover, the state of knowledge on the design of active control schemas for systems using control methods based on ODE and PDE models is examined. Further, the applications of these control algorithms in the suppression of the vibrations of engineering systems are also discussed.

The remainder of this paper is organized into seven sections. In Section 2, the dynamic models of axially moving systems are presented. Section 3 introduces investigations into vibration suppression of axially moving systems based on control methods using ODE models, whereas Section 4 discusses studies on control methods based on PDE models. Studies on hybrid control methods are introduced in Section 5. In Section 6, the applications of vibration control of axially moving systems are examined. Finally, a discussion on future research is presented in Section 7.

2. DYNAMIC MODELS

2.1. Partial differential equations

2.1.1 String model (1 PDE)

The string model is recognized as the simplest and most common model used to analyze the dynamics of axially moving systems (Fig. 5). This model is generally used in the investigation of the system where the bending moment is negligible. The schematic of a translating string with length l and axial velocity v is shown in Fig. 5, where $w(x,t)$ and $u(x,t)$ represent the lateral and longitudinal vibrations of the string. Dynamic analysis of axially moving strings was pioneered in the late fifties by Mahalingam [9],

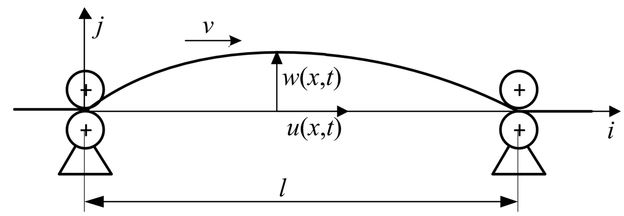

Fig. 5. Axially moving string (or beam) [13].

and the fundamental formulation of the dynamic model of a uniform moving string is represented as follows:

$$\rho A\left(w_{tt}+2vw_{xt}+v^2w_{xx}\right)-P_0w_{xx}=0, \tag{1}$$

where ρ is the mass density, A denotes the cross-sectional area, and P_0 indicates the initial tension. Equation (1) is a linear PDE, in which the first term describes the local string acceleration, the second and third terms correspond to the Coriolis force due to axial movement and the centrifugal acceleration, respectively, and the fourth term represents the pretension. The boundary conditions for a simply supported string model is given as follows:

$$w(0,t)=w(l,t)=0. \tag{2}$$

In other studies on nonlinear vibrations of axially moving strings, the effect of axial deformation on the potential energy of translating strings has been investigated [75–77], and the equation of motion is given as follows [76]:

$$\rho A(w_{tt}+2vw_{xt}+v^2w_{xx})-P_0w_{xx}-\frac{3}{2}EAw_{xx}w_x^2=0, \tag{3}$$

where E is Young's modulus. Pakdemirli *et al*. [78] and Pakdemirli and Ulsoy [79] established a mathematical model for moving strings with varying axial velocity as follows:

$$\rho A(w_{tt}+\dot{v}(t)w_x+2v(t)w_{xt}+v(t)^2w_{xx})-P_0w_{xx}=0. \tag{4}$$

Concerning an axially moving string with time-varying length, Fung *et al*. [80] analyzed the vibration behavior of a string using a set of ordinary differential equations (ODEs) developed based on Hamilton's principle and the variable-domain finite-element method. In Zhu and Ni [81], the lateral vibration of a string with varying length was investigated via the governing equation described by a PDE. The dynamic model of a translating string with varying length is described as follows [81]:

$$\rho Aw_{tt}+\rho A\ddot{l}(t)w_x+2\rho A\dot{l}(t)w_{xt}+\rho A\dot{l}(t)^2w_{xx} \\ -(P(x,t)w_x)_x=0, \tag{5}$$

$$P(x,t)=[m+\rho A(l(t)-x)](g-\ddot{l}(t)). \tag{6}$$

Considering the influence of material damping on the dynamics of an axially moving string, the vibration of a viscoelastic string was also investigated based on viscoelastic strain-stress constitutive relations such as the Kelvin-Voigt model [82–87], standard linear-solid model [77, 88], Burgers model [88], or Boltzmann's superposition principle [89]. In the aforementioned strain-stress relations, the Kelvin-Voigt model is the simplest model that expresses the relationship between the stress σ and the Lagrangian strain ε, namely,

$$\sigma(x,t) = E\varepsilon(x,t) + \mu\varepsilon(x,t), \tag{7}$$

where μ represents the dynamic viscosity of the dashpot of the Kelvin-Voigt model. According to this model, the equation of motion of a viscoelastic axially moving string is given as follows [84]:

$$\begin{aligned}&\rho A w_{tt} + 2\rho A v w_{xt} + \rho A v^2 w_{xx} - P_0 w_{xx} - \frac{3}{2} E A w_{xx} w_x^2 \\ &\quad - \mu A \left(w_x^2 w_{xxt} + 2 w_x w_{xx} w_{xt}\right) \\ &\quad - \mu A v \left(w_x^2 w_{xxx} + 2 w_x w_{xx}^2\right) = 0. \end{aligned} \tag{8}$$

Apart from the free oscillation of a translating string, the vibration behavior of axially moving strings undergoing diverse constraints (e.g., elastic distributed foundation [90, 91], partial nonlinear foundation [82, 92], intermediate spring support [93], multi supports [94–96], mechanical guides [97, 98], etc.) has also received considerable attention.

2.1.2 Beam model (1 PDE)

An axially moving material should be modeled in a beam model if the material's bending stiffness is considerable and cannot be ignored. In such situations, according to Euler-Bernoulli, Timoshenko, or Rayleigh beam theories, investigations of axially moving beams can be performed.

As one of the earliest studies on axially moving beams, Mote [99] utilized the Euler–Bernoulli theory to develop the dynamic model for an axially moving system. This theory is the simplest and most common theory used to describe a flexible beam. Subsequently, a series of studies on axially moving Euler–Bernoulli beam were performed [100–108]. According to the Euler–Bernoulli theory, the governing equation for a translating beam is given as follows [107].

$$\rho A w_{tt} + 2v\rho A w_{xt} + v^2 \rho A w_{xx} - P_0 w_{xx} + EI w_{xxxx} = 0, \tag{9}$$

where EI is the flexural rigidity of the beam. For a simply supported Euler-Bernoulli beam model, the following boundary conditions are considered.

$$w(0,t) = w(l,t) = 0,$$

$$w_{xx}(0,t) = w_{xx}(l,t) = 0. \tag{10}$$

Also, the dynamic model of an axially moving Euler–Bernoulli beam described by an integro-partial different equation was introduced by Wickert [13]. In this work, the author considered both longitudinal and lateral displacements of the beam: However, he assumed that the lateral motion could be decoupled from the longitudinal motion because the lateral waves propagate significantly slower than the longitudinal waves (i.e., it is called the quasi-static stretch assumption). Under this assumption, the mathematical model is given as the following integro-partial different equation [13].

$$\begin{aligned}&\rho A w_{tt} + \rho A \dot{v} w_x + 2v\rho A w_{xt} + v^2 \rho A w_{xx} - P_0 w_{xx} \\ &\quad + EI w_{xxxx} - \frac{1}{2}\frac{EA}{l} w_{xx} \int_0^l w_x^2 dx = 0. \end{aligned} \tag{11}$$

The dynamic behavior of the Euler–Bernoulli beam described by intergro-differential equations of motion was also analyzed in [109–116].

Vibration analysis of axially moving beams based on the Timoshenko beam theory was presented in [117–120]. In the Timoshenko beam theory, the influences of shear deformation and rotational inertia are considered. Consequently, the dynamic model of an axially moving Timoshenko beam is expressed in the following pair of partial differential equations consisting of the lateral vibration $w(x, t)$ and the angle of rotation due to bending $\theta(x, t)$ as follows [117]:

$$\begin{aligned}&\rho A \left(w_{tt} + 2v w_{xt} + v^2 w_{xx}\right) - P_0 w_{xx} - \kappa G A \left(w_{xx} - \theta_x\right) \\ &= 0, \end{aligned} \tag{12}$$

$$\rho I \left(\theta_{tt} + 2v\theta_{xt} + v^2 \theta_{xx}\right) - EI\theta_{xx} - \kappa G A \left(w_x - \theta\right) = 0, \tag{13}$$

where κ and G represent the shear coefficient and the shear modulus, respectively.

The Rayleigh beam theory has also been used to model axially moving beams in [121–123]. This theory considers the influence of the rotary-inertia and ignores the shear deformation of the material. In this theory, the equation of motion of an axially moving viscoelastic beam can be expressed as follows [121]:

$$\begin{aligned}&\rho A \left(w_{tt} + \dot{v} w_x + 2v w_{xt} + v^2 w_{xx}\right) - P_0 w_{xx} + EI w_{xxxx} \\ &\quad - \frac{3}{2} E A w_{xx} w_x^2 - \rho I (w_{xxtt} + 2v w_{xxxt} + \dot{v} w_{xxx} + v^2 w_{xxxx}) \\ &\quad + \mu I w_{xxxxt} - \mu A (w_{xxt} w_x^2 + 2 w_{xt} w_{xx} w_x) = 0. \end{aligned} \tag{14}$$

2.1.3 Coupled model (2 PDEs)

The equations of motion of axially moving systems described by the coupled model consist of the dynamics of both lateral and longitudinal vibrations. Hence, the coupled model is suitable for the cases wherein the distance between the two support points is considerable. Thurman and Mote [18] are recognized as pioneers in the development of a comprehensive dynamic model for a translating strip. In their paper, the following equations of motion describing an axially moving system were established.

$$\rho A\left(w_{tt}+2vw_{xt}+v^2w_{xx}\right)-EAw_{xx}+EIw_{xxxx} + (EA-P_0)\frac{(1+u_x)^2w_{xx}-(1+u_x)w_xu_{xx}}{[(1+u_x)^2+w_x^2]^{3/2}}=0, \tag{15}$$

$$\rho A(u_{tt}+2vu_{xt}+v^2u_{xx})-EAu_{xx} -(EA-P_0)\frac{(1+u_x)w_xw_{xx}-w_x^2u_{xx}}{[(1+u_x)^2+w_x^2]^{3/2}}=0. \tag{16}$$

The boundary conditions for a simply supported coupled model are given as follows.

$$\begin{aligned} &u(0,t)=u(l,t)=0,\\ &w(0,t)=w(l,t)=0,\\ &w_{xx}(0,t)=w_{xx}(l,t)=0. \end{aligned} \tag{17}$$

In their work, the total strain is the synthesis of the elastic strain energy due to the axial force stored in the strip $\varepsilon_{\mathrm{T}}=P_0/EA$ and the disturbance strain $\varepsilon_{\mathrm{d}}(x,t)$ given by the nonlinear geometric relation:

$$\varepsilon_{\mathrm{d}}(x,t)=\sqrt{w_x^2+(1+u_x^2)}-1. \tag{18}$$

To facilitate the dynamical analysis, Wang and Mote [124], Riedel and Tan [125], Sze *et al.* [126], Ding and Chen [15], Ghayesh *et al.* [127, 128] and Ghayesh [129], Yang and Zhang [130], and Suweken and Van Horssen [131] subsequently developed simpler models using the approximated disturbed strain. The coupled equations introduced in [129], wherein the disturbed strain is ignored, are represented as follows:

$$\rho A(w_{tt}+\dot{v}w_x+2vw_{xt}+v^2w_{xx})-P_0w_{xx} -EA\left[w_x\left(u_x+\frac{1}{2}w_x^2\right)\right]_x+EIw_{xxxx}=0, \tag{19}$$

$$\rho A(u_{tt}+\dot{v}(1+u_x)+2vu_{xt}+v^2u_{xx})-EA\left(u_x+\frac{1}{2}w_x^2\right)_x =0. \tag{20}$$

Furthermore, another coupled equations utilizing the Timoshenko beam theory that describes the longitudinal, lateral, and rotational motions of a flexible body were also developed by Ghayesh and Amabili [132, 133] and Farokhi *et al.* [134] namely:

$$\rho A(u_{tt}+2vu_{xt}+v^2u_{xx})-EA\left(u_x+\frac{1}{2}w_x^2\right)_x=0, \tag{21}$$

$$\rho A(w_{tt}+2vw_{xt}+v^2w_{xx})-P_0w_{xx} -EA\left[w_x\left(u_x+\frac{1}{2}w_x^2\right)\right]_x+\kappa GA(w_x+\theta)_x=0, \tag{22}$$

$$\rho I(\theta_{tt}+2v\theta_{xt}+v^2\theta_{xx})-EI\theta_{xx}-\kappa GA\left(w_x+\theta\right)=0. \tag{23}$$

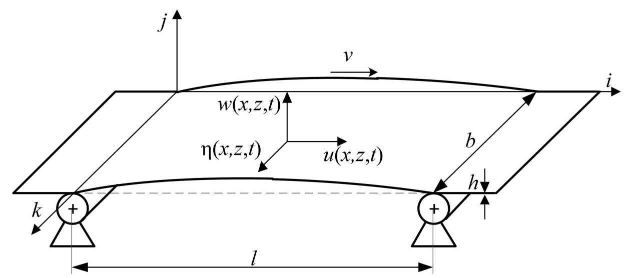

Fig. 6. Axially moving plate [214].

2.1.4 Plate model (3 PDEs)

When the width of the moving strip is sufficiently large, the system should be modeled using a two-dimensional plate equation, and its vibrations should be considered along both *i*- and *k*-axis (Fig. 6). Equivalently, the displacements are functions of the spatial coordinates x and z, and time t.

The earliest study from this aspect by Ulsoy and Mote [135] established the dynamic model for a blade of a band saw and proposed approximate solutions based on both the classical Ritz and finite-element-Ritz methods. In another work on an axially moving plate, Marynowski and Kolakowski [27] introduced a mathematical model for a translating orthotropic plate with width b and thickness h. In their study, both in-plane displacements (i.e., longitudinal and lateral displacements) and out-of-plane displacement (i.e., transverse displacement) were considered. First, the following strain–displacement relations were established.

$$\begin{aligned} &\varepsilon_{\mathrm{X}}=u_x+0.5w_x^2, && \kappa_{\mathrm{X}}=-w_{xx},\\ &\varepsilon_{\mathrm{Z}}=\eta_z+0.5w_z^2, && \kappa_{\mathrm{Z}}=-w_{zz},\\ &\varepsilon_{\mathrm{XZ}}=0.5(u_z+\eta_x+w_xw_z), && \kappa_{\mathrm{XZ}}=-w_{xz}, \end{aligned} \tag{24}$$

where ε_{X}, ε_{Z}, and $\varepsilon_{\mathrm{XZ}}$ represent the strain-tensor components of the middle plate in the x and z coordinates, and κ_{X}, κ_{Z}, and κ_{XZ} are the curvature modification and torsion of the middle surface of the plate. Now, the stress functions σ_{X}, σ_{Z}, and σ_{XZ} and bending moments M_{X}, M_{Z}, and M_{XZ} are given as follows:

$$\begin{aligned} &\sigma_{\mathrm{X}}=\frac{E_{\mathrm{X}}h}{1-\chi\upsilon^2}\left(\varepsilon_{\mathrm{X}}+\chi\upsilon\varepsilon_{\mathrm{Z}}\right),\\ &\sigma_{\mathrm{Z}}=\frac{E_{\mathrm{Z}}h\chi}{1-\chi\upsilon^2}\left(\varepsilon_{\mathrm{Z}}+\chi\varepsilon_{\mathrm{X}}\right),\\ &\sigma_{\mathrm{XZ}}=2Gh\varepsilon_{\mathrm{XZ}}, \end{aligned} \tag{25}$$

$$M_X = -\frac{E_X h}{12(1-\chi\upsilon^2)}(w_{xx} + \chi\upsilon w_{zz}),$$
$$M_Z = -\frac{\chi E_Z h}{12(1-\chi\upsilon^2)}(w_{zz} + \upsilon w_{xx}),$$
$$M_{XZ} = -\frac{Gh^3}{6} w_{xz}, \tag{26}$$

where G denotes the shear modulus of the plate; E_X and E_Z are Young's moduli of the plate along the i- and k-axis, respectively; the ratio $\chi = E_X/E_Z$ represents the orthotropic factor of the plate, and υ is the Poisson's ratio. By Hamilton's principle, the following governing equations describing the vibration of an axially moving plate are obtained as follows [27]:

$$\rho h(w_{tt} + 2vw_{xt} + v^2 w_{xx}) - M_{Xxx} - 2M_{XZxz} - M_{Zzz} - (\sigma_X w_x)_x - (\sigma_Z w_z)_z - (\sigma_{XZ} w_x)_z - (\sigma_{XZ} w_z)_x = F, \tag{27}$$

$$\rho h(u_{tt} + 2vu_{xt} + v^2 u_{xx}) - \sigma_{Xx} - \sigma_{Zz} = 0, \tag{28}$$

$$\rho h(\eta_{tt} + 2v\eta_{xt} + v^2 \eta_{xx}) - \sigma_{Xx} - \sigma_{Zz} = 0, \tag{29}$$

where F is the lateral loading. In the case that the plate does not experience any deflection and torque at the boundaries, the following boundary conditions are given.

$$w(0,z,t) = w(l,z,t) = 0,$$
$$M_Z(0,z,t) = M_Z(l,z,t) = 0. \tag{30}$$

2.2. Approximated model (ODEs)

To facilitate the use of certain techniques used to solve discrete problems to the analysis and control of the vibration of axially moving systems, the approximate model described by a finite set of ODEs is usually established by discretizing the PDE model. In Wickert and Mote [136], an approximate model for a moving string was developed using the classical Galerkin method, wherein the authors assumed that the lateral displacement of the string takes the following form,

$$w(x,t) = \sum_{i=1}^{n} q_i(t)\varphi_i(x), \tag{31}$$

where $q_i(t)$ represents the set of generalized displacements of the string, and $\varphi_i(x)$ represents the set of basis functions given as follows:

$$\varphi_i(x) = \sin\left(\frac{i\pi x}{l}\right). \tag{32}$$

Subsequently, the set of n coupled ordinary differential equations was expressed in the following general form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0, \tag{33}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are global matrices corresponding to the mass, damping coefficient, and string stiffness, respectively, and $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$, and $\mathbf{q}$ indicate the string acceleration, velocity, and deflection vectors, respectively. Approximate models for the moving beam can also be expressed in a similar form based on the Galerkin method [17, 102, 115, 137–139]. Furthermore, the Galerkin method was extended to the two-dimensional model, that is, axially moving plates [27, 140–142]. The lateral displacement of an axially moving plate can be assumed to take the following form:

$$w(x,y,t) = \sum_{i=1}^{n_i}\sum_{j=1}^{n_k} q_{ij}(t)\varphi_i(x)\psi_j(z), \tag{34}$$

where $q_{ij}(t)$ are unknown functions of time; $\varphi_i(x)$ and $\psi_j(z)$ are basis functions; and n_i and n_k represent the total number of basis functions corresponding to the lateral displacements along the i and k directions, respectively. In the extended Galerkin method, basis functions are only required to satisfy the displacement conditions at the boundaries (i.e., the essential boundary condition), and it is not necessary to meet the force and stress conditions at the boundaries (i.e., the natural boundary condition). The basis function $\varphi_i(x)$ can be chosen similarly to the case of a simply supported beam, whereas $\psi_j(z)$ has the same form as the case of a free-free beam. According to the extended Galerkin method and (34), the approximate model for axially moving plates that are described by a set of $n_i \times n_k$ coupled ODEs can be expressed in the matrix form.

3. CONTROL BASED ON ODE MODEL

In early studies on vibration suppression of axially moving systems, the PDEs describing the system vibration were often converted to a low-dimensional system of ODEs to facilitate the use of the available classical control methods. The design of the control law and its stability analysis were performed based on the ODE model.

3.1. Model-based feedback control

One of the earliest studies devoted to the vibration control of an axially moving system based on the reduced-order model was performed by Ulsoy [42]. In this paper, a pointwise controller for suppressing the lateral vibration of a moving string was presented. The approximate model described by a matrix form (i.e., similar to (33)) was expressed in the state space form as follows [42]:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{f}, \tag{35}$$

where $\mathbf{f}$ is the control input vector,

$$\mathbf{X} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}. \tag{36}$$

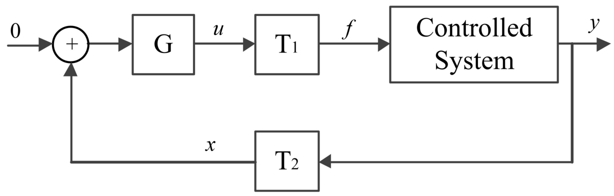

Fig. 7. Vibration control scheme for axially moving system [42].

Using the observer-based state feedback control method, the author introduced a control scheme, as shown in Fig. 7. Furthermore, the following equation of the closed-loop system was obtained.

$$\dot{\mathbf{X}} = (\mathbf{A} - \mathbf{B}\mathbf{T}_1\mathbf{G}\mathbf{T}_2)\mathbf{X}, \tag{37}$$

where $\mathbf{T}_1$ and $\mathbf{T}_2$ are the matrices that correspond to the locations of the actuators and sensors, respectively, and $\mathbf{G}$ is the control gain matrix that depends on the axial velocity. The closed-loop system with the designed controller, however, can lead to a spillover phenomenon because the reduced-order model ignores the high-frequency modes although the sensors are affected by these modes (i.e., observation spillover) that are excited by the actuators (i.e., control spillover) [143]. The spillovers due to the use of a reduced-order control in axially moving systems can cause instability. In their study, the authors also discussed the effects of observation and control spillovers and the enhancement of control performance via the use of a comb filter to eliminate the observation spillover.

3.2. Sliding mode control

The variable structure control (VSC) and its particular type—sliding mode control (SMC)—is recognized as a powerful tool in the design of a robust controller. Contrary to the conventional feedback control system that only has a fixed control structure throughout the control process, the VSC system is a synthesis of subsystems with separate control structures wherein each subsystem corresponds to a specified region of system behavior [144]. As an example of a typical VSC, the sliding mode control is characterized by a switching function and a feedback control law. The switching function is chosen so that the sliding motion satisfies the required specifications. The control law, in which the control gains switch during the control process, is designed to move the systems' state towards the vicinity of the switching function. A remarkable trait of the sliding mode control is its robustness. The response of the closed-loop system is insensitive to parameter variations: Therefore, the sliding mode control is regarded as a useful robust control technique.

In the early works on vibration suppression of an axially moving system based on a sliding mode control, Fung and Liao [48] attenuated the lateral vibrations of a translating string with a periodically varying tension and viscous damping based on the sliding mode technique and the independent model space control (IMSC). First, they adopted the Galerkin method to obtain ODEs for the string. The ODE model expressed in a matrix form was then transformed into the state space form. Based on this model, the reaching law method was used to design the sliding regime to satisfy the reaching condition. To reduce the numbers of sensors and actuators, they designed a controller to deal with a limited set of modes that only contained the modeled modes. The controller included two components: (i) A state estimator in which the inputs are sensor measurements, and the output is an estimated state. A modal filter [145] was employed to determine the modal displacements and velocities based on the displacement and velocity measured by the sensors; (ii) a linear state variable feedback control based on SMC and the estimated state to produce the control forces. Furthermore, the IMSC method (i.e., each mode is independently controlled) was combined with the SMC method to design a controller. Based on the numerical results, the authors concluded that the created control force could quickly reduce the lateral vibration of the string. Moreover, the closed-loop response of the system was insensitive to parametric uncertainty and external disturbances.

Later, Fung *et al.* [146] improved the controller based on the usual VSC using integral and proportional compensation: Consequently, two novel control methods were introduced—integral variable structure control (IVSC) and proportional variable structure control (PVSC). In accordance with the former, an integral compensator was utilized along with the VSC controller to deal with time-history errors, whereas the PVSC controller is a synthesizer of the VSC controller with a proportional gain. The authors then combined these two controllers in turn with the conventional VSC controller and two new controllers were referred to as the modified IVSC controller and the modified PVSC controller, respectively. The concept associated with the use of the modified IVSC and PVSC controllers is to switch the controllers at a suitable time to exploit their advantages, to enhance control performance. The modified IVSC controller employed the IVSC controller to improve its transient response as well as the VSC controller to minimize overshoot. The main traits of the modified PVSC controller include the reduction of the rising time of the PVSC controller and the stabilization of the steady-state responses by the VSC controller. The authors also demonstrated the effectiveness of these controllers in the alleviation of the influence of the lateral vibration of both linear and nonlinear moving strings.

In addition to the axially moving string with constant length, the sliding mode control was also used for vibration suppression of strings with varying length in Fung *et al.* [39]. This study will be introduced in detail in Section 6.

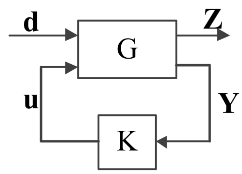

Fig. 8. H_∞ control scheme [49].

3.3. H_∞ control

Another robust control method called the H_∞ method has also been employed for vibration control in an axially moving system in the presence of disturbances by Wang *et al*. [49]. In this study, the lateral vibration of a moving cantilever beam with tip mass was suppressed via a noncontact magnet vibration exciter. The authors used the Galerkin method to convert the PDE model that describes the system into an ODE model, and the state-space equations were then obtained by transforming the ODE model. The state-space equations describe a control scheme based on an H_∞ close-loop system (Fig. 8) given as follows [49]:

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{AX}+\mathbf{B}_1\mathbf{d}+\mathbf{B}_2\mathbf{u},\\ \mathbf{Z} &= \mathbf{C}_1\mathbf{X}+\mathbf{D}_1\mathbf{u},\\ \mathbf{Y} &= \mathbf{C}_2\mathbf{X}+\mathbf{D}_2\mathbf{d}.\end{aligned} \tag{38}$$

In this case, $\mathbf{X} = \begin{bmatrix}\dot{\mathbf{q}} & \mathbf{q}\end{bmatrix}^T$, $\mathbf{d}$ is exogenous input including the reference signal and disturbances, $\mathbf{u}$ is the control input, $\mathbf{Z}$ denotes the error signal, $\mathbf{Y}$ is the sensed output, and $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{D}_1$, and $\mathbf{D}_2$ are the state-space matrices. The concept of this method is to design controller $\mathbf{H}$ to minimize the H_∞ norm of the closed-loop transfer function of the system. An experimental system was employed to prove the effectiveness of the designed controller, and the results indicated that the response of the system under the proposed controller was excellent.

4. CONTROL BASED ON PDE MODEL

As previously indicated, control strategies based on the ODE model can lead to the spillover phenomenon. To address this problem, researchers have investigated the use of the PDE model in the design of a controller for axially moving systems. In this section, control methods using the PDE model directly will be introduced.

4.1. Frequency domain analysis

4.1.1 Transfer function method

According to the distributed transfer function method originated by Butkovskiy [147], the dynamic behavior of a closed-loop system can be predicted based on the information included in the transfer function. Accordingly, by analyzing the transfer function, the control law of axially moving systems can be designed in the frequency domain [148–152].

In the pioneering studies on the development of a stable feedback controller for vibration suppression of axially moving strings, Yang and Mote [150,152] established a transfer function containing the dynamics of the string, sensors, actuators, and control law. In their studies, the following control force was formulated in the frequency domain [150]:

$$f(x,s) = \delta(x-x_a)G_a(s)G_c(s)G_s(s)w(x_s,s), \tag{39}$$

where s is the Laplace variable, $G_a(s)$ and $G_s(s)$ are the transfer functions of the actuator located at x_a and the sensor at x_s, respectively, and $G_c(s)$ is the transfer function of the designed controller. Stability analysis was performed in the frequency domain by investigating the root loci of the closed-loop system. Consequently, the two stability criteria were established. Based on these stability criteria, the feedback controllers were designed for two cases; collocated and non-collocated positions of the sensors and actuators. In the latter, the time delay technique can be employed to handle the influence of the phase lag effect on the on-line vibration control. Besides, it was shown that the problem of spillover instability could be solved and all the modes of vibration can be stabilized. In Yang and Mote [150], the practicability of the proposed controller was experimentally validated. At the approximately same time, Yang and Mote [151] introduced a feedback control for a translating string based on both the stability criteria determined via the root loci method and the generalized Nyquist stability criteria. Later, Yang [150] used an exact form method to analyze transfer functions of constrained axially moving beams.

4.1.2 Wave cancellation method

The concept of the wave cancellation method is to attenuate the vibration energy of axially moving systems by absorbing traveling waves. In this method, the system can be stabilized by eliminating all reflected waves and preventing the accumulation of vibration energy. This method was used to decay vibrations via the pointwise technique [50,51,63] and the boundary control technique [37].

With regard to the implementation of the wave cancellation method, Chung and Tan [37] established boundary feedback control laws to suppress the oscillations of a moving string subject to external forces in two cases: unconstrained and constrained forces. In their investigation, the exact closed-form that describes the transfer function of the closed-loop system that contains the dynamics of the string, sensors, actuators, and a feedback boundary control law was derived and analyzed. According to the wave cancellation method, the boundary control force is expressed in the frequency domain as follows [37]:

$$f(x,s) = -\lambda_1\left(1-\left(v\frac{\rho}{P}\right)^2\right)se^{-s\lambda_1(1-x_s)}w(x_s,s), \tag{40}$$

where λ_1 is the proportional gain and x_s is the location of the sensor. Subsequently, based on transfer function analysis, the authors showed that the controller could eliminate all reflected waves at the right boundary and the system had no pole in the right half-plane under the proposed control law: Hence, the controlled system was stable with no resonance. Based on numerical simulations, they demonstrated the effectiveness of the wave cancellation method in vibration suppression of an axially moving string. Ying and Tan [51] presented a controller based on the pointwise control technique for reducing the oscillation in a region near the boundary (i.e., the downstream part of a string) of an axially moving string with excitations. In their paper, the following pointwise control force was designed:

$$f(x,s) = \frac{-2s[w(x_1,s) - w(x_2,s)e^{-\lambda_1(x_2-x_1)}]e^{\lambda_2(a-x_1)}}{1-e^{(\lambda_2-\lambda_1)(x_2-x_1)}} - C(s)\frac{2sw(x_3,s)}{1-e^{(\lambda_2-\lambda_1)(1-x_3)}}. \quad (41)$$

Control force (41) was determined via the synthesis of two controllers; a feed-forward controller (i.e., the first term) and a feedback controller (i.e., the second term). The feed-forward controller consisting of a velocity sensor and time delay functions was employed to eliminate the vibrations of the string, whereas the feedback controller, which acts as a low-pass filter was designed to suppress the oscillations due to undesired disturbances in the downstream region. The numerical results showed that the controllers worked effectively under both sinusoidal and random excitations.

In all the works above, the moving string was investigated only for the case of fixed boundaries, and the stability of the feed-forward control was not considered. Concerning a moving system with unfixed boundary conditions, Tan and Ying [50] discussed wave cancellation control for an axially moving string in which the general boundary conditions were considered. A pointwise feed-forward controller using two control forces was designed to suppress the vibration of the string in both the upstream and downstream regions. Furthermore, to enhance the stability and robustness of the controlled system, they supplemented control forces with a stabilization coefficient without changing the controller structure. The effectiveness of the proposed controller was experimentally validated on both the moving belt drive and the automotive engine chain drive systems. In another study on wave cancellation control, Zhang and Chen [63] designed a feedback control law for a serpentine belt system. In their paper, the serpentine belt-driven system was modeled as a moving string, and a tensioner arm was used to suppress the lateral vibration of the string. The equations of motion that contain the dynamics of the string and tensioner were transformed using the Laplace transform, and the resulting equations were used to design the controller.

4.2. Time-domain analysis

4.2.1 Lyapunov method

The Lyapunov method has received considerable attention in the development of controllers for axially moving systems in either linear or nonlinear formulation [153]. According to the Lyapunov method, the stability of a system can be analyzed based on the concept of energy attenuation; namely, the system is stable when its total energy is continuously decaying. The main point of the design of a controller is to establish an appropriate Lyapunov function candidate associated with the system energy. Moreover, the control law is designed in such a way that the candidate function becomes a Lyapunov function for the system (i.e., the time derivative of the candidate is negative semi-definite or negative definite). As such, the stability of the system under the designed controller is naturally archived.

Based on the Lyapunov direct method, Lee and Mote [41] investigated the implementation of right/left boundary control laws to suppress the vibration energy of an axially moving string. In their paper, the total mechanical energy of the system was used as a Lyapunov functional candidate as follows:

$$V(t) = \frac{1}{2}\rho\int_0^l (w_t(x,t)+vw_x(x,t))^2 dx + \frac{1}{2}P_0\int_0^l w_x(x,t)^2 dx. \quad (42)$$

Based on the Lyapunov method, boundary control forces were proposed to stabilize the vibration of the string via passive control using a viscous damper or velocity feedback active control. For example, the right force control law with the local velocity feedback was expressed as follows [41]:

$$f(t) = -k_f w_t(l,t), \;\; 0 < k_f < 1/v, \quad (43)$$

where $w_t(l,t)$ is the lateral velocity at the right boundary of the string. Also, the right force control law with the material velocity feedback was given as follows [41]:

$$f(t) = -k_d(vw_x(l,t)+w_t(l,t)), \;\; k_d > 0, \quad (44)$$

where $w_t(l,t)$ is the slope at the right boundary of the string. Furthermore, they also obtained the time-optimal control gain for the maximum dissipation of vibration energy by minimizing the reflected energy at the boundaries. The maximum time to decay the vibration energy of the system to zero was also archived. The proofs of asymptotic and exponential stabilities of the closed-loop system were then pursued based on the invariance principle and the semigroup theory. The authors showed the effectiveness and optimality of the designed boundary control via numerical results using a finite differential scheme.

Vibration suppression of axially moving Kirchhoff strings with varying tension was presented in [154–

158]. In these studies, the lateral vibration of translating Kirchhoff strings with differential non-constant tensions was suppressed by implementing linear boundary control forces. Linear control inputs, which were the negative feedback of the lateral velocity at the string boundary, were given as follows [154]:

$$f(t) = -kw_t(l,t), \;\; k > 0. \tag{45}$$

The stability of the system controlled by (45) was then verified based on the Lyapunov method. Subsequently, Li *et al*. [159] expanded the results of the previous works by using the exact strain to describe the geometric nonlinearity due to the finite lateral deformation. The authors implemented the control law (45) on a string boundary to reduce the lateral vibrations of the system, and they also showed that these lateral vibrations exponentially tend to zero as time tends to infinity. An extension of the adaptive boundary control based on the negative velocity feedback control law to dissipate the vibration of the Kirchhoff string was also investigated by Kim *et al*. [160].

In another study on the implementation of the Lyapunov method, Fung *et al*. [161] proposed a linear boundary feedback control law to suppress the linear vibration of a damped moving string with a mass-damper-spring (MDS) mechanism located at the right boundary. According to the Lyapunov method and the semigroup theory, they proved that the total mechanical energy of the controlled system is exponentially attenuated. Concurrently a study on nonlinear vibration suppression of a damped moving string with MDS was performed by Fung *et al*. [162]. In this work, they considered the influence of axial deformation on the potential energy of the string. Under this consideration, the lateral vibration of the string became nonlinear, and the vibration suppression was performed by applying the boundary control force $f(t)$ on the MDS system as follows [162]:

$$f(t) = \begin{cases} -kw_t(l,t) - \delta_A w_x(l,t)^4/w_t(l,t), & \\ & w_t(l,t) \neq 0, \\ -kw_t(l,t), & w_t(l,t) = 0, \end{cases} \tag{46}$$

where δ_A is the linear/nonlinear coefficient of the feedback control law. The stability of the system under the controller above was verified based on the following Lyapunov function candidate [162].

$$\begin{aligned} V(t) =& \frac{1}{2}\int_0^l \Big[\rho(w_t(x,t) + vw_x(x,t))^2 + P_0 w_x(x,t)^2 \\ &+ \frac{1}{4}EAw_x(x,t)^4\Big]dx + \frac{1}{2}[mw_t(x,t)^2 + k_e w(x,t)^2], \end{aligned} \tag{47}$$

where m and k_e are the mass and stiffness of the MDS. Based on the investigated results, the authors concluded that the system using the linear control law (i.e., $\delta_A = 0$) was asymptotically stable: However, they could not prove that this system is exponentially stable due to the presence of nonlinear terms. To address this problem, the nonlinear feedback control law that dominates the nonlinear terms was used to decay the vibration. Under the nonlinear control law, both the asymptotic and exponential stabilities of the closed-loop system were proven.

These studies focused on the suppression of only the lateral vibration of a system without examining the longitudinal vibration. The use of the Lyapunov method to design a controller to suppress the longitudinal vibration of axially moving systems was investigated in several studies [163–165]. Furthermore, Nguyen and Hong [166] controlled both lateral and longitudinal vibrations and the translating velocity of an axially moving string system by regulating the control torques of two rollers of the system and applying an external control force via a hydraulic actuator at the right boundary. The authors established a coupled PDE-ODE model containing longitudinal and lateral oscillations, axial velocity, and the motion of the actuators. Based on this model, they observed that vibration suppression and velocity control are coupled because the control torques of the two rollers affected the axial velocity and the longitudinal dynamics as well as the tension. Also, the lateral dynamics of the string was affected by the longitudinal displacement. A control scheme using the following control torques and control force was designed to suppress both longitudinal and lateral oscillations as well as to control the axial velocity to track the desired profile [166].

$$\begin{aligned} f(t) =& -\frac{k_1k_2lP_0}{2(k_1v+2k_2l)^2}w_t(l,t) - \frac{k_2lP_0}{2(k_1v+2k_2l)}w_x(l,t) \\ &- \rho v^2 w_x(l,t) + (c_a - \rho v)w_t(l,t) \\ &- (v + 2k_2l/k_1)mw_{xt}(l,t), \end{aligned} \tag{48}$$

$$\begin{aligned} \tau_1(t) =& \frac{EAR}{k_1v}\Bigg\{\frac{k_1v(T_{b1}-T_0)}{EA} + k_2lEAu_x(l,t)w_x(l,t)^2 \\ &- \frac{k_2EA}{2l}\left(1-\frac{1}{2k_{\tau1}}\right)u(l,t)^2 - \frac{EA}{2lk_1}u(l,t)u_t(l,t) \\ &- k_1(v+u_t+vu_x)(u_x+w_x^2/2)\Big|_{x=0}^{x=l} \\ &+ \frac{Jk_1}{R^2}\left[\left(\frac{k_1v}{EA} - u_t(0,t)\right)u_{tt}(0,t) - u_t(l,t)u_{tt}(l,t)\right]\Bigg\} \\ &- k_2EAlu_x(l,t)^2 + k_1vu_x(0,t)) \\ &+ k_1mw_x(l,t)w_{xt}(l,t + k_2\rho l(w_t(l,t) \\ &+ vw_x(l,t))^2 - \frac{u_t(0,t)^2 + u_t(l,t)^2}{k_{\tau1}} \\ &- k_2\rho l(v + u_t(l,t) + vu_x(l,t)), \end{aligned} \tag{49}$$

$$\begin{aligned} \tau_2(t) =& (\rho lR + 2J/R)\dot{v}_d - k_{\tau2}(v - v_d) - \tau_1(t) \\ &+ (T_{b1} - T_{b2})R, \end{aligned} \tag{50}$$

where k_1 and k_2 are the positive constants, $k_{\tau1}$ and $k_{\tau2}$

are the control gains, J and R are the inertia moment and the radius of the two rollers, respectively, T_{b1} and T_{b2} are constants that are related to the material tensions in the respective adjacent spans, c_a is the damping coefficient of the actuator, and v_d denotes the desired axial velocity. The asymptotical stability of the system under the designed control law was proven using the Lyapunov method, wherein the Lyapunov function candidate was defined based on the modified total mechanical energy of the string as follows [166]:

$$\begin{aligned}V(t) =& k_1\left\{\frac{1}{2}\int_0^l\left[\rho\left(v+u_t+vu_x\right)^2+\rho\left(w_t+vw_x\right)^2\right.\right.\\&\left.+EAu_x^2+P_0w_x^2+\frac{1}{4}EAw_x^4\right]dx+\frac{1}{2}\left(v-v_d\right)^2\\&+\frac{EA}{4l}u(l,t)^2+\frac{m}{2}\left(w_t(l,t)+\left(v+\frac{2k_2l}{k_1}\right)w_x(l,t)\right)^2\\&+\frac{mw_x(l,t)^2}{2}+\frac{J}{2R^2}\left(u_t\left(0,t\right)^2+u_t(l,t)^2\right)\\&\left.+\frac{1}{2}\int_0^l EAu_xw_x^2dx\right\}\\&+2k_2\rho\int_0^l x[u_x(v+u_t+vu_x)+w_x(w_t+vw_x)]dx.\end{aligned} \tag{51}$$

Apart from the studies on vibration reduction for translating elastic materials, control and stability analyses for axially moving viscoelastic materials were also investigated in [167–174]. In these studies, the integral constitutive law — Boltzmann's principle — was employed to describe the viscoelastic material properties. Therefore, axially moving systems were described using integro-partial differential equations. Kelleche *et al*. [173, 174] demonstrated that the stabilization of a viscoelastic string could be guaranteed due to the extra damping produced by the string's movement. Subsequently, Kelleche and Tatar [172] proposed a boundary control of a moving viscoelastic Kirchhoff string using a hydraulic actuator at the right boundary as follows:

$$f(t)=(\eta_{\mathrm{a}}-v)w_t(l,t)-mvw_{xt}(l,t), \tag{52}$$

where m and η_{a} are the mass and damping coefficient of the hydraulic actuator. The uniform stability of the controlled system was proven via stability analysis and the multiplier method. Subsequently, Kelleche and Tatar [171] introduced a boundary controller for a translating beam and showed that the system controlled by the proposed controller was exponentially stable. In another study, an exponentially stable adaptive boundary control, in which an adaptive technique was employed to handle the boundary disturbance, was designed by Kelleche [167]. After that, Kelleche and Saedpanah [169] expanded the work in [167] to the string under a spatiotemporally varying tension. In their work, stability analysis and the multiplier method were also employed to evaluate the exponential stability of the system under the designed adaptive boundary control law.

4.2.2 Adaptive control

In many practical systems, difficulties may be encountered in the control system design due to the presence of system uncertainties. Their presence can negatively influence control performance as well as causing instability. One of the widely discussed techniques to address this problem is adaptive control [175–180]. The central concept of adaptive control is to estimate the unknown parameters online using the signals available in the system, followed by the design of an adaptive controller based on the estimated parameters. Two ways to develop adaptive controllers are via the model-reference and self-tuning adaptive control. In accordance with the model reference adaptive control, a reference model is first established, and a controller is then designed using an adaptation law, wherein this law adjusts controller parameters such that the error between the response of the controlled real model and the reference model converges to zero. Contrary to the model-reference adaptive control, the self-tuning adaptive control estimates the unknown parameters in the plant using the input/output data online (i.e., tune the system parameters first), and then use the estimated parameters in an established control method.

1) Model reference adaptive control

De Queiroz *et al*. [175] pioneered the implementation of an adaptive technique for vibration control of axially moving systems with unknown parameters. In this study, the authors investigated a hybrid system including an axially moving string and a mechanical guide located within the string span, in which several system parameters (e.g., guide mass and inertial and string tension) were unmeasured. To attenuate the lateral vibration of the string, a control force and a control torque or equivalently, two control input forces $f_1(t)$ and $f_2(t)$ were applied to the string via the mechanical guide. The Lyapunov method was first used to design an exponentially stable control law based on exact knowledge of the system parameters (i.e., guide mass m, guide inertial J, string density mass ρ, and string tension P_0), namely,

$$\mathbf{F}=-\mathbf{A}(t)\boldsymbol{\Theta}-\mathbf{K}_s\begin{bmatrix}w_t(l_1,t)+w_x(l_1,t)\\w_t(l_2,t)+w_x(l_2,t)\end{bmatrix}. \tag{53}$$

In this case, we have:

$$\mathbf{A}(t)=\begin{bmatrix}A_{11} & A_{12} & A_{13} & A_{14}\\A_{21} & A_{22} & A_{23} & A_{24}\end{bmatrix}, \tag{54}$$

where

$$\begin{cases} A_{11} = \dfrac{w_{xt}(l_1,t) - w_{xt}(l_2,t)}{4}, \\ A_{12} = \dfrac{w_{xt}(l_1,t) + w_{xt}(l_2,t)}{(l_2-l_1)^2}, \\ A_{13} = w_t(l_1,t) - w_t(l_2,t), \\ A_{14} = -\dfrac{w(l_1,t) - w(l_2,t)}{l_2-l_1} - w_x(l_1,t), \\ A_{21} = \dfrac{w_{xt}(l_1,t) - w_{xt}(l_2,t)}{4}, \\ A_{22} = -\dfrac{w_{xt}(l_1,t) + w_{xt}(l_2,t)}{(l_2-l_1)^2}, \\ A_{23} = w_t(l_1,t) - w_t(l_2,t), \\ A_{24} = \dfrac{w(l_1,t) - w(l_2,t)}{l_2-l_1} - w_x(l_2,t). \end{cases} \tag{55}$$

$\mathbf{K}_s \in \mathbb{R}^{2\times 2}$ is a positive-definite diagonal matrix that contains the control gains, and Θ denotes the vector of unknown parameters, namely,

$$\mathbf{\Theta} = \begin{bmatrix} m+\rho(l_2-l_1) & J+\frac{\rho(l_2-l_1)^3}{12} & \rho v & P_0 \end{bmatrix}. \tag{56}$$

The control law was redesigned using the model-reference adaptive control scheme to compensate for the uncertainties associated with these system parameters. The redesigned control law is given as follows:

$$\mathbf{F} = -\mathbf{A}(t)\hat{\mathbf{\Theta}} - \mathbf{K}_s \begin{bmatrix} w_t(l_1,t)+w_x(l_1,t) \\ w_t(l_2,t)+w_x(l_2,t) \end{bmatrix}, \tag{57}$$

where $\hat{\mathbf{\Theta}}$ is the vector of the estimated parameters that were determined online based on the following adaptive law:

$$\dot{\hat{\mathbf{\Theta}}} = \mathbf{K}_{\mathrm{a}}\mathbf{A}^{\mathrm{T}}(t) \begin{bmatrix} w_t(l_1,t)+w_x(l_1,t) \\ w_t(l_2,t)+w_x(l_2,t) \end{bmatrix}, \tag{58}$$

where $\mathbf{K}_{\mathrm{a}}$ is the adaptive gain matrix. They also demonstrated the asymptotic stability of the system under the new controller and validated the control law for its stability. Li and Rahn [176] used an adaptive control scheme to control the lateral vibration of a translating beam. In their paper, the beam was divided into two spans — a controlled span and an uncontrolled span subjected to a distributed bounded disturbance — using a pivoting roller actuator located within the beam. Based on the Lyapunov method and the model reference adaptive control, they presented an asymptotically stable adaptive controller to isolate the controlled span from disturbances in the uncontrolled span; namely, to dissipate the undesired lateral vibration in the controlled span. They also performed experiments to verify the effectiveness of the proposed controller for vibration suppression. In their subsequent study, Li *et al.* [177] extended the adaptive control method in [176] to isolate the controlled span from both distributed disturbance and boundary disturbance in the uncontrolled span. The experimental results demonstrated the efficiency of the designed adaptive controller. Chen and Zhang [55] investigated vibration suppression in a similar model, wherein a tensioner arm was used instead of a mechanical guide. The tensioner arm, which was a part of a serpentine belt drive system, included a tensioner spring with unknown stiffness, a rotation arm with uncertain inertia, and a roller. A control law was established based on the Lyapunov method and adaptive control technique, and the asymptotical stability of the controlled system was also subsequently proven.

2) Self-tuning adaptive control

Fung *et al.* [40] investigated the control scheme for a translating string with a mass-spring-damper mechanism at the boundary using the self-tuning approach. In this paper, the authors succeeded in extending the adaptive computed-torque control algorithm used in the lumped parameter system to handle the distributed parameter system; namely, the axially moving string system. Based on these algorithms, they developed an adaptive boundary controller for vibration suppression and then proved that the designed controller asymptotically decayed the lateral oscillation of the system. The control law based on the adaptive computed-torque controller is given as follows [40]:

$$\begin{aligned} f(t) = &-\hat{m}(k_1 w_t(l,t)+k_2 w(l,t)) + (\hat{c}_{\mathrm{e}} - \hat{\rho}v)w_t(l,t) \\ &+\hat{k}_{\mathrm{e}} w(l,t) + (\hat{P}_0 - \hat{\rho}v^2)w_x(l,t), \end{aligned} \tag{59}$$

and adaptive laws can be stated as follows:

$$\begin{cases} \dot{\hat{m}} = \dfrac{k_{\mathrm{a1}}}{4\hat{m}}\Psi(t)w_{tt}(l,t), \\ \dot{\hat{c}}_{\mathrm{e}} = \dfrac{k_{\mathrm{a2}}}{4\hat{m}}\Psi(t)w_t(l,t), \\ \dot{\hat{\rho}} = -\dfrac{k_{\mathrm{a3}}}{4\hat{m}}\Psi(t)\left[vw_t(l,t)+v^2 w_x(l,t)\right], \\ \dot{\hat{P}}_0 = \dfrac{k_{\mathrm{a4}}}{4\hat{m}}\Psi(t)w_x(l,t), \\ \dot{\hat{k}} = \dfrac{k_{\mathrm{a5}}}{4\hat{m}}\Psi(t)w(l,t), \\ \dot{\hat{c}} = 0, \end{cases} \tag{60}$$

where

$$\Psi(t) = w_{\mathrm{d}}(t) - w(l,t) + 2\left(w_{\mathrm{d}t}(t) - w_t(l,t)\right). \tag{61}$$

m, c_e, and k_e are the mass, damping, and stiffness of the mass-damper-spring system, respectively, c is the damping coefficient of the string, and $w_d(t)$ denotes the desired trajectory. Considering (59)-(61), they concluded that the obtained boundary control could be conveniently applied in practice because the control input and the adaptive laws only require the displacement, velocity, and slope of the string at the boundary.

3) Robust adaptive control

Adaptive control is a powerful technique for coping with uncertainties for constant or slowly varying parameters. However, in the presence of unknown disturbance, noise, and unmodeled dynamics, a controller based on adaptive control can lead to instabilities due to the lack of robustness of the adaptive controller [181]. In these situations, robust control that can deal with such disturbance, noise, and unmodeled dynamics should be employed in conjunction with adaptive control. In the studies of an axially moving system with unknown bounded disturbances, robust adaptive control is often used to design the controller. In these instances, robust control addresses the disturbance rejection while the adaptive control is employed to estimate the unknown parameter as well as the bound of the disturbance.

In [44], an axially moving string with varying tension was separated into two spans by a lateral force actuator located within the string span. To decay lateral vibrations in the controlled span of the string, the authors developed a boundary control law based on the Lyapunov method. Given that the oscillation in the uncontrolled span caused disturbances in the actuator, a robust control strategy was proposed to deal with these unknown disturbances: Consequently, the following control law was designed [44].

$$\begin{aligned} f(t) =& (c_a-\hat{\rho}v)w_t(l,t)-\hat{\rho}v^2w_x(l,t)+k_3w_x(l,t) \\ &-\frac{m}{k_1}(k_1v+2k_2l)w_{xt}(l,t)-k_4w_t(l,t)-f_d(t), \end{aligned} \tag{62}$$

where the last term is determined using a robust control strategy, namely:

$$f_d(t)=\frac{\hat{\mu}_d(t)^2\left[k_1w_t(l,t)+(k_1v+2k_2l)\,w_x(l,t)\right]}{\varepsilon+\hat{\mu}_d(t)\left|k_1w_t(l,t)+(k_1v+2k_2l)\,w_x(l,t)\right|}, \tag{63}$$

with $\varepsilon>0$, and the adaptive laws used to estimate the bound of the disturbance and mass destiny of the string are given by

$$\begin{aligned} \dot{\hat{\mu}}_d(t) =& k_{a2}|k_1w_t(l,t)+(k_1v+2k_2l)\,w_x(l,t)| \\ &-k_{a1}\hat{\mu}_d(t), \end{aligned} \tag{64}$$

$$\begin{aligned} \dot{\hat{\rho}} =& k_{a3}v[w_t(l,t)+vw_x(l,t)] \\ &\times[k_1w_t(l,t)+(k_1v+2k_2l)w_x(l,t)]. \end{aligned} \tag{65}$$

Subsequently, based on the semigroup theory, the authors showed that the system associated with the controller (62)-(65) is asymptotically stable. Nguyen and Hong [182] suppressed the vibration of a moving string with spatiotemporal tension using a system that included a hydraulic actuator and a damper at the boundary. Based on the Lyapunov redesign method, a robust adaptive boundary controller in which adaptation laws were used to estimate the uncertain parameters (i.e., the density of the string, actuator mass, and damping coefficient of the damper) and the bound in the boundary disturbance was presented. The authors proved that the controlled span was asymptotically stable and demonstrated the effectiveness of the controller via numerical results.

4.2.3 Optimal control

Besides the aforementioned control methods, optimal control methods have also been successfully implemented in axially moving systems. In [38], an optimal boundary controller for a translating string with a mass-damper-spring at the boundary was developed based on the output feedback method and the maximum principle theory. The control input design based on the output feedback control law requires the values of the lateral displacement and velocity at the boundary. By employing the maximum principle theory that represents the controller in terms of an adjoint variable, the designed control input only included the adjoint variable at the boundary, the mass of the MDS mechanism, the mass density, and the string length.

5. HYBRID CONTROL METHODS

Hybrid control methods have been developed based on the synthesis of different control methods, in which the advantages of each control method are exploited to enhance the control performance of the system. Concerning the vibration suppression of axially moving systems, several hybrid control approaches have been proposed [59, 60, 183–186].

In [183], the authors applied a hybrid control approach — fuzzy sliding mode control — for a flexible cable, which was modeled as a string with varying length. In this paper, the PDE model of the system was established and converted to a multi-dimension dynamic system using the third-order truncated Galerkin method. An active control technique based on the combination of sliding mode control and fuzzy logic theory was then developed to suppress the significant amplitude vibrations of the string. The effectiveness of this control strategy was illustrated with numerical results of the string during extrusion. Chao and Lai [59] and Huang *et al*. [60] combined the sliding mode control with intelligent control techniques that were developed based on the emulation of the characteristics of human intelligence; namely, fuzzy control and neural networks. Fuzzy control is a powerful tool for the development of controllers when the system information is uncertain, imprecise, or ambiguous, using logical rules and fuzzy set theory. In the neural-networks method, artificial neural networks inspired by the human neural system have been established. These artificial neural networks are mathematical models with learning capacity that can be employed to determine the input-output relationship of a system. In Huang *et al.* [60], a sliding-mode control law was first developed to control the lateral vibrations of an axially moving string via the variation of the string's tension. They also utilized two intelligent control approaches—fuzzy sliding-mode control and fuzzy neural network to handle the discontinuity and non-analyticity of the control input when the vibration is small. In these fuzzy logic-based approaches, the switching function and its derivative were used as inputs while a tension variation was considered as the output. Chao and Lai [59] also designed a boundary controller based on these approaches for vibration control of a translating string with a mass-damper-spring at the boundary. In another work, Ma *et al.* [185] used a fuzzy PD controller to suppress the vibrations of a translating cantilever beam. Subsequently, Ma *et al.* [186] extended this approach to a non-uniform cantilever beam.

Apart from the aforementioned studies, the combination of adaptive control with other methods to handle parametric uncertainties has also been received considerable attention. For example, Fung *et al.* [40] developed an adaptive computed torque control, whereas Yang *et al.* [44] designed a robust adaptive control based on the Lyapunov method. These works were introduced in detail in the previous section.

6. CONTROL APPLICATIONS

6.1. Lifting systems

Axially moving systems with varying length were investigated due to their wide applications in various systems such as elevator cables [39, 64, 182, 187], mining cable elevators [188, 189], container cranes [190, 191], and drilling risers [192–194].

In one of the first studies on vibration control for axially moving systems with varying length, Fung *et al.* [39] developed an active control strategy based on the sliding mode control to suppress the lateral vibrations of a moving elevator cable using a permanent magnet (PM) synchronous servo motor. A set of nonlinear partial differential equations including the dynamic model of a string with varying length and the rotor of the motor were discretized using the Galerkin method with time-dependent basis functions. The lateral vibration of the string was reduced by controlling the current of the PM synchronous servo motor because the motion of the string and the rotor was coupled. A sliding mode control algorithm in which the sliding mode was designed by using the reaching law method was used to control the current of the motor. As a result, lateral vibrations could be suppressed. Based on numerical analyses, the authors demonstrated the effectiveness of the proposed controller. The lateral vibration was suppressed, and the total energy of the elevator cable was reduced during both extrusion and retraction, and the system was also stabilized.

In another study in this aspect, Zhu and Ni [64] designed a general control law for pointwise controllers to decay the lateral vibrations of axially moving systems with varying length and an attached mass-spring at the boundary (i.e., both beam and string model). The exponential stability of the system under the pointwise controller was analyzed via the Lyapunov method. Furthermore, they also determined the optimal control gains for the controller that led to the fastest rates of reduction of the energy associated with the vibration of the controlled system. Later, Zhu and Chen [187] presented a novel experimental method to verify the theoretical prediction for the vibration behavior of the uncontrolled and controlled elevator cables based on a scaled elevator.

In [190], the model of the translating string with varying length was utilized to analyze the vibration response of a hosting cable used to host up and lower loads in a container crane system. In this paper, a coupled PDE-ODE model that described the dynamics of the hybrid system including a trolley (gantry), a hosting cable, and a load (i.e., container and spreader) was developed. Based on this model, the authors designed a Lyapunov-based boundary controller, wherein a control force was applied to the trolley to move the load to the desired position and suppress the lateral vibrations of the load when the trolley attained the desired position. An experimental evaluation was performed using the InTeCo 3D Crane system to validate the feasibility and effectiveness of the proposed controller and to compare the control performances of this controller with the one proposed by Rahn *et al.* [195] in the lift-up process. Subsequently, Ngo and Hong [191] investigated a similar container crane system in which an unknown disturbance force affected the trolley. Based on the synthesis of the Lyapunov method and the adaptive control technique, an adaptive boundary control was designed to control the system in which an adaptive technique was employed to estimate the amplitude of the unmeasured disturbance. The efficiency of the system with the designed controller was also experimentally examined.

Another approach for the application of boundary control for axially moving systems with varying length was proposed by He *et al.* [193]. In their work, the authors proposed a boundary controller using two hydraulic actuators to suppress the lateral vibrations of a vertical string with a tip mass. A static hydraulic actuator was attached at the top boundary, and a moving one with the same axial speed as the tip mass applied a control force to the tip mass. According to the integral-barrier Lyapunov function, it was demonstrated that the controlled system is exponentially stable when the disturbance is ignored. In the case in which the boundary disturbances affect the tip mass, a disturbance observer was employed. Subsequently, based on the aforementioned work, He *et al.* [194] investigated the vibration problem of a moving flexible drilling riser in a drilling system. In this paper, the drilling riser was modeled as an axially moving beam with varying length, and the authors only used a moving hydraulic actuator attached to the tip mass to suppress lateral vibration. In a later study, Gou *et al*. [192] designed a boundary control for the drilling riser when subjected to both the boundary disturbance and the distributed disturbances caused by ocean currents.

In addition to the lateral vibration, the longitudinal vibration control of an axially moving cable with varying length was also investigated in [188, 189]. Wang *et al*. [188] developed the equation of motion for a mining cable used to lift up and down a cage. Subsequently, they proposed an observer-based output feedback control law for dissipating the longitudinal vibration of the cable using the state observer. The exponential stability of the controlled system was validated via Lyapunov analysis. Later, Wang *et al*. [189] improved the results of their study by addressing the problem of anti-collocated disturbance caused by airflow in the cage. They designed a disturbance estimator to dissipate the harmonic disturbances with uncertain amplitudes and frequencies. An output feedback control was then designed based on the estimated disturbance for suppressing the longitudinal vibration of the mining cable with airflow disturbance.

6.2. Steel industry

Zinc galvanizing is the process of coating a steel strip with zinc. In this process, the oscillation of the axially moving steel strip can affect the uniformity of the zinc layer coated on the strip surface. Therefore, to improve the quality of the coated zinc, the undesirable vibration of the strip should be suppressed. Implementation of a boundary control on the translating steel strip in the zinc galvanizing line was introduced in [43, 44, 66, 196–201]. In these situations, the steel strip can be modeled using a string model [44, 197, 198], a beam model [43, 196, 199, 200], or a coupled model [66], depending on the considered distance between two boundaries.

In [66], the authors used a coupled model to describe the strip: Therefore, both the lateral and longitudinal vibrations of the strip were investigated. The coupled equations of lateral and longitudinal vibrations were decoupled using the quasi-static stretch assumption [13]. Under this assumption, the lateral oscillation of the strip can be controlled using a single actuator. Based on the Lyapunov method and the quasi-static stretch assumption, a nonlinear right boundary control law was derived. It was also proved that the axially moving strip is exponentially stable under the proposed control law. In another study [196], the steel strip was modeled as an Euler–Bernoulli beam, and its nonlinear oscillations were decayed via passive damping and active control. A control force was applied at the right boundary of the strip via a hydraulic touch-roll actuator. Based on the Lyapunov method, a boundary control law was implemented, wherein both the strip and actuator dynamics were considered. The authors determined that the designed control law only requires the measured value of the strip slope at the right boundary and the damping coefficient of the actuator, which is a design parameter of the actuator. They also validated the exponential stability of the closed-loop system using the semigroup theory. Later, Yang *et al*. [43] extended this work [196] by considering the axial tension of the strip as a spatiotemporally varying function. In practical situations, the tension is a periodic bounded function in time due to the eccentricity of the support rollers. Also, the gravitational force, which depends on the spatial variable, can be considered as an additional tension on the strip. The authors developed a boundary control law for a translating beam with spatiotemporal tension and subsequently verified the asymptotic and exponential stability of the system. In a later study, Kim *et al*. [198] revisited the system introduced in the aforementioned studies. In their paper, vibration control for an axially moving string with varying velocity and tension was investigated. Based on the Lyapunov method, a boundary control law was derived for suppressing nonlinear vibrations and ensuring the exponential stability of the closed-loop system. Furthermore, robust adaptive boundary control for moving strips in the zinc galvanizing line was also studied in [44, 199]. These studies were introduced in the previous section.

6.3. Flexible electronics

High-speed roll-to-roll systems are widely employed in applications such as rewinding processes, material transport processes, and electrical device manufacturing processes. In these processes, the productivity of the system is often enhanced via the vibration suppression of the moving substrate using the boundary, pointwise, or distributed control techniques. These techniques control the vibration of moving material via the application of external forces. This can damage the surface of the material under certain circumstances, particularly in the case of roll-to-roll systems that manufacture large-area electronics devices. To overcome this problem, Nguyen and Hong [202] proposed a new vibration control algorithm based on the regulation of axial velocity. Unlike boundary and distributed control techniques, the control technique developed in this investigation directly used the system parameter—axial velocity—to control the lateral vibration of the beam. In their paper, the authors utilized the moving velocity to control a large-area high-throughput roll-to-roll system that was described as an axially moving beam. To quickly decay the vibration energy, they regulated the axial velocity to track an appropriate profile. The technique of using the moving velocity in their proposed algorithm is considered innovative in the literature. By observing the state-space equation of the system, the authors identified that the linear operator in their system depended on the axial transport velocity. This allowed the eigenvalues of the linear operator as well as the lateral vibrations to be regulated by adjusting the axial velocity. As such, a control algorithm based on the regulation of the axial velocity was designed for quickly dissipating the vibration energy and eliminating lateral oscillation when the axial velocity is zero. The control algorithm adjusted the axial velocity to track a velocity profile consisting of several slopes instead of the conventional constant-deceleration profile. To obtain this profile, an optimal control problem, in which an energy-like function was considered as a cost function, and the axial velocity was used as a control input, was proposed and solved using the conjugate gradient method [203]. The effectiveness of this new control algorithm was also examined via numerical analysis. Later, Nguyen and Hong [204] extended the control algorithm by regulating the axial velocity to the two-dimensional model; namely, the axially moving web model with lateral vibration, depending on two different spatial variables. They successfully demonstrated that the proposed vibration control algorithm effectively addressed the control problem of the translating web.

In addition to vibration control by regulation of the axial velocity, Nguyen *et al*. [205] presented an active control strategy based on the adjustment of the axial tension to dissipate the vibration energy of a translating string driven by two rollers at the boundaries. To suppress the lateral vibration and to control the axial speed, they designed two control torques that were applied to the rollers via the Lyapunov method. Furthermore, they also obtained the exponential convergence of the lateral oscillation and the axial speed tracking error. In another study on vibration control of the roll-to-roll system, Nguyen *et al*. [206] investigated vibration control in a rewinding process. This study considered the rewinding system as an axially moving beam driven by fixed rollers at the left boundary and a rewind roller at the right boundary. A control force generated by a hydraulic actuator located near the right boundary was implemented to suppress the lateral vibrations of the beam, while a control torque at the rewind roller was used to regulate the axial speed and the radius of the rewind roller. A hybrid PDE-ODE model that involves the dynamics of the translating-beam, the rewind-roller, and the actuator was initially presented. Based on this model and the Lyapunov method, an exponentially stable controller was developed. Besides, an adaptive technique was utilized to handle the uncertain bearing friction coefficient in the shaft of the rewind roller and the disturbance with an unknown bound at the rewind roller. The feasibility and effectiveness of the control strategy were shown based on simulated results.

Surface mount technology (SMT) is a method for mounting electronic components to the surface of a printed circuit board. Belt systems concerning SMT were studied in [56–58, 207–213]. In this system, the belt not only moves at high-acceleration/deceleration but is also subjected to unknown disturbances. In the first study on the belt SMT system [57], the belt was modeled as an axially moving string under bounded disturbances—a spatiotemporally varying distributed disturbance along the length of the string and a time-varying boundary disturbance. Furthermore, the profile of axial velocity and acceleration/deceleration of the belt was designed using an S-curve acceleration/deceleration process. Based on the back-stepping technique and the Lyapunov method, a boundary controller with a disturbance observer was used to decay the oscillations of the system, in which the disturbance observer was employed to handle unknown boundary disturbances. The authors proved that the lateral vibration of the system was bounded when time goes to infinity. They also discussed the design of control gain to reduce the compact boundary size and to enhance control performance. In addition to the boundedness property of the vibration, they showed that the closed-loop system under the proposed controller was exponentially stable in the free vibration case. In another paper, Liu *et al*. [208] suggested that several belt system parameters such as belt tension, actuator mass, damping coefficient, and the mass density of the belt are uncertain. To compensate for these unknown parameters, the authors designed an adaptive boundary controller with a disturbance observer by synthesizing the adaptive technique and the Lyapunov-based back-stepping method. Subsequently, they proved the existence, uniqueness, and convergence of the solution of the closed-loop system for the designed controller via Sobolev spaces.

The input saturation that exists in the belt system of SMT in practice due to the limitation of the actuator or physical constraints of the system was considered in [56, 207]. In several cases, the input saturation can adversely affect the control performance and cause the system to become unstable. To overcome this problem, the authors used an auxiliary system to eliminate the effect of the input saturation. Furthermore, based on this auxiliary system, the Lyapunov method, and the robust adaptive control technique, a boundary controller was designed to suppress the lateral vibrations of the belt system. The well-posedness and the uniform bounded stability were also validated.

In the previous studies, the state signals such as lateral displacement and slope angles at the boundary as well as their first-order time derivatives were obtained using sensors or through algorithms. However, in practice, the accurate measurement of these terms is challenging due to the noise from the sensor, in particular, from the first-order time-derivative terms. In the case whereby these states are not accurately measurable, Zhao *et al*. [210,212,213] used output feedback boundary control in which the unmeasured terms can be estimated using high-gain observers for vibration control of the axially moving string with the restricted input. The effectiveness of the proposed controller was validated via numerical results. In [210], the authors investigated a non-uniform belt system with spatial-varying mass density and spatiotemporally varying tension for the first time.

7. CONCLUSIONS AND FUTURE PROSPECTS

In this paper, a detailed review of active vibration controls of axially moving systems was carried out. Mathematical models of axially moving systems were presented in Section 2. Active vibration control strategies based on the ODE model was introduced in Section 3, whereas control methods based on the PDE model were present in Section 4. Moreover, hybrid control methods were discussed in Section 5. In Section 6, the applications of techniques for suppressing vibration in engineering systems were discussed in detail. Several important studies on vibration control of axially moving system were summarized in Table 1. Based on this survey, the following aspects are proposed for further research on vibration control of axially moving systems.

Table 1. Important contributions in the field of vibration control of axially moving systems.

Reference	Model	Contributions
Control based on ODE model		
Ulsoy (1984)	String	Development a controller based on the observer-based state feedback control method for lateral vibration suppression
Fung and Liao (1995)	String with small periodic tension	Development of a control law based on variable structure control
Fung *et al.* (1997)	String with varying length	Development of a control law based on variable structure control
Zhu and Ni (2001)	String and beam with varying length	Development of a pointwise control law for lateral vibration suppression
Nguyen and Hong (2011)	Beam	Development of a control method based on the regulation of axial velocity for vibration reduction
Nguyen and Hong (2012b)	Plate	Usage of a regulated axial velocity profile for vibration suppression
Control based on PDE model		
Yang and Mote (1991)	String	Usage of the transfer function formulation for designing the control law
Chung and Tan (1995)	String	Usage of the wave cancellation method for reducing the lateral vibration
Lee and Mote (1996)	String	Development of a Lyapunov-based control law for suppressing lateral vibration
Queiroz *et al.* (1999)	String with a guide	Development of an adaptive boundary control law
Lee and Mote (1999)	Beam	Usage of the Lyapunov method for developing a boundary control law
Fung and Tseng (1999)	String with a mass-damper spring actuator	Development of a hybrid boundary control law based on the Lyapunov method and sliding mode control
Nagakatii *et al.* (2000)	String	Usage of the Lyapunov method for suppressing the longitudinal vibration
Li *et al.* (2002)	String with a mechanical guide	Investigation on an adaptive control based on both theory and experiment
Choi *et al.* (2004)	Beam	Development of a boundary control law for control of a translating steel strip of zinc-galvanizing line
Hong *et al.* (2004)	Coupled model	Development of a Lyapunov-based controller for suppressing the lateral vibration
Yang *et al.* (2004)	Beam with varying tension	Usage of the Lyapunov method for reducing the lateral vibration
Yang *et al.* (2005b)	String with varying tension	Development of a robust adaptive boundary controller based on Lyapunov method
Yang *et al.* (2005c)	String with varying tension	Development of a robust adaptive boundary controller using a PR transfer function
Nguyen and Hong (2010)	String with varying tension	Usage of the Lyapunov redesign method for developing a robust adaptive boundary control
Nguyen and Hong (2012a)	String with varying tension	Development of a Lyapunov-based controller for suppressing the longitudinal and lateral vibrations and tracking the desired axial velocity
He *et al.* (2015)	String with varying length and output constraint	Usage of the Lyapunov method for developing a controller for a translating string with varying length and output constraint
Zhao *et al.* (2016)	String	Development of an adaptive boundary control law for suppressing the vibration of a translating string with high acceleration/deceleration
Liu *et al.* (2017)	String with varying length and input constraint	Development of the boundary control for control of a flexible aerial refueling hose
Kelleche and Tatar (2017)	Viscoelastic string	Development of a boundary control law for suppressing the lateral vibration of a viscoelastic axially moving string
Review papers		
		A study on active control

Wang and Li (2004)		methods for suppressing the linear lateral vibration of axially moving systems
Chen (2005)		A survey on dynamics and control of axially moving strings

Timoshenko beams: The studies on vibration control of axially moving beams hitherto have focused on Euler–Bernoulli beams. Vibration suppression of moving Timoshenko beam has not been investigated to date. The Timoshenko beam theory, which considers the influences of both shear deformation and rotational inertia, is more suitable for predicting the behavior of thick and short beams as well as the frequencies of the high modes of vibration. Given these advantages, the control scheme for suppressing vibration of axially moving materials based on Timoshenko beam theory should be developed in the future.

Two-dimensional model: Another aspect of axially moving systems that should be addressed is vibration suppression of the two-dimensional model; namely, an axially moving plate. Most studies have considered the one-dimension modal (i.e., string, beam, and coupled models) and there are limited investigations on vibration control in axially moving plates due to the complexity of the associated dynamic model. Therefore, researches on this aspect must be pursued in the future.

Laminated composite material model: The laminated composite material is a material type that includes two or more layers of orthotropic materials with different properties. Recently, the use of laminated composite materials in axially moving systems has received considerable attention. The design of control algorithms for axially moving systems with laminated composite materials is one of the most anticipated fields.

Influence of harsh environments: Many works on the vibration control of flexible systems under harsh environments were performed [215,216]; however, vibration suppression of axially moving systems under harsh environments such as vortex-including vibrations in the ocean and high temperature is limited. There is a need to develop more efficient control schemes for these systems because of their broad applications.

Development of controllers for hybrid systems: The motion of practical hybrid systems is often a synthesis of different motions. For example, the dynamic model of a container crane [190,191] consists of the axial movement of the cable and the translational motion of the trolley, and the motion of roll-to-roll processes is the combination of the axial motions of the material.

Experimental studies: Lastly, apart from numerical simulation, an advanced innovative system should be developed to verify the effectiveness of the control algorithms.

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Keum-Shik Hong received his B.S. degree in Mechanical Design and Production Engineering from Seoul National University in 1979, his M.S. degree in Mechanical Engineering from Columbia University, New York, in 1987, and both an M.S. degree in Applied Mathematics and a Ph.D. in Mechanical Engineering from the University of Illinois at Urbana-Champaign (UIUC) in 1991. He joined the School of Mechanical Engineering at Pusan National University (PNU) in 1993. His Integrated Dynamics and Control Engineering Laboratory was designated a National Research Laboratory by the Ministry of Science and Technology of Korea in 2003. In 2009, under the auspices of the World Class University Program of the Ministry of Education, Science and Technology (MEST) of Korea, he established the Department of Cogno-Mechatronics Engineering, PNU. Dr. Hong served as Associate Editor of Automatica (2000-2006), as Editor-in-Chief of the Journal of Mechanical Science and Technology (2008-2011), and is serving as Editor-in-Chief of the International Journal of Control, Automation, and Systems. He was a past President of the Institute of Control, Robotics and Systems (ICROS), Korea, and is President-Elect of Asian Control Association. He was the Organizing Chair of the ICROS-SICE International Joint Conference 2009, Fukuoka, Japan. He is an IEEE Fellow, a Fellow of the Korean Academy of Science and Technology, an ICROS Fellow, a Member of the National Academy of Engineering of Korea, and many other so cieties. He has received many awards including the Best Paper Award from the KFSTS of Korea (1999), the F. Harashima Mechatronics Award (2003), the IJCAS Scientific Activity Award (2004), the Automatica Certificate of Outstanding Service (2006), the Presidential Award of Korea (2007), the ICROS Achievement Award (2009), the IJCAS Contribution Award (2010), the Premier Professor Award (2011), the JMST Contribution Award (2011), the IJCAS Contribution Award (2011), the IEEE Academic Award of ICROS (2016), etc. Dr. Hong's current research interests include brain-computer interface, nonlinear systems theory, adaptive control, distributed parameter systems, autonomous vehicles, and innovative control applications in brain engineering.

Phuong-Tung Pham received his B.S. and M.S degrees in Mechanical Engineering from Ho Chi Minh City University of Technology, in 2016 and 2018, respectively. He is currently a Ph.D. candidate in the School of Mechanical Engineering, Pusan National University, Korea. His research interests include nonlinear control, adaptive control, vibration control, and control of distributed parameter systems.