Delay-Independent Exponential Stability Criteria for Time-Varying Discrete Delay Systems

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***Abstract*—In this paper we derive delay-independent exponential stability conditions for linear/nonlinear time-varying discrete delay systems. Since these conditions are of delay-independence and easily verifiable, they may provide handy tools for the stability analysis.**

I. Introduction

In the stability analysis of time-delay systems, two different approaches have been adopted among the researchers. One approach is to contrive the stability conditions which do not depend upon the delay [3]–[5], [8], and the other is to take it into account [6]. The first direction, delay-independent stability criteria, may provide a handy way to investigate the stability of a time-delay system at the first stage.

It has been proven in [4] and [8] that in the case of continuous-time delay systems such that

$$\dot{x}_i(t) = -a_{ii}x_i(t) + \sum_{j=1, j\neq i}^{N} a_{ij}x_j(t - T_{ij}), \qquad i = 1, 2, \cdots, N \tag{1.1}$$

where $t \in [0, \infty)$, a_{ij} and $T_{ij} \geq 0$ are constants, the quasi-diagonal dominance of the interconnection matrix, i.e.,

$$d_i a_{ii} + \sum_{j=1,\ j\neq i}^{N} d_j|a_{ij}| < 0 \tag{1.2}$$

is a sufficient condition for exponential stability of (1.1). Thus, the corresponding delay free system is insensitive to delays occurring in the off-diagonal terms.

In this paper, we derive delay-independent stability conditions for linear discrete-time time-delayed (simply, discrete delay) systems (Theorems 1–4) and nonlinear discrete delay systems (Theorem 5). Consider a linear nonautonomous discrete delay system as

$$x_i(n+1) = \sum_{j=1}^{N} a_{ij}(n)x_j(n - T_{ij}), \qquad i = 1, 2, \cdots, N \tag{1.3}$$

when $n \in Z^+$ and each T_{ij} is an arbitrary nonnegative integer. Note that delays are allowed in the diagonal terms. The solution $x(n)$, $x(n) \in R^N$ of the discrete delay system (1.3) depends on the specification of initial conditions $\{x_0(t)\colon t = 0, -1, -2, \cdots, -T\}$, where $T = \max_{i,j} T_{ij}$.

Definition 1: The null solution of the discrete delay system (1.3) is said to be exponentially stable if there exist constants $C > 0$ and η, $0 \leq \eta < 1$, such that $\|x(n)\| \leq C\eta^n\|x_0\|_\infty$, where $\|x_0\|_\infty = \max_{-T\leq t\leq 0}\{\|x_0(t)\|\}$.

Obtaining exponential stability of a control system is sometimes of great importance. In adaptive control it is known that an exponentially stable adaptive system can tolerate a certain amount of disturbances and unmodeled dynamics. Recently several exponential stability results in relation to adaptive control for different types of dynamic systems have been investigated [1], [2].

II. Delay Insensitive Systems

If all the eigenvalues of the coefficient matrix A of the discrete system

$$x(n+1) = Ax(n), \qquad n \in Z^+, \qquad x \in R^N \tag{2.1}$$

are inside the unit circle, then the system is exponentially stable. This is not always so, however, for the discrete delay systems. For example, a discrete delay system

$$\begin{aligned} x_1(n+1) &= x_2(n) \\ x_2(n+1) &= -0.5x_1(n) - 0.8x_2(n-1)p \end{aligned} \tag{2.2}$$

is not stable (note that without delay it is exponentially stable). This is easily seen from the equivalent system obtained by replacing $x_2(n-1) = x_1(n)$ in the second equation of (2.2), which gives the characteristic equation $\lambda^2 + 1.3 = 0$.

The following theorem provides delay-independent exponential stability criterion for the time-varying discrete system (1.3) with arbitrary delays.

Theorem 1: Consider a discrete delay system (1.3). Suppose that there exist positive constants $d_1, d_2, \cdots, d_N$ such that

$$\sum_{j=1}^{N}\frac{d_j}{d_i}|a_{ij}(n)| \leq \delta < 1 \tag{2.3}$$

for all $n \in Z^+$ and $i = 1, 2, \cdots, N$. Then the discrete delay system (1.3) is exponentially stable.

Proof: For all $x \in R^N$ define a norm

$$\|x\| = \max_{1\leq k\leq N}\{d_k^{-1}|x_k|\}. \tag{2.4}$$

Then

$$\begin{aligned} \|x(n+1)\| &= \max_{1\leq i\leq N}\{d_i^{-1}|x_i(n+1)|\} \\ &= \max_{1\leq i\leq N}\left\{d_i^{-1}\left|\sum_{j=1}^{N}a_{ij}(n)x_j(n-T_{ij})\right|\right\} \\ &\leq \max_{1\leq i\leq N}\left\{d_i^{-1}\sum_{j=1}^{N}|a_{ij}(n)||x_j(n-T_{ij})|\right\} \\ &= \max_{1\leq i\leq N}\left\{\sum_{j=1}^{N}\frac{d_j}{d_i}|a_{ij}(n)|d_j^{-1}|x_j(n-T_{ij})|\right\} \\ &\leq \left\{\max_{1\leq i\leq N}\sum_{j=1}^{N}\frac{d_j}{d_i}|a_{ij}(n)|\right\} \\ &\quad \cdot\left\{\max_{1\leq i,\, j\leq N}(d_j^{-1}|x_j(n-T_{ij})|)\right\} \\ &\leq \delta d_{j_1}^{-1}|x_{j_1}(n-T_{i_1j_1})| \end{aligned}$$

where $d_{j_1}^{-1}|x_{j_1}(n-T_{i_1j_1})| = \max_{1\leq i,\,j\leq N}(d_j^{-1}|x_j(n-T_{ij})|)$. Therefore

$$\|x(n+1)\| \leq \delta\|x(n-T_{i_1j_1})\| \tag{2.5}$$

and

$$\|x(n)\| \leq \delta\|x(n-(T_{i_1j_1}+1))\|. \tag{2.6}$$

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Repeating this procedure r times obtains

$$\|x(n)\| \leq \delta^r \left\| x\left(n - \sum_{t=1}^{r}(T_{i_t j_t} + 1)\right)\right\|. \tag{2.7}$$

Let

$$0 \geq n - \sum_{t=1}^{r}(T_{i_t j_t} + 1) \geq n - r(T+1) \tag{2.8}$$

where $T = \max_{1\leq i,\, j \leq N}\{T_{ij}\}$. Then for $r \geq [n/(T+1)]$, where $[p]$ is the smallest integer $\geq p$, (2.7) becomes

$$\begin{aligned}\|x(n)\| &\leq \delta^{\left[\frac{n}{T+1}\right]}\|x_0\|_\infty = \left(\delta^{\left[\frac{n}{T+1}\right]\frac{1}{n}}\right)^n \|x_0\|_\infty \\ &\leq \left(\delta^{\frac{1}{T+1}}\right)^n \|x_0\|_\infty = \eta^n \|x_0\|_\infty \end{aligned} \tag{2.9}$$

where $\|x_0\|_\infty = \max_{-T\leq t \leq 0}\|x_0(t)\|$, $\{x_0(t)\colon\ -T \leq t \leq 0\}$ is the initial data, and $\eta = \delta^{\frac{1}{T+1}}$, $0 \leq \eta < 1$. Also the following inequality for $n = a(T+1) + b$, $0 \leq b \leq T$

$$\left[\frac{n}{T+1}\right]\frac{1}{n} = \frac{a+1}{a(T+1)+b} \geq \frac{a+1}{a(T+1)+T+1} = \frac{1}{T+1}$$

has been used in (2.9). Therefore, it follows that the delay system is exponentially stable. Q.E.D.

Remark 1: Theorem 1 remains valid for the following system

$$x_i(n + s_i) = \sum_{j=1}^{N} a_{ij} x_j(n - T_{ij}), \qquad i = 1, 2, \cdots, N \tag{2.10}$$

where s_i is a positive integer, $n \in Z^+$, and T_{ij} is an arbitrary nonnegative integer. It follows then that (2.10) can be converted to the form (1.3) by defining $n + s_i = k + 1$.

Example 1: Consider a discrete delay system

$$\begin{aligned} x_1(n+1) &= -0.4x_1(n-1) + 0.5x_2(n) \\ x_2(n+1) &= 0.5x_1(n) + 0.2x_2(n). \end{aligned} \tag{2.11}$$

If choosing $d_1 = d_2 = 1$, condition (2.3) of Theorem 1 is satisfied. So (2.11) is exponentially stable. This can be also seen from the equivalent system obtained by introducing $x_3(n) = x_1(n-1)$ such that

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \\ x_3(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -0.4 \\ 0.5 & 0.2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix}. \tag{2.12}$$

The characteristic polynomial of (2.12) is $-\lambda^3 + 0.2\lambda^2 - 0.15\lambda + 0.08$. Setting $\lambda = -p$, it is noted that the polynomial $p^3 + 0.2p^2 + 0.15\lambda + 0.08$ has decreasing coefficients. Therefore, by Theorem 5 of [10], zeroes are inside the unit circle and the system is exponentially stable.

Using the argument similar to that given in the proof of Theorem 1, the following is established.

Theorem 2: Consider the discrete system (1.3). Suppose $A(n) = [a_{ij}(n)]$, $A(n) \in R^{N\times N}$ and $|a_{ij}(n)| \leq m_{ij}$ for all $n \in Z^+$. If $M = [m_{ij}]$ is stable, then the discrete system (1.3) is exponentially stable.

III. Partitioned Systems

The exponential stability of partitioned discrete delay systems is considered in this section.

Theorem 3: Consider a partitioned system of (1.3). Let $N = N_1 + N_2$, $x_{(i)} \in R^{N_i}$, $i = 1, 2$, $A_{ij} \in R^{N_i\times N_j}$, and $\|A_{ij}\| = \sup_{\|x_{(j)}\|_j=1}\|A_{ij}x_{(j)}\|_i$, where the subscript i in the notation $\|\cdot\|_i$ represents a (arbitrary) norm for the subsystem $x_{(i)} \in R^{N_i}$. Then the discrete delay system

$$\begin{aligned} x_{(1)}(n+1) &= A_{11}x_{(1)}(n - T_{11}) + A_{12}x_{(2)}(n - T_{12}) \\ x_{(2)}(n+1) &= A_{21}x_{(1)}(n - T_{21}) + A_{22}x_{(2)}(n - T_{22}) \end{aligned} \tag{3.1}$$

where

$$x_{(1)}(n - T_{k1}) = \begin{bmatrix} x_1(n - T_{k1}) \\ x_2(n - T_{k2}) \\ \cdots \\ x_{N_1}(n - T_{kN_1}) \end{bmatrix},$$

and

$$x_{(2)}(n - T_{k2}) = \begin{bmatrix} x_{N_1+1}(n - T_{k,\,N_1+1}) \\ x_{N_1+2}(n - T_{k,\,N_1+2}) \\ \cdots \\ x_{N_1+N_2}(n - T_{k,\,N_1+N_2}) \end{bmatrix}, \qquad k = 1, 2,$$

is exponentially stable if $\delta = \max\{\|A_{11}\| + \|A_{12}\|,\ \|A_{21}\| + \|A_{22}\|\} < 1$.

Proof: For all $x_{(i)} \in R^{N_i}$, $i = 1, 2$, where $N = N_1 + N_2$, let $x(n) = \begin{bmatrix} x_{(1)}(n) \\ x_{(2)}(n)\end{bmatrix}$, and define $\|x(n)\| \triangleq \max\{\|x_{(1)}(n)\|_1,\ \|x_{(2)}(n)\|_2\}$. Then

$$\begin{aligned} &\|x(n+1)\| \\ &\leq \max\{\|A_{11}\|\|x_{(1)}(n - T_{11})\|_1 + \|A_{12}\|\|x_{(2)}(n - T_{12})\|_2, \\ &\qquad \|A_{21}\|\|x_{(1)}(n - T_{21})\|_1 + \|A_{22}\|\|x_{(2)}(n - T_{22})\|_2\} \\ &\leq \max\{\|A_{11}\| + \|A_{12}\|,\ \|A_{21}\| + \|A_{22}\|\} \\ &\quad \cdot \max\{\|x_{(1)}(n - T_{11})\|_1,\ \|x_{(1)}(n - T_{21})\|_1, \\ &\qquad \|x_{(2)}(n - T_{12})\|_2,\ \|x_{(2)}(n - T_{22})\|_2\} \\ &\leq \delta\|x_{(i_1)}(n - T_{p_1 q_1})\|_{i_1} \end{aligned}$$

where $\|x_{(i_1)}(n - T_{p_1q_1})\|_{i_1} \triangleq \max\{\|x_{(1)}(n - T_{11})\|_1,\ \|x_{(1)}(n - T_{21})\|_1,\ \|x_{(2)}(n - T_{12})\|_2,\ \|x_{(2)}(n - T_{22})\|_2\}$. Therefore

$$\|x(n+1)\| \leq \delta\|x(n - T_{p_1q_1})\|$$

and

$$\|x(n)\| \leq \delta\|x(n - (T_{p_1q_1} + 1))\|.$$

Repeating this procedure r times (see the proof of Theorem 1) gives

$$\|x(n)\| \leq \delta^{\left[\frac{n}{T+1}\right]}\|x_0\|_\infty$$

where $\|x_0\|_\infty = \max_{-T\leq t\leq 0}\|x_0(t)\|$, $\{x_0(t)\colon\ -T \leq t \leq 0\}$ is the initial data. The rest of the proof follows the proof of Theorem 1. Therefore, it follows that the delay system is exponentially stable. Q.E.D.

Remark 2: The method of proof used in Theorem 3 is also applicable when the system is partitioned into p parts, $p \geq 3$, and when each A_{ij} is a function of n instead of constant.

IV. State Feedback with Delay

The following theorem provides conditions for the exponential stability of a discrete feedback control system

$$x(n+1) = A(n)x(n) + B(n)u(n) \tag{4.1}$$

where $x \in R^N$, $A \in R^{N\times N}$, in which the state feedback involves an arbitrary delay such that

$$u(n) = Kx(n - t). \tag{4.2}$$

Theorem 4: Suppose $x \in R^N$ and $A(n)$ and $B(n)K \in R^{N\times N}$. If there exists a positive integer M, a number δ, $0 \le \delta < 1$, and a norm $\|\cdot\|$ in R^N such that

$$\|A(n)\| + \|B(n)K\| \le \delta \tag{4.3}$$

for all $n \ge M$, then the discrete delay system

$$x(n+1) = A(n)x(n) + B(n)Kx(n-t) \tag{4.4}$$

where t is an integer, $t \ge 1$, is exponentially stable.

Proof: It follows that

$$\|x(n+1)\| \le \|A(n)\|\|x(n)\| + \|B(n)K\|\|x(n-t)\|. \tag{4.5}$$

Setting $\|x(n)\| = x_n$ and rewriting the equation obtains

$$x_{n+1} \le \|A(n)\|x_n + \|B(n)K\|x_{n-t}.$$

Define $y(a, b) = \max\{x_a, x_{a+1}, \cdots, x_b\}$. Then

$$x_{n+1} \le (\|A(n)\| + \|B(n)K\|)y(n-t, n) \tag{4.6}$$

and for all $n \ge \max(t, M)$,

$$x_{n+1} \le \delta y(n-t, n)$$

or

$$x_n \le \delta y(n-(t+1), n-1). \tag{4.7}$$

Repeating this procedure r times obtains

$$x_n \le \delta^r y(n - r(t+1), n-r). \tag{4.8}$$

For r a positive integer such that

$$n - r(t+1) \ge M$$

we set

$$r = \left\lceil \frac{n-M}{t+1} \right\rceil - 1 \tag{4.9}$$

where $\lceil\rho\rceil$ is the smallest integer $\ge \rho$. On the other hand, for the (bounded) initial data $x_0 = \|x(0)\|$, $x_1 = \|x(1)\|, \cdots, x_t = \|x(t)\|$

$$y(n-r(t+1), n-r) \le \mu \max\{x_0, x_1, \cdots, x_t\} \le \alpha$$

where μ is some positive constant depending on the starting part of the evolution system (4.4), and α is some positive number. Therefore for all k, $k = 0, 1, \cdots$

$$x_k \le \alpha$$

and furthermore from (4.8)

$$x_n \le \alpha\delta^r. \tag{4.10}$$

Since $n \to \infty$ implies $r \to \infty$, $x_n \to 0$. Thus the system is exponentially stable. Q.E.D.

Example 2: Consider a discrete system

$$x(n+1) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix} x(n) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(n)$$

with a particular feedback control $u(n) = [-0.18 \;\; 0]x(n-t)$, where t is an arbitrary delay. Since

$$\|A\|_\infty + \|BK\|_\infty = \left\| \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix} \right\|_\infty + \left\| \begin{bmatrix} -0.18 & 0 \\ 0 & 0 \end{bmatrix} \right\|_\infty = 0.8 + 0.18 = 0.98$$

$x(n) \to 0$ as $n \to \infty$ exponentially by Theorem 4.

V. Nonlinear Systems

Theorem 5: Consider the following nonlinear delay system

$$x_1(n+1) = f_1(x_1(n-T_{11}), x_2(n-T_{12}), \cdots, x_N(n-T_{1N}))$$
$$x_2(n+1) = f_2(x_1(n-T_{21}), x_2(n-T_{22}), \cdots, x_N(n-T_{2N}))$$
$$\cdots$$
$$x_N(n+1)P = f_N(x_1(n-T_{N1}), x_2(n-T_{N2}), \cdots, x_N(n-T_{NN})). \tag{5.1}$$

where f: $R^N \to R^N$, C^1-function, and $f(0) = 0$. Then (5.1) is exponentially stable if there exists $\delta > 0$ such that

$$\|\nabla f_i(z)\|_1 \le \delta < 1 \tag{5.2}$$

for every $z \in R^N$ and every $i = 1, 2, \cdots, N$, where the norm $\|\cdot\|_1$ is defined as $\|x\|_1 = \sum_{i=1}^N |x_i|$.

We state the following lemma to be used in the proof of Theorem 5.

Lemma: Let f: $R^N \to R^N$ be continuously differentiable. Then for every x, $y \in R^N$, there exist $z \in \overline{xy}$ such that

$$f(y) - f(x) = \langle \nabla f(z), y - x \rangle \tag{5.3}$$

where $\overline{xy} = \{z \in R^N\colon z = \alpha x + (1-\alpha)y,\ 0 \le \alpha \le 1\}$.

Proof of Theorem 5: For every $x \in R^N$, define

$$V(x) = \|x\| \triangleq \max_{1\le i\le N} |x_i|. \tag{5.4}$$

Let the maximum in (5.4) be achieved at the i_1th component, then

$$\|x(n+1)\| = |x_{i_1}(n+1)| = |f_{i_1}(x_1(n-T_{i_1 1}), x_2(n-T_{i_1 2}), \cdots, x_N(n-T_{i_1 N})|. \tag{5.5}$$

Now let $x = 0$, and $y = [x_1(n-T_{i_1 1}) \;\; x_2(n-T_{i_1 2}) \;\; \cdots \;\; x_N(n-T_{i_1 N})]^T$, then by the condition of Theorem 5 and the Lemma, there exists $z \in \overline{0y}$ such that

$$\begin{aligned}
\|x(n+1)\| &= |\langle \nabla f_{i_1}(z), y\rangle| \\
&= \left| \sum_{j=1}^N \frac{\partial f_{i_1}}{\partial z_j}(z) y_j \right| \\
&\le \sum_{j=1}^N \left| \frac{\partial f_{i_1}}{\partial z_j}(z) \right| |y_j| \\
&\le \|y\|_\infty \sum_{j=1}^N \left| \frac{\partial f_{i_1}}{\partial z_j}(z) \right| \\
&= \|y\|_\infty \|\nabla f_{i_1}(z)\|_1 \\
&\le \delta \|y\|_\infty \\
&= \delta \max_{1\le j\le N} \{|x_j(n - T_{i_1 j})|\} \\
&\le \delta \|x(n - T_{i_1 j_1})\|
\end{aligned} \tag{5.6}$$

where $\|x(n-T_{i_1 j_1})\| = \max_{1\le j\le N}\{|x_j(n-T_{i_1 j})|\}$. Therefore

$$\begin{aligned}
\|x(n)\| &\le \delta \|x(n - T_{i_1 j_1} - 1)\| \\
&\le \delta^2 \|x(n - T_{i_1 j_1} - T_{i_2 j_2} - 2)\|.
\end{aligned}$$

Repeating this r times obtains

$$\|x(n)\| \le \delta^r \left\| x\left(n - \sum_{k=1}^r (T_{i_k j_k} + 1)\right) \right\|. \tag{5.7}$$

Noting that (5.7) is of the form (2.7), the rest of the proof follows the proof of Theorem 1. Q.E.D.

Corollary 1: Consider the system (5.1). Suppose that there exist $d_1, d_2, \cdots, d_N$ such that for every i, $1 \le i \le N$, and every $x \in R^N$

$$\sum_{j=1}^{N} d_j \left| \frac{\partial f_i}{\partial x_j}(X) \right| \le \delta < 1 \tag{5.8}$$

then the system (5.1) is exponentially stable.

Proof: The proof can be easily modified from the proof of Theorem 5.

Example 3: Consider

$$\begin{aligned} x_1(n+1) &= \frac{\sin x_2(n-T_{12})}{3(1+x_1^2(n-T_{11}))} = f_1(x_1 - T_{11},\, x_2 - T_{12}) \\ x_2(n+1) &= \frac{1}{2}\log\left(1 + x_1^2(n-T_{21}) + x_2^2(n-T_{22})\right) \\ &= f_2(x_1 - T_{21},\, x_2 - T_{22}). \end{aligned}$$

Then

$$\begin{aligned} \|\nabla f_1\|_1 &= \frac{2|\sin x_2|}{3(1+x_1^2)^2}|x_1| + \frac{|\cos x_2|}{3(1+x_1^2)} \\ &\le \frac{2|x_1|}{3(1+x_1^2)^2} + \frac{1}{3(1+x_1^2)} \\ &= \frac{1}{3}\left(\frac{1+|x_1|}{1+x_1^2}\right)^2 \\ &\le \frac{1}{3}\left(\frac{1+\sqrt{2}-1}{1+(\sqrt{2}-1)^2}\right)^2 \\ &< 1 \end{aligned}$$

where the function $f(x) = (1+x)/(1+x^2)$, $x \in [0, \infty)$, achieves the maximum at $\sqrt{2}-1$. Similarly

$$\|\nabla f_2\|_1 = \frac{|x_1| + |x_2|}{1 + x_1^2 + x_1^2} \le \frac{1}{\sqrt{2}} < 1.$$

By the theorem, the system is globally exponentially stable and independent of the delay size.

VI. Conclusion

In this paper several stability criteria, which are independent of the delay, for discrete delay systems are derived. The obtained results are simple and easy to apply.

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