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Observer-based leader-following consensus of one-sided Lipschitz multi-agent systems over input saturation and directed graphs

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Abstract

This paper investigates a local observer-based leader-following consensus control of one-sided Lipschitz (OSL) multi-agent systems (MASs) under input saturation. The proposed consensus control scheme has been formulated by using the OSL property, input saturation, directed graphs, estimated states, and quadratic inner-boundedness condition by attaining the regional stability. It is assumed that the graph always includes a (directed) spanning tree with respect to the leader root to develop matrix inequalities for investigating parameters of the proposed observer and consensus protocols. Further, a new observer-based consensus tracking method for MASs with saturation, concerning independent topologies for communicating outputs and estimates over the network, is explored to deal with a more perplexing and realistic situation. In contrast to the traditional methods, the proposed consensus approach considers output feedback and deals with the input saturation for a generalized class of nonlinear systems. The efficiency of the obtained results is illustrated via application to a group of five moving agents in the Cartesian coordinates.

KEYWORDS

input saturation, leader-based consensus, multi-agent systems, observer-based consensus, one-sided Lipschitz function

1 | INTRODUCTION

Multi-agent systems (MASs) consist of several subsystems that perform together to accomplish particular tasks, and these subsystems communicate with each other, based on certain communication links. Consensus control is needed for MASs to achieve the same behavior in the outputs of subsystems [1], resulting in an agreement between the agents. Owing to the abundant applications of consensus control in robotics, unmanned aerial and ground vehicles, flocking, formation, surveillance, military missions, sensor networks, rendezvous, and microgrids [2–4], researchers in various control communities have acknowledged compelling attention in the recent years. In a leader-based consensus, the followers track the activities of the leader by employing and manipulating the control signals. Consensus control has been explored from different perspectives of control strategies, the agents' dynamics, constraints on topologies, design techniques, robustness to uncertainties, adaptation of parameters, and communication delays. Many strategies have been adapted for the consensus of linear multi-agents [4–6]. Control of MASs with linear dynamics has been discussed in [7] for handling external disturbances and input saturation. An application of consensus for resource allocation problems has been addressed in [8]. ABoth leaderless and leader-following finite-time consensus for the second-order linear MASs with input saturation were suggested in [9]. A finite-time consensus protocol has been designed for linear time-invariant MASs with mismatched disturbance constraint in [10]. The semi-global consensus for discrete-time MASs subject to external disturbances and input saturation over switching networks was explored in [11]. A survey of consensus of MASs with time-invariant and dynamically varying information exchange in topologies has been provided in [12]. A theoretical framework for the consensus of multi-agent networked systems was studied in the work of [13].

Owing to the applicability limitation of linear models, researchers have revealed vigorous interests in the development of consensus methods for the nonlinear MASs. The consensus problem for Lipschitz nonlinear MASs was addressed in [4, 14–18]. Recently, the consensus of systems having a more generalized form than the Lipschitz one, called the one-sided Lipschitz (OSL) dynamics, has been studied in [19–21]. OSL systems are less conservative than the conventional Lipschitz systems because the former form has a smaller magnitude of the nonlinearity parameter, and the relevant nonlinearity constant can have either a positive or a negative sign [22]. Using the OSL constraint and quadratic inner-boundedness (QIB) constraint, the observer-based consensus of MASs was studied in [23], nonlinear observer design was formulated in [24], and observer-based control was investigated in [25]. A distributed consensus of the nonlinear systems was addressed in [26] for incorporating the switching of communication links, which was further extended to an adaptive and fully distributed consensus of OSL systems [27]. Consensus control of the OSL switching MASs was attained in the studies [28, 29] by extending the directions of [26]. The works in [26–29] have limited applicability because of the requirement of states for feedback, which is not preferred in practical applications. An important scenario of consensus of a generalized form of nonlinear agents was derived in [30] for the time-varying linkages by applying a general linear incremental nonlinear function concept. However, this approach has a major apprehension of the obtainment of control parameters using convex formulations.

Input saturation complications result in performance degradation, lag, uncertainty, unpredictability, instability, overshoot, and undershoot of the desired output. Consensus of nonlinear agents with OSL dynamics under input saturation was studied in [31], where a new approach for dealing with saturation effects was studied to obtain a bounded region-based error. It is significant to note that consensus control schemes, for example, [22, 26, 27], assume that the states of each agent are accessible to other agents. However, the accessibility of states of all agents by employing additional sensors is expensive when all states of MASs are not accessible for the implemented consensus mechanism. Under such conditions, the static or dynamic output feedback and observer-based consensus schemes have been investigated in the works [30, 32, 33], respectively. Nonetheless, these strategies do not accommodate the input saturation; while in almost all of the applications, control signals must satisfy lower and upper bounds critically, giving rise to the actuator saturation. Once an actuator saturates, these conventional methodologies may fail to deliver the desired performance and stability attributes in the closed-loop responses. Consensus control subject for the nonlinear MASs with OSL dynamics and input saturation by considering the unavailability of the states of the agents is an essential problem, demanding serious commitments. As per our observation, this paper investigates the local design approach for the observer-based consensus of the leader-following MASs retaining OSL nonlinearities under input saturation for the first time.

In this paper, the design of an observer-based consensus protocol for the OSL MASs with input saturation, integrated with a directed communication topology, has been investigated. The observers are used to estimate the states of the agents, and the estimated states along with the differences between the measured and observed outputs are shared between the agents via a communication topology. A design condition for evaluating the gains of the observers and consensus protocols has been formulated for the leader-following output-based consensus. This design condition ensures the consensus of MASs for a bounded region of initial conditions on the consensus error and state estimation error. It is worth noting that very limited work is available on the output feedback-based consensus under input saturation of the agents via a guaranteed regional stability.

The proposed approach has been revisited for the consensus control through independent topologies for state observation and consensus control. This complicated scenario for the consensus under input saturations of MASs has been revealed owing to the reason that it is not necessary for two topologies, observation and control, to be the same for all time. It also allows us to use different communication channels for sending and receiving both control and observation information. Further, a particular agent with limited communication resources (like bandwidth and power) can also take part in the consensus control by sending and receiving information of either estimated states or observation errors or both. In contrast to the existing works on the consensus of OSL systems in [19, 23–25, 27–31], the proposed approach deals with the consensus of OSL agents using the output feedback method by considering the input saturation. In addition, the proposed approach considers a directed graph for addressing the leader-following consensus compared with the existing work [31]. The cone complementary linearization (CCL) arrangement is applied for dealing with the nonlinear constraints to evaluate a feasible solution of the matrix inequalities through convex optimization. The main contributions of this paper are summarized as follows:

1. This work provides an observer-based consensus control for the leader-based consensus of OSL MASs under input saturation. To the best of our knowledge, the observer-based consensus for the OSL nonlinear MASs with saturating inputs and unmeasured states of the followers is addressed for the first time.
2. A new observer-based consensus control of MASs with input saturation having independent graph topologies for consensus controller and observer is further explored to deal with limited communication resources for an agent.
3. In contrast to [31, 34], a more complicated scenario for the region of stability by addressing observation errors and observer agents has been revealed in the present work.

The proposed consensus methodology has been employed to five agents under motion, and simulation results are provided in the end.

This paper has been organized into five sections. Section 2 provides preliminaries and describes the dynamics of systems under input saturation and observers under a directed topology. Two types of observer-based control synthesis schemes for the same and independent graph topologies under saturating actuators are formulated in Section 3. Simulations of five OSL agents via nonlinear observer-based protocol are addressed in Section 4. Section 5 ends the paper with conclusions.

Notations. Constants, n column vectors and $m\times n$ matrices are represented via $z\in\mathbb{R}$, $z\in\mathbb{R}^n$, and $Z\in\mathbb{R}^{m\times n}$, respectively, where $\mathbb{R}$ defines real numbers. The minimum eigenvalue of a symmetric matrix is denoted via $\lambda_{\min}(Z)$. $\langle z,x\rangle$ defines a scalar product between two vectors, and $\otimes$ symbolizes the Kronecker product. 1_N defines an N entries vector of ones. Various definiteness properties of a square matrix Z are mentioned as $Z>0, Z\geq 0, Z<0$, and $Z\leq 0$ for different scenarios of positive definiteness, semi-positive definiteness, negative definiteness, and semi-negative definiteness, respectively. $A_{(l)}$ and $A_{\{m\}}$ defines l^{th} row and m^{th} column of a matrix A, respectively.

2 | SYSTEM DESCRIPTION

Consider a class of $N+1$ nonlinear MASs consisting of a leader agent (indexed as 0) and N follower agents, given by

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + B\varphi(u_i(t)) + H\Phi(x_i(t),t),\\ y_i(t) &= Cx_i(t),\ \ \forall i=0,1,\dots,N,\end{aligned}\tag{1}$$

where $(A,H)\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times p}$, $C\in\mathbb{R}^{q\times n}$ are constant matrices. $x_i(t)$, $y_i(t)$, $u_i(t)$, and $\Phi(x_i(t),t)$ are the state, output, control input, and nonlinear function of the i^{th} agent, respectively. The control input saturation is defined by the vector $\varphi(u_i(t)) = \left[\mathrm{sat}(u_{i1}) \cdots \mathrm{sat}(u_{ip})\right]^T$, where

$$\mathrm{sat}(u_{ij}) = \begin{cases}\overline{u}_j, & \text{if } u_{ij}\geq \overline{u}_j,\\ u_{ij}, & \text{if } -\overline{u}_j < u_{ij} < \overline{u}_j,\\ -\overline{u}_j, & \text{if } u_{ij}\leq -\overline{u}_j,\end{cases}\quad \begin{aligned}&\forall i=1,2,\dots,N,\\ &\forall j=1,2,\dots,p,\end{aligned}$$

And the bound on a saturation function is denoted by $\overline{u}_j$.

The agents V of MASs communicate with each other through links E, and the communication system can be denoted via a directed graph $\mathrm{G}=(\mathrm{V},\mathrm{E})$, with adjacency matrix $\mathcal{A}=\left[a_{ij}\right]_{(N+1)\times(N+1)}$, where a_{ij} describes the flow of signals from an agent j to another agent i. If an agent i is connected to all other agents in the graph, the corresponding graph is said to have a directed spanning tree (DST) with respect to the agent i. Self-loops are ignored in the graph (i.e., $a_{ii}=0$), and the leader node is assumed to send information only (leading to $a_{0j}=0$ and $u_0=0$). The Laplacian of the graph G is defined as $L=\left[L_{ij}\right]_{(N+1)\times(N+1)}$, where $L_{ij}=-a_{ij}$, when $i\neq j$ and $L_{ii}=\sum_{j=0, j\neq i}^{N} a_{ij}$.

The following nonlinear observer is incorporated for estimating the agents' states:

$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + B\varphi(u_i(t)) + H\Phi(\tilde{x}_i(t),t) + \alpha\sum_{j=0}^{N}\left(a_{ij}F\left(\sigma_j(t)-\sigma_i(t)\right)\right), \quad (2)$$
$$\tilde{y}_i(t) = C\tilde{x}_i(t), \quad \forall i=1,\ldots,N,$$

where $\tilde{x}_i(t)$, $\tilde{y}_i(t)$, and $\Phi(\tilde{x}_i(t),t)$ are the observer state, observer output, and nonlinear function for the i^{th} observer agent. For observer dynamics $\alpha \in \mathbb{R}$ and $F \in \mathbb{R}^{n\times q}$ are unknowns, which are required to be investigated. We define the error $\sigma_i(t)$ between the outputs of observer and agent, given as

$$\sigma_i(t) = \tilde{y}_i(t) - y_i(t) = C(\tilde{x}_i(t) - x_i(t)). \quad (3)$$

We know that the leader agent acts as a reference point for the follower agents; therefore, we assume that the states of the leader are known, that is, $\sigma_0(t)=0$. If the states are unknown, these states can be estimated by using a Luenberger-type observer as follows:

$$\dot{\tilde{x}}_0(t) = A\tilde{x}_0(t) + H\Phi(x_0(t),t) - M(\tilde{y}_0(t) - y_0(t)), \quad (4)$$
$$\tilde{y}_0(t) = C\tilde{x}_0(t),$$

where $\tilde{x}_0$ and $\tilde{y}_0$ are the state and output of the observer for the leader, and the gain M can be selected using [24].

To design an observer-based consensus control for the leader-based consensus of OSL MASs under input saturation, we consider the subsequent assumptions.

Assumption 1. [19, 24, 28]: Suppose $\Phi(x_i(t),t)$ validates the condition, given as

$$(\Phi(x_a,t)-\Phi(x_b,t))^T(x_a-x_b) \leq \rho(x_a-x_b)^T(x_a-x_b), \quad (5)$$

where $(x_a,x_b) \in \mathbb{R}^n$ and $\rho \in \mathbb{R}$ is the OSL constant.

Assumption 2. [19, 24, 28]: Let the function $\Phi(x_i(t),t)$ satisfy the relation, given as

$$(\Phi(x_a,t)-\Phi(x_b,t))^T(\Phi(x_a,t)-\Phi(x_b,t)) \leq \varepsilon(x_a-x_b)^T(x_a-x_b) + \eta(x_a-x_b)^T(\Phi(x_a,t)-\Phi(x_b,t)), \quad (6)$$

where $(x_a,x_b) \in \mathbb{R}^n$ and $(\varepsilon,\eta) \in \mathbb{R}$ are the QIB constants.

Compared with a simple Lipschitz condition that has a positive Lipschitz constant, conditions (5) and (6) cover a broader class of nonlinearity. It should be noted that the OSL and QIB constants can be positive, negative, or zero [28]. The OSL nonlinearities are used to develop less conservative controllers and observers, while the QIB condition is employed because of its attributes related to controller synthesis. The OSL nonlinearity covers a broader range of nonlinearity than the Lipschitz one, which can be shown by using the Cauchy–Euler property. The QIB condition for $\eta=0$ results in a simple Lipschitz condition. This type of condition is usually attained for achieving feasible and computable results while dealing with the OSL condition, that is, QIB is used as a supportive constraint. The main difference in QIB is the consideration of the nonlinear quadratic term $\|\Phi(x_a,t)-\Phi(x_b,t)\|^2$ for attaining the feasible results. The constants ε and η are employed to express the upper bound on the nonlinear quadratic term in terms of linear norm $\|x_a-x_b\|^2$ and inner product $\langle(x_a-x_b),(\Phi(x_a,t)-\Phi(x_b,t))\rangle$ between linear and nonlinear components.

Assumption 3. [10, 19, 28]: The topology G has a DST with respect to the leader agent 0.

The consensus of MASs (1) with OSL dynamics and under saturating inputs can be achieved by using a control protocol as

$$u_i(t) = sK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{x}_j(t)-\tilde{x}_i(t)\right)\right), \quad i=1,\ldots,N, \quad (7)$$

where $s>0$ is the weight and $K \in \mathbb{R}^{p\times n}$ represents the gain matrix. Under Assumption 3 and the provided description of graph topology G, the Laplacian L has a specific structure, given as

$$L = \begin{bmatrix} 0 & O^T \\ w & \tilde{L} \end{bmatrix}, \quad (8)$$

where $w \in \mathbb{R}^N$ is a part of Laplacian matrix L for representing the communication from the leader to the followers, $O \in \mathbb{R}^N$ has all elements as zero because the leader does not receive information from the followers, and $\tilde{L} \in \mathbb{R}^{N\times N}$ is a component of L for considering interactions between the follower agents.

Remark 1. Note that protocol (7) does not employ the exact state information and is based on the estimated states $\tilde{x}_i(t)$. In addition, the control signals to the agents are bounded because of saturation nonlinearities.

In general, the consensus control approaches via output feedback under input saturation for guaranteeing a region of stability are limited in the existing works. And, specifically, consensus of OSL nonlinear systems under output feedback subjected to control signal saturation is addressed.

We need to formulate a strategy for reaching an agreement between OSL MASs (1), exposed to the input saturation, by accounting protocol (7) and observer (2) with appropriate values of scalars $s>0$ and $\alpha>0$ along with matrices $F\in\mathbb{R}^{n\times q}$ and $K\in\mathbb{R}^{p\times n}$.

3 | MAIN RESULTS

In this section, we formulate the conditions for investigating the unknown parameters of observer and control protocols to accomplish a leader-based consensus for the OSL MASs (1) subjected to the input saturation. For leader-based consensus, we know that the leader agent is not affected by the follower agents, we have $u_0(t)=0, \forall t\geq 0$, and from (1), we obtain

$$\dot{x}_0(t)=Ax_0(t)+H\Phi(x_0(t),t). \tag{9}$$

Adding and subtracting $B\varphi\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)\right)$ and substituting (7) into (1), it leads to

$$\begin{aligned}\dot{x}_i(t)=Ax_i(t)+B\varphi\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{x}_j(t)-\tilde{x}_i(t)\right)\right)\right)\\+B\varphi\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)\right)+H\Phi(x_i(t),t)\\-B\varphi\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)\right),\end{aligned} \tag{10}$$

$\forall i=1,\ldots,N$. To incorporate the input saturation, we take

$$\left|\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)\right)_{(j)}\right|\leq\bar{u}_j, \tag{11}$$

$$\left|\left(sK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{x}_j(t)-\tilde{x}_i(t)\right)\right)\right)_{(j)}\right|\leq\bar{u}_j. \tag{12}$$

By considering (10), (11), and (12), we attain

$$\begin{aligned}\dot{x}_i(t)=Ax_i(t)+sBK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{x}_j(t)-\tilde{x}_i(t)\right)\right)\\+sBK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)+H\Phi(x_i(t),t)\\-sBK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right).\end{aligned} \tag{13}$$

Rearranging the terms in (13) leads to

$$\begin{aligned}\dot{x}_i(t)=Ax_i(t)+sBK\sum_{j=0}^{N}\left(a_{ij}\left(x_j(t)-x_i(t)\right)\right)\\+sBK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{x}_j(t)-x_j(t)-\tilde{x}_i(t)+x_i(t)\right)\right)\\+H\Phi(x_i(t),t).\end{aligned} \tag{14}$$

Let us define the error equations as $\tilde{e}_i(t)=x_i(t)-\tilde{x}_i(t)$ and $e_i(t)=x_i(t)-x_0(t), \forall i=1,\ldots,N$, the above dynamics gives

$$\begin{aligned}\dot{x}_i(t)=Ax_i(t)+sBK\sum_{j=0}^{N}\left(a_{ij}\left(e_j(t)-e_i(t)\right)\right)\\+H\Phi(x_i(t),t)-sBK\sum_{j=0}^{N}\left(a_{ij}\left(\tilde{e}_j(t)-\tilde{e}_i(t)\right)\right).\end{aligned} \tag{15}$$

By selecting the vectors $e=\left[e_1{}^T\ e_2{}^T\ \cdots\ e_N{}^T\right]^T$ and $\Phi(x(t);t)=\left[\Phi\left(x_1^T(t),t\right)\ \ldots\ \Phi\left(x_N^T(t),t\right)\right]^T$, applying (9) and (15), and using attributes of the graph theory and Kronecker product, one finds

$$\begin{aligned}\dot{e}(t)=\left(I_N\otimes A-s\left(\tilde{L}\otimes BK\right)\right)e(t)+s\left(\tilde{L}\otimes BK\right)\tilde{e}(t)\\+\left(I_N\otimes H\right)\left(\Phi(x(t);t)-\left(1_N\otimes\Phi(x_0(t),t)\right)\right).\end{aligned} \tag{16}$$

Similarly, for conditions $\tilde{e}(t)=\left[\tilde{e}_1^T(t)\ \cdots\ \tilde{e}_N^T(t)\right]^T$, $\Phi(\tilde{x}(t);t)=\left[\Phi\left(\tilde{x}_1^T(t),t\right)\ \ldots\ \Phi\left(\tilde{x}_N^T(t),t\right)\right]^T$, (1), and (2), it attains

$$\begin{aligned}\dot{\tilde{e}}(t)=\left[I_N\otimes A-\alpha\left(\tilde{L}\otimes FC\right)\right]\tilde{e}(t)\\+\left(I_N\otimes H\right)\left(\Phi(x(t);t)-\Phi(\tilde{x}(t);t)\right).\end{aligned} \tag{17}$$

To obtain the error system, we define the augmented vector as $\hat{e}=\left[e^T\ \tilde{e}^T\right]^T$, which further gives

$$\dot{\widetilde{e}} = \widehat{A}\widehat{e} + (I_{2N} \otimes H)\widehat{\Phi}, \tag{18}$$

$$\widehat{A} = \begin{bmatrix} I_N \otimes A - s(\widetilde{L} \otimes BK) & s(\widetilde{L} \otimes BK) \\ 0 & I_N \otimes A - \alpha(\widetilde{L} \otimes FC) \end{bmatrix},$$
$$\widehat{\Phi} = \begin{bmatrix} \Phi(x(t);t) - 1_N \otimes \Phi(x_0(t),t) \\ \Phi(x(t);t) - \Phi(\widetilde{x}(t);t) \end{bmatrix} = \begin{bmatrix} \Phi(x(t) - x_0(t);t) \\ \Phi(x(t) - \widetilde{x}(t);t) \end{bmatrix}. \tag{19}$$

The concept of a regional consensus has been applied in the works [31, 34]. In the regional consensus, an agreement between the agents can be achieved if the error signals at time $t=0$ are bounded in an ellipsoid. More specifically, if the error $e(t)$ converges to the origin for a region $\widetilde{e}^T(0)(I_N \otimes S)\widetilde{e}(0) + \iota e^T(0)(I_N \otimes R^{-1})e(0) \leq \delta^{-1}$ under a positive-definite matrices R and S and a positive scalar δ, then a regional consensus between the leader and followers in (1) is achieved.

Next, we formulate theorems for determining the unknown matrices K and F in addition to scalar weights s and α with a defined region of stability for which the agreement of the agents' states will be achieved. In the first subsection, we aim to derive a local design approach for observer-based consensus of OSL MASs under input saturation. In the next subsection, this problem is further extended when there are independent topologies of MASs for different purposes.

3.1 | Observer-based consensus control

The proposed observer-based consensus control condition over input saturation is provided herein.

Theorem 1. *For MASs (1), let Assumptions 1–3 be satisfied, and the regional consensus is attained between the agents by employing observer-based control protocol (2) and (7) under input saturation, if there exist scalars $v_k > 0$, for $k = 1,2,3,4$, $c_h > 0$, for $h = 1,2$, $\beta > 0$, and $\gamma > 0$, and matrices $R > 0 \in \mathbb{R}^{n\times n}$ and $S > 0 \in \mathbb{R}^{n\times n}$ such that the following inequalities hold:*

$$\begin{bmatrix} \mathrm{E} & SH - v_1 I_n + v_2\eta I_n \\ * & -2v_2 I_n \end{bmatrix} \leq 0,$$
$$\mathrm{E} = \beta S + SA + A^T S - c_1 C^T C + 2(v_1\rho I_n + v_2 \varepsilon I_n), \tag{20}$$

$$\begin{bmatrix} R & B_{\{j\}} \\ * & 4\delta \overline{u}_j^2 \overline{\lambda}_{\max}^{-1} s^{-2} \end{bmatrix} \geq 0, \tag{21}$$

$$\begin{bmatrix} S & R^{-1}B_{\{j\}} \\ * & 4\delta \overline{u}_j^2 \overline{\lambda}_{\max}^{-1} s^{-2} \end{bmatrix} \geq 0, \forall j = 1,\ldots,p, \tag{22}$$

with the parameters $\overline{\lambda}_{\max} = \lambda_{\max\limits_{i=1,\ldots,N}}\left(\left(\widetilde{L}_{(i)}\right)^T\left(\widetilde{L}_{(i)}\right)\right)$ and $\lambda_o = \lambda_{\min}\left(\widetilde{L}\right)$. Moreover, at least one of the three cases based on the sign of $v_3\rho + v_4\varepsilon$ needs to have a feasible solution:

i. If $(v_3\rho + v_4\varepsilon) > 0$, then

$$\begin{bmatrix} Z & H - v_3 E + v_4\eta R & R\sqrt{2v_3\rho I_n + 2v_4\varepsilon I_n} \\ * & -2v_4 I_n & 0 \\ * & * & -I_n \end{bmatrix} \leq 0,$$
$$Z = \gamma R + AR + RA^T - c_2 BB^T. \tag{23}$$

ii. If $(v_3\rho + v_4\varepsilon) = 0$, then

$$\begin{bmatrix} \gamma R + AR + RA^T - c_2 BB^T & H - v_3 R + v_4\eta R \\ * & -2v_4 I_n \end{bmatrix} \leq 0. \tag{24}$$

iii. If $(v_3\rho + v_4\varepsilon) < 0$, then

$$\begin{bmatrix} \Delta & H - v_3 R + v_4\eta R \\ * & -2v_4 I_n \end{bmatrix} \leq 0,$$
$$\Delta = \gamma R + AR + RA^T - c_2 BB^T - U,$$
$$U = -2R^T(v_3\rho I_n + v_4\varepsilon I_n)R. \tag{25}$$

For an exponential rate $\chi = \min\{\beta, \gamma\}$, the convergence of the two error signals e and $\widetilde{e}$ is achieved under initial conditions, validating

$$\widetilde{e}^T(0)(I_N \otimes S)\widetilde{e}(0) + \iota e^T(0)(I_N \otimes R^{-1})e(0) \leq \delta^{-1},$$
$$\delta > 0, S < \lambda_1 I_n, R > \lambda_2 I_n. \tag{26}$$

The proposed consensus protocol gains can be determined via relations $K = B^T R^{-1}$, $F = S^{-1} C^T$, $\alpha > c_1/\lambda_o$, and $s > c_2/\lambda_o$.

Proof. Consider a Lyapunov function as

$$V(t)=\tilde{e}^T(t)(I_N\otimes S)\tilde{e}(t)+\iota e^T(t)(I_N\otimes R^{-1})e(t), \quad (27)$$

where $R=R^T>0$, $S=S^T>0$ and ι is a scalar. Taking the time derivative of (27) along (18) results in

$$\begin{aligned}\dot{V}(t)=&\tilde{e}^T(t)\Big(I_N\otimes(SA+A^TS)-2\alpha\big(\tilde{L}\otimes SFC\big)\Big)\tilde{e}(t)\\&+2\tilde{e}^T(t)(I_N\otimes SH)\Phi(x(t)-\tilde{x}(t);t)+\\&\iota\Big(2se^T(t)\big(\tilde{L}\otimes R^{-1}BK\big)\tilde{e}(t)+e^T(t)\big(I_N\otimes\\&(R^{-1}A+A^TR^{-1})-2s\big(\tilde{L}\otimes R^{-1}BK\big)\big)e(t)\\&+2e^T(t)(I_N\otimes R^{-1}H)\Phi(x(t)-x_0(t);t)\Big).\end{aligned} \quad (28)$$

■

By using Assumptions 1–2 and Kronecker product, it results

$$\begin{aligned}&2v_1\Phi^T(x(t)-\tilde{x}(t);t)(I_N\otimes I_n)\tilde{e}(t)\\&-2v_1\rho\tilde{e}^T(t)(I_N\otimes I_n)\tilde{e}(t)\leq 0,\end{aligned} \quad (29)$$

$$\begin{aligned}&2v_2\Phi^T(x(t)-\tilde{x}(t);t)(I_N\otimes I_n)\Phi(x(t)-\tilde{x}(t);t)\\&-2v_2\varepsilon\tilde{e}^T(t)(I_N\otimes I_n)\tilde{e}(t)\\&-2v_2\eta\tilde{e}^T(t)(I_N\otimes I_n)\Phi(x(t)-\tilde{x}(t);t)\leq 0.\end{aligned} \quad (30)$$

$$\begin{aligned}&2v_3\Phi^T(x(t)-x_0(t);t)(I_N\otimes I_n)\tilde{e}(t)\\&-2v_3\rho\tilde{e}^T(t)(I_N\otimes I_n)\tilde{e}(t)\leq 0,\end{aligned} \quad (31)$$

$$\begin{aligned}&2v_4\Phi^T(x(t)-x_0(t);t)(I_N\otimes I_n)\Phi(x(t)-x_0(t);t)\\&-2v_4\varepsilon\tilde{e}^T(t)(I_N\otimes I_n)\tilde{e}(t)\\&-2v_4\eta\tilde{e}^T(t)(I_N\otimes I_n)\Phi(x(t)-x_0(t);t)\leq 0.\end{aligned} \quad (32)$$

By accounting (28)–(32) after multiplying (31) and (32) by a small positive scalar ι results into

$$\begin{aligned}\dot{V}(t)\leq&\tilde{e}^T(t)(I_N\otimes(SA+A^TS+2v_2\varepsilon I_n+2v_1\rho I_n)-2\alpha\times\\&\big(\tilde{L}\otimes SFC\big)\big)\tilde{e}(t)+2\tilde{e}^T(t)(I_N\otimes(SH+v_2\eta I_n-v_1I_n))\\&\times\Phi(x(t)-\tilde{x}(t);t)-2v_2\Phi^T(x(t)-\tilde{x}(t);t)(I_N\otimes I_n)\\&\Phi(x(t)-\tilde{x}(t);t)+\iota\big(2se^T(t)\big(\tilde{L}\otimes R^{-1}BK\big)\tilde{e}(t)\\&+e^T(t)(I_N\otimes(R^{-1}A+A^TR^{-1}+2v_4\varepsilon I_n+2v_3\rho I_n)\\&-2s\big(\tilde{L}\otimes R^{-1}BK\big)\big)e(t)+2e^T(t)(I_N\otimes(R^{-1}H+v_4\eta I_n\\&-v_3I_n))\Phi(x(t)-x_0(t);t)-2v_4\Phi^T(x(t)-x_0(t);t)\\&\times(I_N\otimes I_n)\Phi(x(t)-x_0(t);t)\big).\end{aligned} \quad (33)$$

By using $F=S^{-1}C^T$, $K=B^TR^{-1}$, $s>c_2/\lambda_o$, and $\alpha>c_1/\lambda_o$ into (33), and for $\lambda_o=\lambda_{\min}\big(\tilde{L}\big)$, we attain

$$\begin{aligned}\dot{V}(t)\leq&\tilde{e}^T(t)(I_N\otimes(SA+A^TS+2v_2\varepsilon I_n+2v_1\rho I_n-c_1C^T\times\\&C))\tilde{e}(t)+2\tilde{e}^T(t)(I_N\otimes(SH+v_2\eta I_n-v_1I_n))\times\\&\Phi(x(t)-\tilde{x}(t);t)-2v_2\Phi^T(x(t)-\tilde{x}(t);t)(I_N\otimes I_n)\\&\times\Phi(x(t)-\tilde{x}(t);t)+\iota\big(2se^T(t)\big(\tilde{L}\otimes R^{-1}BK\big)\tilde{e}(t)\\&+e^T(t)(I_N\otimes(R^{-1}A+A^TR^{-1}-c_2R^{-1}BB^TR^{-1}+2v_4\varepsilon I_n\\&+2v_3\rho I_n))e(t)+2e^T(t)(I_N\otimes(R^{-1}H+v_4\eta I_n-v_3I_n))\\&\Phi(x(t)-x_0(t);t)-2v_4\Phi^T(x(t)-x_0(t);t)(I_N\otimes I_n)\\&\Phi(x(t)-x_0(t);t)\big).\end{aligned} \quad (34)$$

By defining $\Xi_1(t)=\big[\tilde{e}^T(t)\ \ \Phi^T(x(t)-\tilde{x}(t);t)\big]^T$ and $\Xi_2(t)=\big[e^T(t)\ \ \Phi^T(x(t)-x_0(t);t)\big]^T$, condition (34) can be converted into the following matrix inequality via application of the Schur complement:

$$\begin{aligned}\dot{V}\leq&\Xi_1(t)^T\begin{bmatrix}I_N\otimes \mathrm{P} & I_N\otimes(SH+v_2\eta I_n-v_1I_n)\\ * & -I_N\otimes 2v_2I_n\end{bmatrix}\Xi_1(t)\\&+\iota\Bigg(2se^T(t)\big(\tilde{L}\otimes R^{-1}BB^TR^{-1}\big)\tilde{e}(t)\\&+\Xi_2(t)^T\begin{bmatrix}I_N\otimes \mathrm{T} & I_N\otimes(R^{-1}H+v_4\eta I_n-v_3I_n)\\ * & -I_N\otimes 2v_4I_n\end{bmatrix}\Xi_2(t)\Bigg),\end{aligned} \quad (35)$$

$$\begin{aligned}&\mathrm{P}=SA+A^TS-c_1C^TC+2v_2\varepsilon I_n+2v_1\rho I_n,\\&\mathrm{T}=R^{-1}A+A^TR^{-1}-c_2R^{-1}BB^TR^{-1}+2v_4\varepsilon I_n+2v_3\rho I_n.\end{aligned}$$

To get $\dot{V}\leq 0$, we need

$$\begin{bmatrix}\mathrm{P} & SH+v_2\eta I_n-v_1I_n\\ * & -2v_2I_n\end{bmatrix}\leq 0, \quad (36)$$

$$\begin{bmatrix}\mathrm{T} & R^{-1}H+v_4\eta I_n-v_3I_n\\ * & -2v_4I_n\end{bmatrix}\leq 0. \quad (37)$$

If there exist four scalars β, $\tilde{\beta}$, γ, and $\tilde{\gamma}$, such that $\beta\gg\tilde{\beta}$ and $\gamma\gg\tilde{\gamma}$, and a feasible solution of (20) and (25) exists, (35) leads to

$$\begin{aligned}\dot{V}<&-\big(\beta+\tilde{\beta}\big)\tilde{e}^T(t)(I_N\otimes S)\tilde{e}(t)\\&+\iota\Big(2se^T(t)\big(\tilde{L}\otimes R^{-1}BB^TR^{-1}\big)\tilde{e}(t)\\&-(\gamma+\tilde{\gamma})e^T(t)(I_N\otimes R^{-1})e(t)\Big).\end{aligned} \quad (38)$$

Using $\hat{e}=\big[e^T\ \ \tilde{e}^T\big]^T$, (38) can be converted to

$$\dot{V}(t) < -\beta\tilde{e}^T(t)(I_N \otimes S)\tilde{e}(t) - \iota\gamma e^T(t)(I_N \otimes R^{-1})e(t) + \hat{e}^T(t)\begin{bmatrix} -\tilde{\beta}I_N \otimes S & \iota s\tilde{L} \otimes R^{-1}BB^TR^{-1} \\ * & -\iota\tilde{\gamma}I_N \otimes R^{-1}\end{bmatrix}\hat{e}(t). \tag{39}$$

For $\gamma > 0$ and $R > 0$, we attain $(-\iota\tilde{\gamma}I_N \otimes R^{-1}) < 0$, $\left(\tilde{\beta}I_N \otimes S\right) > 0$, and $\left(s\tilde{L} \otimes R^{-1}BB^TR^{-1}\right)^T(\tilde{\gamma}I_N \otimes R^{-1})^{-1} \times \left(s\tilde{L} \otimes R^{-1}BB^TR^{-1}\right) \geq 0$, we can find ι by

$$\iota < \min_{i \in \{1,\ldots,s\}}\left(\frac{1}{\lambda_{\max}\left\{\left(\tilde{\beta}I_N \otimes S\right)^{-1}\mathrm{T}^T(\tilde{\gamma}I_N \otimes R^{-1})^{-1}\mathrm{T}\right\}}\right), \quad \mathrm{T} = \left(s\tilde{L} \otimes R^{-1}BB^TR^{-1}\right), \tag{40}$$

which leads to $\left(\tilde{\beta}I_N \otimes S\right) > \iota\mathrm{T}^T\left(s\tilde{L} \otimes R^{-1}BB^TR^{-1}\right)\mathrm{T}$. By means of the Schur complement, we arrive at

$$\begin{bmatrix} -\tilde{\beta}I_N \otimes S & \sqrt{\iota}s\tilde{L} \otimes R^{-1}BB^TR^{-1} \\ * & -\iota\tilde{\gamma}I_N \otimes R^{-1}\end{bmatrix} < 0. \tag{41}$$

With (41), (39) leads to

$$\dot{V}(t) < -\beta\tilde{e}^T(t)(I_N \otimes S)\tilde{e}(t) - \iota\gamma e^T(t)(I_N \otimes R^{-1})e(t), \tag{42}$$

and substituting (27) into (42) gives

$$\dot{V}(t) < -\min\{\beta,\gamma\}V(t). \tag{43}$$

If (26) and (43) hold, then

$$\tilde{e}^T(t)(I_N \otimes S)\tilde{e}(t) + \iota e^T(t)(I_N \otimes R^{-1})e(t) \leq \delta^{-1}. \tag{44}$$

By squaring and adding (11) and (12), we attain

$$\left(sK\sum_{j=0}^{N}(a_{ij}(\tilde{x}_j(t) - \tilde{x}_i(t)))\right)_{(j)}^T\left(sK\sum_{j=0}^{N}(a_{ij}(\tilde{x}_j(t) - \tilde{x}_i(t)))\right)_{(j)} + \left(sK\sum_{j=0}^{N}(a_{ij}(x_j(t) - x_i(t)))\right)_{(j)}^T\left(sK\sum_{j=0}^{N}(a_{ij}(x_j(t) - x_i(t)))\right)_{(j)} \leq 2\overline{u}_j^2. \tag{45}$$

By combining set (44) and region (45), we reach the following condition:

$$e^T(t)(I_N \otimes R^{-1})e(t) + \tilde{e}^T(t)(I_N \otimes S)\tilde{e}(t) - \delta^{-1}\left(sK\sum_{j=0}^{N}(a_{ij}(\tilde{x}_j(t) - \tilde{x}_i(t)))\right)_{(j)}^T\left(sK\sum_{j=0}^{N}(a_{ij}(\tilde{x}_j(t) - \tilde{x}_i(t)))\right)_{(j)} \times (2\overline{u}_j)^{-2} - \delta^{-1}\left(sK\sum_{j=0}^{N}(a_{ij}(x_j(t) - x_i(t)))\right)_{(j)}^T \times \left(sK\sum_{j=0}^{N}(a_{ij}(x_j(t) - x_i(t)))\right)_{(j)}(2\overline{u}_j)^{-2} \geq 0. \tag{46}$$

Replacing $K = B^TR^{-1}$, it produces

$$e^T(t)(I_N \otimes R^{-1})e(t) + \tilde{e}^T(t)(I_N \otimes S)\tilde{e}(t) - \delta^{-1}s^2(2\overline{u}_j)^{-2} e^T(t)\left(\left(\tilde{L}_{(i)}\right)^T\left(\tilde{L}_{(i)}\right) \otimes R^{-1}B_{\{j\}}B_{\{j\}}^TR^{-1}\right)e(t) - \delta^{-1}s^2(2\overline{u}_j)^{-2} \tilde{e}^T(t)\left(\left(\tilde{L}_{(i)}\right)^T\left(\tilde{L}_{(i)}\right) \otimes R^{-1}B_{\{j\}}B_{\{j\}}^TR^{-1}\right)\tilde{e}(t) \geq 0, \tag{47}$$

by using the properties of the Kronecker product and the Laplacian matrix. Condition (47) can be validated, if the following two inequalities hold:

$$e^T(t)\left(\left(I_N \otimes R^{-1}\right) - \delta^{-1}s^2(2\overline{u}_j)^{-2}\left(\left(\tilde{L}_{(i)}\right)^T\left(\tilde{L}_{(i)}\right) \otimes R^{-1}B_{\{j\}}B_{\{j\}}^TR^{-1}\right)\right)e(t) \geq 0, \tag{48}$$

$$\tilde{e}^T(t)\left(\left(I_N \otimes S\right) - \delta^{-1}s^2(2\overline{u}_j)^{-2}\left(\left(\tilde{L}_{(i)}\right)^T\left(\tilde{L}_{(i)}\right) \otimes R^{-1}B_{\{j\}}B_{\{j\}}^TR^{-1}\right)\right)\tilde{e}(t) \geq 0. \tag{49}$$

The inequalities (21) and (22) are attained by applying the Schur complement and the congruence transformation via $\mathrm{diag}(R, I_N)$ to (48) and using the Schur complement to (49), respectively. This ends the proof.

The conditions in Theorem 1 are not based on convex constraints. The nonlinear terms involved in inequalities (22) and (25) can be simplified and resolved via CCL formulation by assigning $\overline{R} = R^{-1}$. For instance, the nonlinear inequality (23) or (24) can be solved by the following nonlinear optimization scheme (see details in [24, 28, 29]) with unpretentious constraints:

$$\begin{cases} \min \mathrm{Trace}(R\overline{R}), \\ \text{subjected to,} \\ \begin{bmatrix} R & I_n \\ I_n & \overline{R}\end{bmatrix} > 0, \\ \text{inequalities (20)} - \text{(22) and any of (23) or (24).}\end{cases} \tag{50}$$

Similarly, the nonlinear inequality (25) can be converted to the following optimization problem:

$$\begin{cases} \min \text{Trace}\big(0.5\overline{U}U + R\overline{R} - \overline{U}\big(R^T \times \\ \quad (v_3\rho I_n + v_4\varepsilon I_n)R\big)\big), \\ \text{subjected to} \\ \begin{bmatrix} -2(v_3\rho I_n + v_4\varepsilon I_n) & \overline{R} \\ * & \overline{U} \end{bmatrix} \geq 0, \begin{bmatrix} U & I_n \\ * & \overline{U} \end{bmatrix} \geq 0, \\ \begin{bmatrix} R & I_n \\ * & \overline{R} \end{bmatrix} \geq 0, \\ \text{inequalities}\,(20)-(22)\,\text{and}\,(25)\,\text{with}\,\overline{U} = U^{-1}. \end{cases} \tag{51}$$

The conditions in (50) and (51) are linear constraints with nonlinear objective function optimization, which can be solved via recursive LMI optimization through CCL (see details in [24, 28, 29]). The optimization problems require some global knowledge of topology in terms of scalar parameters $\overline{\lambda}_{\max}$ and λ_o. This deficiency can be overcome in future studies through the adaptive fully distributed mechanisms.

Remark 2. The consensus of OSL systems by using state feedback is discussed in [19, 27, 28] for distributed consensus, adaptive distributed consensus, and consensus under switching topologies, respectively. The drawback of [19, 27, 28] is that they do not use output feedback and we do not always have states available because of sensors' limitations. The papers [23–25] discussed the consensus of OSL systems through the output feedback method, nonlinear observer, and observer-based control. The problem of input saturation is not discussed in [23–25], which is a common issue owing to the actuator (or input) limitations.

Remark 3. Using OSL and QIB properties, consensus of nonlinear MASs under input saturation is discussed in [31]. The drawback of [31] is that it follows a state feedback mechanism. In Theorem 1, outputs of MASs are used for feedback and to acquire the estimates of the systems' states. This study builds a consensus framework for directed graphs in agents, rather than using an undirected subgraph between the followers as in [31]. Furthermore, unlike [31], we use the notion of exponential stability for designing control and observer protocols by establishing a feasible solution of the provided matrix inequalities.

Remark 4. Unlike works on input saturation with semi-global methods [7, 11, 35], we provide a region of stability (26) in terms of initial states (initial conditions on the error bound). The conventional methods do not investigate the allowable initial conditions, for which their design can be validated. The present work ensures the reliability of the consensus protocol for a clear understanding about the applicability of the developed method. The work [26] considers a region of stability for a simple scenario, where we only have the agents' states and do not incorporate the idea of estimated states. Presently, we consider both the system's and observer's state vectors for investigating a more complicated scenario of the region of stability.

Remark 5. Unlike [7, 11, 35], Theorem 1 ensures a linear region of control input by defining a region of stability in the form of an invariant set. We can increase the range of initial conditions by minimizing scalars δ and λ_1 and by maximizing the scalar λ_2 in the region of initial condition $\tilde{e}^T(0)(I_N \otimes S)\tilde{e}(0) + \iota e^T(0)(I_N \otimes R^{-1})e(0) \leq \delta^{-1}$ in order to avoid the nonlinear control input region of saturation nonlinearity.

3.2 | Consensus under independent topologies

In Theorem 1, we considered that the consensus protocol and observers exchange information for different agents using a single graph topology. In practice, graph topologies for controllers and observers can be independent for each other. The weights and links in the systems' and observers' communication graph can be different. Examples include networked systems where we might have bandwidth issues such that the information of observer and system states cannot be transferred over the same link. Inspired by practical limitations of computation and storage issues, we consider a new observer-based design of the consensus protocol for the OSL MASs by employing the concept of independent communication topologies. Independent topologies for observers are chosen such that there is a DST present with respect to the leader agent.

Assumption 4. Let the two independent topologies $\breve{\mathrm{G}}$ and $\overline{\mathrm{G}}$ of observer agents and observer output difference, respectively, possess a DST with respect to agent 0.

The proposed nonlinear observer for estimating the states of the agents under independent topologies is given as

$$\begin{aligned}\dot{\widetilde{x}}_i(t) &= A\widetilde{x}_i(t) + B\varphi(u_i(t)) + H\Phi(\widetilde{x}_i(t),t)\\ &\quad + \alpha\sum_{j=0}^{N}\left(\breve{a}_{ij}F\left(\sigma_j(t)-\sigma_i(t)\right)\right),\\ \widetilde{y}_i(t) &= C\widetilde{x}_i(t), \quad \forall i=1,\ldots,N.\end{aligned} \tag{52}$$

The consensus protocol has a different topology, given by

$$u_i(t) = sK\sum_{j=0}^{N}\left(\overline{a}_{ij}\left(\widetilde{x}_j(t)-\widetilde{x}_i(t)\right)\right), \ \forall i=1,\ldots,N, \tag{53}$$

where $\breve{a}_{ij}$ and $\overline{a}_{ij}$ are the entries of adjacency matrices for two graphs with respect to observer agent and observer outputs difference $\sigma_i(t)$ and estimated state $\widetilde{x}_i(t)$, respectively. Laplacian matrices for both topologies $\breve{L}$ and $\overline{L}$ can be divided as

$$\breve{L} = \begin{bmatrix} 0 & O^T \\ \breve{w} & \widetilde{\breve{L}} \end{bmatrix}, \tag{54}$$

$$\overline{L} = \begin{bmatrix} 0 & O^T \\ \overline{w} & \widetilde{\overline{L}} \end{bmatrix}. \tag{55}$$

Under Assumption 4, (1), (52), and (53), the error equation (18) can be rewritten as

$$\dot{\widehat{e}}(t) = \widetilde{\widehat{A}}\widehat{e}(t) + \left(I_{2N}\otimes H\right)\widehat{\Phi}(\widehat{e}(t);t), \tag{56}$$

where

$$\widetilde{\widehat{A}} = \begin{bmatrix} (I_N\otimes A) - s\left(\widetilde{\overline{L}}\otimes BK\right) & s\left(\widetilde{\overline{L}}\otimes BK\right) \\ 0 & (I_N\otimes A) - \alpha\left(\widetilde{\breve{L}}\otimes FC\right) \end{bmatrix}. \tag{57}$$

In Theorem 1, we formulate inequalities-based conditions to guarantee the local consensus between MASs under input saturation by application of the nonlinear observers. The outcomes of Theorem 1 may not be useful to attain consensus of MASs if these agents have independent topologies. Consequently, in Theorem 2, a new condition has been formulated for the computation of observer-based protocol to reach an agreement in MASs under independent topologies.

Theorem 2. *For MASs (1) under input saturation, let Assumptions 1, 2, and 4 be satisfied. The regional consensus is attained between agents by employing observer-based control protocol (52) and (53), if there exist scalars* $\breve{v}_k>0$, *for* $k=1,2,3,4$, $\breve{c}_h>0$, *for* $h=1,2$, $\breve{\beta}>0$, *and* $\breve{\gamma}>0$, *and matrices* $\breve{R}>0\in\mathbb{R}^{n\times n}$ *and* $\breve{S}>0\in\mathbb{R}^{n\times n}$ *such that*

$$\begin{bmatrix} \mathrm{E} & \breve{S}H - \breve{v}_1 I_n + \breve{v}_2\eta I_n \\ * & -2\breve{v}_2 I_n \end{bmatrix} \le 0,$$
$$\mathrm{E} = \breve{\beta}\breve{S} + \breve{S}A + A^T\breve{S} - \breve{c}_1 C^T C + 2\left(\breve{v}_1\rho I_n + \breve{v}_2\varepsilon I_n\right), \tag{58}$$

$$\begin{bmatrix} \breve{R} & B_{\{j\}} \\ * & 4\delta\overline{u}_j^2\overline{\lambda}_{\max}^{-1}s^{-2} \end{bmatrix} \ge 0, \tag{59}$$

$$\begin{bmatrix} \breve{S} & \breve{R}^{-1}B_{\{j\}} \\ * & 4\delta\overline{u}_j^2\overline{\lambda}_{\max}^{-1}s^{-2} \end{bmatrix} \ge 0, \forall j=1,\ldots,p, \tag{60}$$

where $\overline{\lambda}_{\max} = \lambda_{\max\limits_{i=1,\ldots,N}}\left(\left(\widetilde{\overline{L}}_{(i)}\right)^T\left(\widetilde{\overline{L}}_{(i)}\right),\left(\widetilde{\breve{L}}_{(i)}\right)^T\left(\widetilde{\breve{L}}_{(i)}\right)\right)$, $\breve{\lambda}_o = \lambda_{\min}\left(\widetilde{\breve{L}}\right)$, *and* $\overline{\lambda}_o = \lambda_{\min}\left(\widetilde{\overline{L}}\right)$. *Moreover, at least one of the three cases based on the sign of* $\breve{v}_3\rho + \breve{v}_4\varepsilon$ *needs to have a feasible solution:*

i. If $\breve{v}_3\rho + \breve{v}_4\varepsilon > 0$, then

$$\begin{bmatrix} \mathrm{B} & H - \breve{v}_3\breve{R} + \breve{v}_4\eta\breve{R} & \breve{R}\sqrt{2\breve{v}_3\rho I_n + 2\breve{v}_4\varepsilon I_n} \\ * & -2\breve{v}_4 I_n & 0 \\ * & * & -I_n \end{bmatrix} \le 0,$$
$$\mathrm{B} = \breve{\gamma}\breve{R} + A\breve{R} + \breve{R}A^T - \breve{c}_2 BB^T. \tag{61}$$

ii. If $\breve{v}_3\rho + \breve{v}_4\varepsilon = 0$, then

$$\begin{bmatrix} \breve{\gamma}\breve{R} + A\breve{R} + \breve{R}A^T - \breve{c}_2 BB^T & H - \breve{v}_3\breve{R} + \breve{v}_4\eta\breve{R} \\ * & -2\breve{v}_4 I_n \end{bmatrix} \le 0. \tag{62}$$

iii. If $\breve{v}_3\rho + \breve{v}_4\varepsilon < 0$, then

$$\begin{bmatrix} \breve{\gamma}\breve{R} + A\breve{R} + \breve{R}A^T - \breve{c}_2 BB^T - U & H - \breve{v}_3\breve{R} + \breve{v}_4\eta\breve{R} \\ * & -2\breve{v}_4 I_n \end{bmatrix} \le 0,$$
$$U = -2\breve{R}^T\left(\breve{v}_3\rho I_n + \breve{v}_4\varepsilon I_n\right)\breve{R}. \tag{63}$$

With an exponential rate $\breve{\chi} = \min\left\{\breve{\beta},\breve{\gamma}\right\}$, the convergence of the two error signals e and $\widetilde{e}$ is achieved for all initial conditions validating

$$\breve{e}^T(0)\left(I_N \otimes \breve{S}\right)\breve{e}(0) + \iota e^T(0)\left(I_N \otimes \breve{R}^{-1}\right)e(0) \le \delta^{-1}, \quad (64)$$
$$\delta > 0, \breve{S} < \lambda_1 I_n, \breve{R} > \lambda_2 I_n.$$

The proposed consensus protocol gains can be determined via relations $K = B^T \breve{R}^{-1}$, $F = \breve{S}^{-1} C^T$, $\alpha > \breve{c}_1/\overline{\lambda}_o$ and $s > \breve{c}_2/\overline{\lambda}_o$.

Proof. The proof is similar to Theorem 1; therefore, it can be omitted. ∎

Remark 6. In Theorem 1, [28, 35], observer-based consensus of OSL MASs under input saturation, consensus of OSL MASs under switching topologies, and relative-output based semi-global consensus are discussed, where the agents possess the same communication topology for errors and agents. In some real situations, it is desirable to have different topologies for different signals because of computation or storage issues. As a result, in Theorem 2, we express a method to compute the parameters of the observer-based controller, capable of dealing with two communication graphs. An observer-based consensus of OSL MASs under input saturation with a guaranteed regional stability, having association with different topologies for errors and agents, is developed for the first time.

4 | SIMULATION RESULTS

Consider the five mobile agents in the Cartesian coordinates for the leader-following consensus under input saturation and the dynamics [28] of individual agent is modeled by (1) with $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = [1 \;\; 0], \Phi(x_i, t) = \left[-x_{i1}\left(x_{i1}^2 + x_{i2}^2\right) - x_{i2}\left(x_{i1}^2 + x_{i2}^2\right)\right]^T$, where $x_i(t) = (x_{i1} \;\; x_{i2})^T$ with $i = 0, 1, \ldots, 4$ and $x_0(t)$ is the leader agent.

The connectivity between agents for interchanging communication signals is demonstrated in Figure 1. The value of the Laplacian matrix is given as

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -2 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 2 \end{bmatrix}.$$

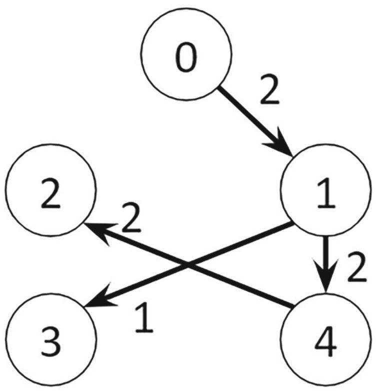

FIGURE 1 Graph topology with a directed spanning tree (DST) for the agents having motions in the Cartesian coordinates.

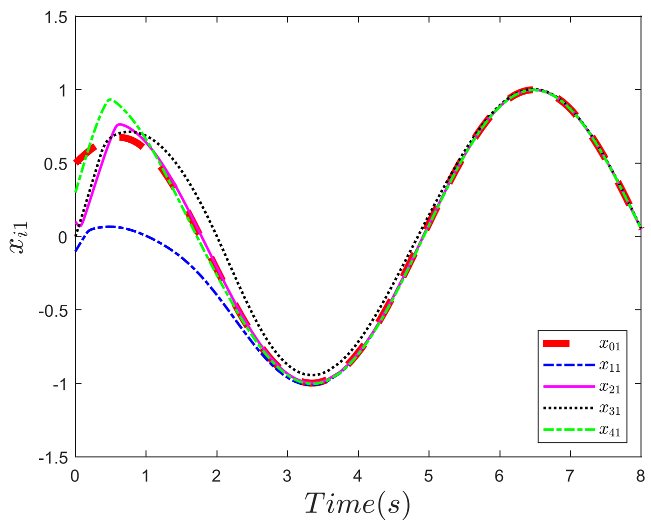

FIGURE 2 States x_{i1} for $i = 0, 1, \ldots, 4$ under input saturation for observer-based protocol.

The OSL and QIB constants are taken as $\rho = 0, \varepsilon = 0$, and $\eta = 40$ (see [36] for calculation of OSL and QIB constants). We select $c_1 = 2, c_2 = 5$, $\nu_2 = 3$, $\nu_4 = 2$, and $\nu_1 = \nu_3 = 1$ in this study.

The exponential stability constants for observer and controller are taken as $\beta = 1$ and $\gamma = 3$. We solve the conditions in Theorem 1 for feasibility to obtain unknown parameters K, F, α, and s. For $\delta = 0.5$, we determine a feasibility solution for matrices R and S as

$$R = \begin{bmatrix} 0.2453 & 0.0278 \\ 0.0278 & 0.2447 \end{bmatrix}, S = \begin{bmatrix} 0.7569 & 0.0584 \\ 0.0584 & 0.4786 \end{bmatrix}.$$

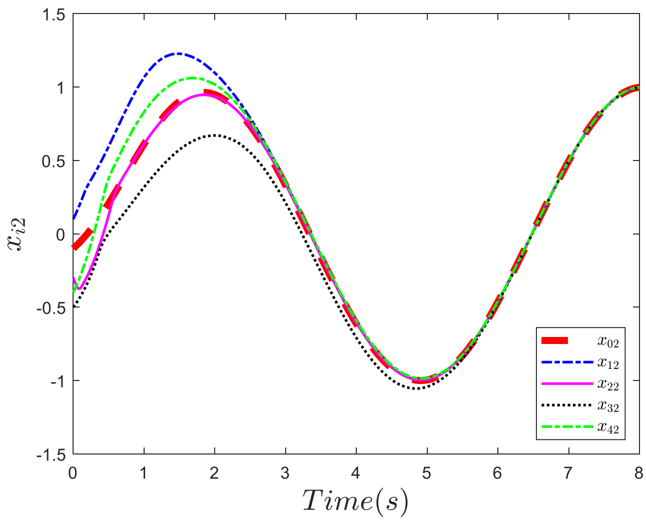

FIGURE 3 States x_{i2} for $i=0,1,\ldots,4$ under input saturation for observer-based protocol.

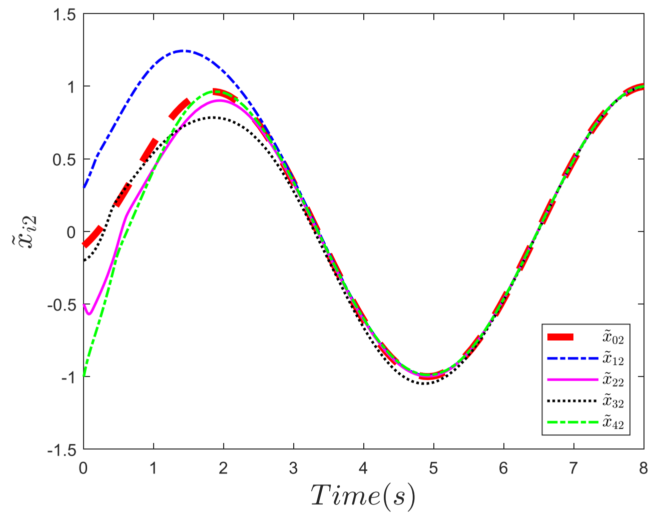

FIGURE 5 States $\tilde{x}_{i2}$ for $i=0,1,\ldots,4$ under input saturation for observer-based protocol.

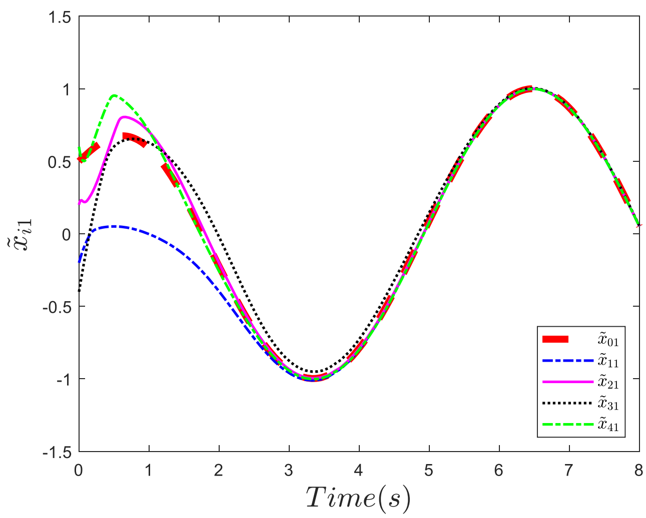

FIGURE 4 States $\tilde{x}_{i1}$ for $i=0,1,\ldots,4$ under input saturation for observer-based protocol.

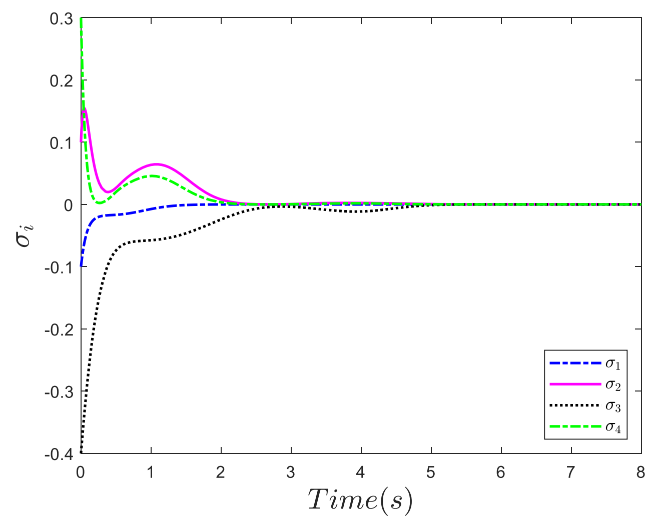

FIGURE 6 Error $\sigma_i(t)$ between the outputs of the observer and the agents.

It further provides the controller gains as

$$K=[3.6601 \quad 3.6717], F=[1.3337 \quad -0.1628]^T.$$

And, furthermore, coupling weights are obtained as

$$\alpha=2>c_1/\lambda_o \, s=5>c_2/\lambda_o, \text{where } \lambda_o=1.$$

For testing the proposed observer-based consensus protocol, the initial condition of the leader has been taken as $x_0(0)=\tilde{x}_0(0)=[0.5 \quad -0.1]$, and the initial conditions for the follower agents are chosen as

$$x_1(0)=[-0.1 \quad 0.1], x_2(0)=[0.1 \quad -0.3],$$

$$x_3(0)=[0 \quad -0.5], x_4(0)=[0.3 \quad -0.4],$$

$$\tilde{x}_1(0)=[-0.2 \quad 0.3], \tilde{x}_2(0)=[0.2 \quad -0.5],$$

$$\tilde{x}_3(0)=[-0.4 \quad -0.2], \; \tilde{x}_4(0)=[0.6 \quad -1].$$

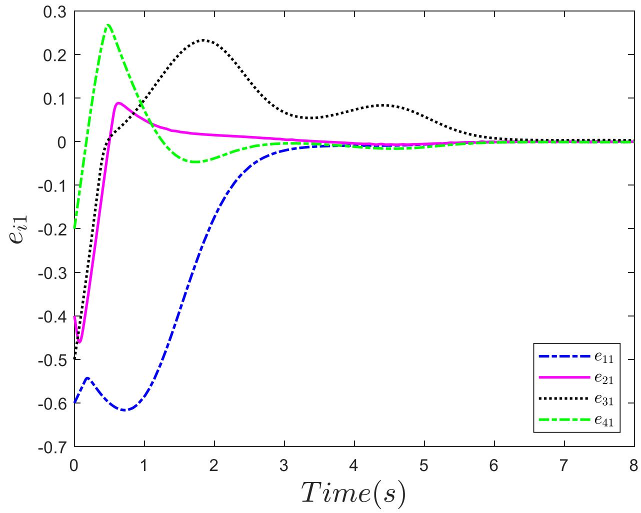

FIGURE 7 Error $e_{i1}(t)$ between the state $x_{i1}(t)$ of the agents and the followers.

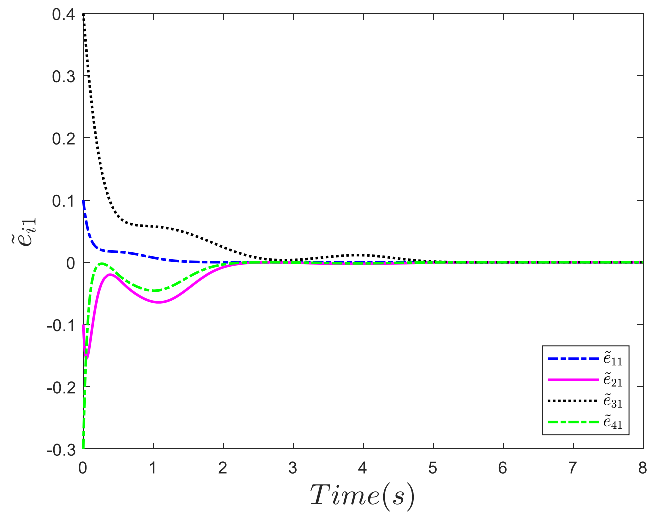

FIGURE 9 Error $\widetilde{e}_{i1}(t)$ between the state $x_{i1}(t)$ of the agents and the state $\widetilde{x}_{i1}(t)$ of the followers.

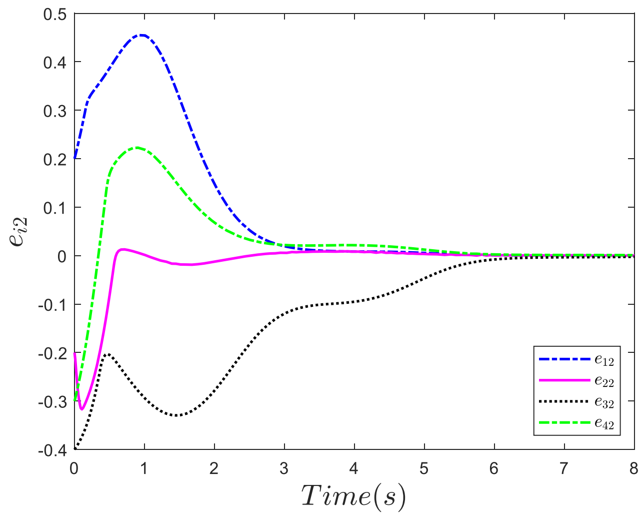

FIGURE 8 Error $e_{i2}(t)$ between the state $x_{i2}(t)$ of the agents and the followers.

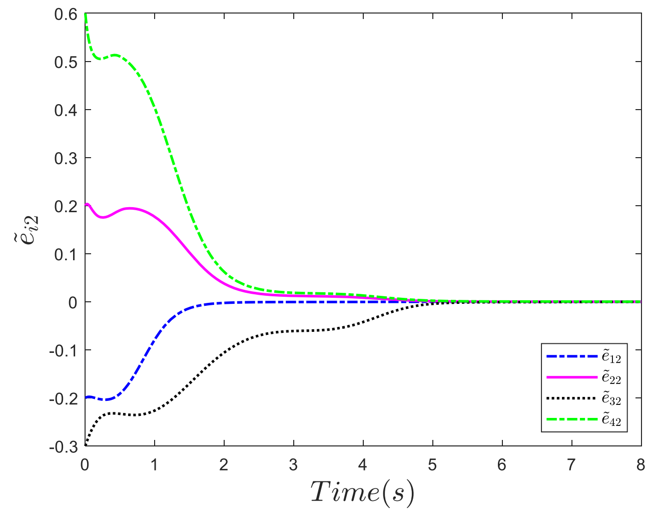

FIGURE 10 Error $\widetilde{e}_{i2}(t)$ between the state $x_{i2}(t)$ of the agents and the state $\widetilde{x}_{i2}(t)$ of the followers.

The region of stability comes out to be $\widetilde{e}^T(0)(I_N \otimes S)\widetilde{e}(0) + \iota e^T(0)(I_N \otimes R^{-1})e(0) = 1.59 \leq 2 = \delta^{-1}$. In addition, the saturation bound for inputs is taken to be $\overline{u}_1 = 1$. The responses of the mobile systems by means of the developed observer-based constrained consensus method are shown in Figures 2 and 3. It reveals that the states of the agents by application of the proposed observer-based control approach for the consensus under input saturations are converging to common values. Figures 4 and 5 provide observer states, which are converging to the states of the agents. Figure 6 provides the output error plots, Figures 7 and 8 show the error between the states of the leader and the followers, and Figures 9 and 10 show the error between the states of the system and its observer. All of these errors are converging to the origin. The work of [28] considers a state-feedback approach without input saturation and achieves consensus after 3 s. In contrast, an agreement between the states of the agents has been achieved in the steady state within 7 s. Further, the example has been extended for the independent topologies,

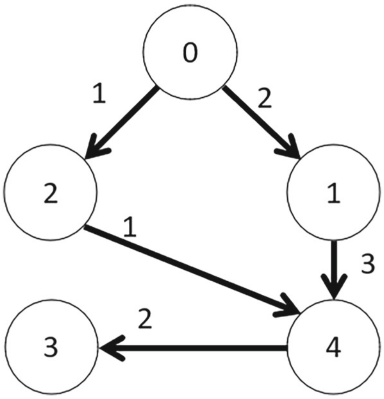

FIGURE 11 Graph topology with a directed spanning tree (DST) for the observer agents having motions in the Cartesian coordinates.

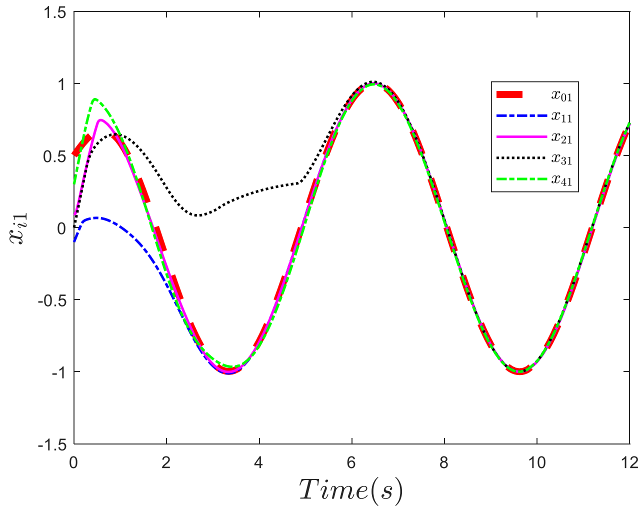

FIGURE 12 States x_{i1} for $i=0,1,\ldots,4$ under input saturation with independent topologies.

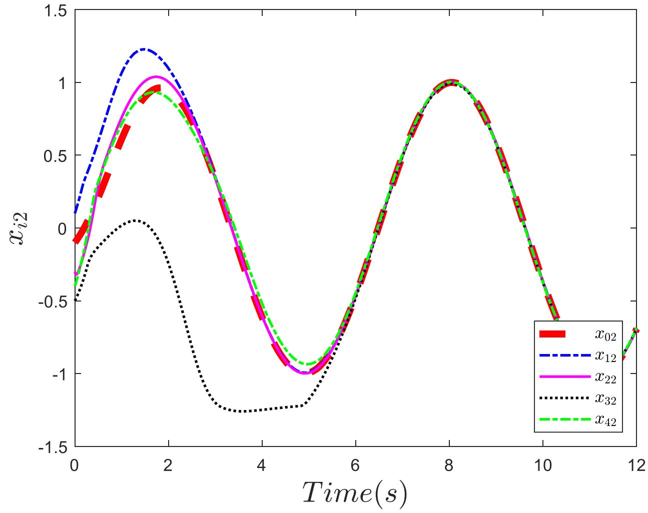

FIGURE 13 States x_{i2} for $i=0,1,\ldots,4$ under input saturation with independent topologies.

where the agents of the observer communicate using the graph in Figure 11. The simulations in Figures 12 and 13 with independent topologies show that the states converge using the proposed protocol in Theorem 2. Hence, the proposed methodology can be beneficial for consensus of OSL agents under saturations using the proposed observer-based scheme.

5 | CONCLUSIONS

In this paper, we have considered the observer-based consensus using a leader-based schema for the OSL MASs by accounting for the input saturation nonlinearity. Observer-based consensus protocols were derived by means of the QIB inequality, OSL condition, saturation bounds, convex constraints, graph theory, and CCL formulation. Nonlinear inequalities-based design mechanisms were derived to compute the parameters of the nonlinear observer and consensus controller. In contrast to the existing consensus protocols, the proposed observer-based approach, dealing with the input saturation nonlinearities, can be applied to the consensus among MASs even though the states of the agents are unavailable for a feedback tool. Further, the proposed result has been extended for the independent topologies where different graphs for the measured and estimated outputs error and estimated states are regarded to attain a matter-of-fact consensus control methodology. The existing works are limited to the semi-global methods, not dealing with a region of stability, or attaining a region of stability for a simple state feedback consensus controller. Consequently, a complex scenario for investigating the region of stability by regarding the observer and protocol, employing estimation of the states, has been investigated in contrast to the existing works. A simulation study on five mobile OSL agents under input saturation has been explored to represent the effectiveness of the resultant observer-based consensus methodology. Future work will include fuzzy schemes for observer-based consensus under frequent occurrence of DST in graph topologies, jointly connected tree, and disturbances.

AUTHOR CONTRIBUTIONS

Muhammad Ahsan Razaq: Data curation; investigation; methodology; software; writing-original draft. **Muhammad Rehan:** Conceptualization; formal analysis; investigation; methodology; writing-review and editing. **Muntazir Hussain Bangash:** Conceptualization; data curation; validation; writing-review and editing. **Shakeel Ahmed:** Formal analysis; investigation; software; visualization. **Keum-Shik Hong:** Supervision; validation; visualization; writing-review and editing.

CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

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