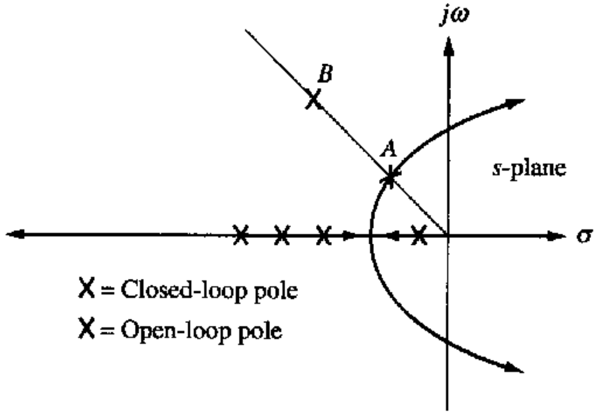
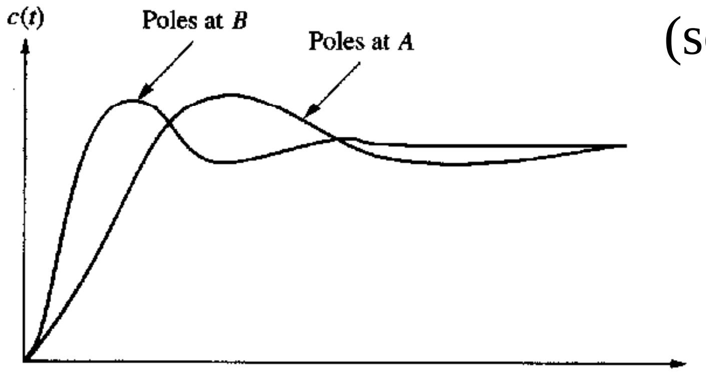
* Improving Transient Response



(a)



(b)

A goal is to speed up the transient response at A to that of B without affecting the percent overshoots.

(solution1) Augment or compensate the system with additional poles and zeros - chapter 9

(solution2) Design or change the location of all the closed-loop poles without additional poles and zeros.
- controllers and observers
- chapter 12

* Compensation Techniques

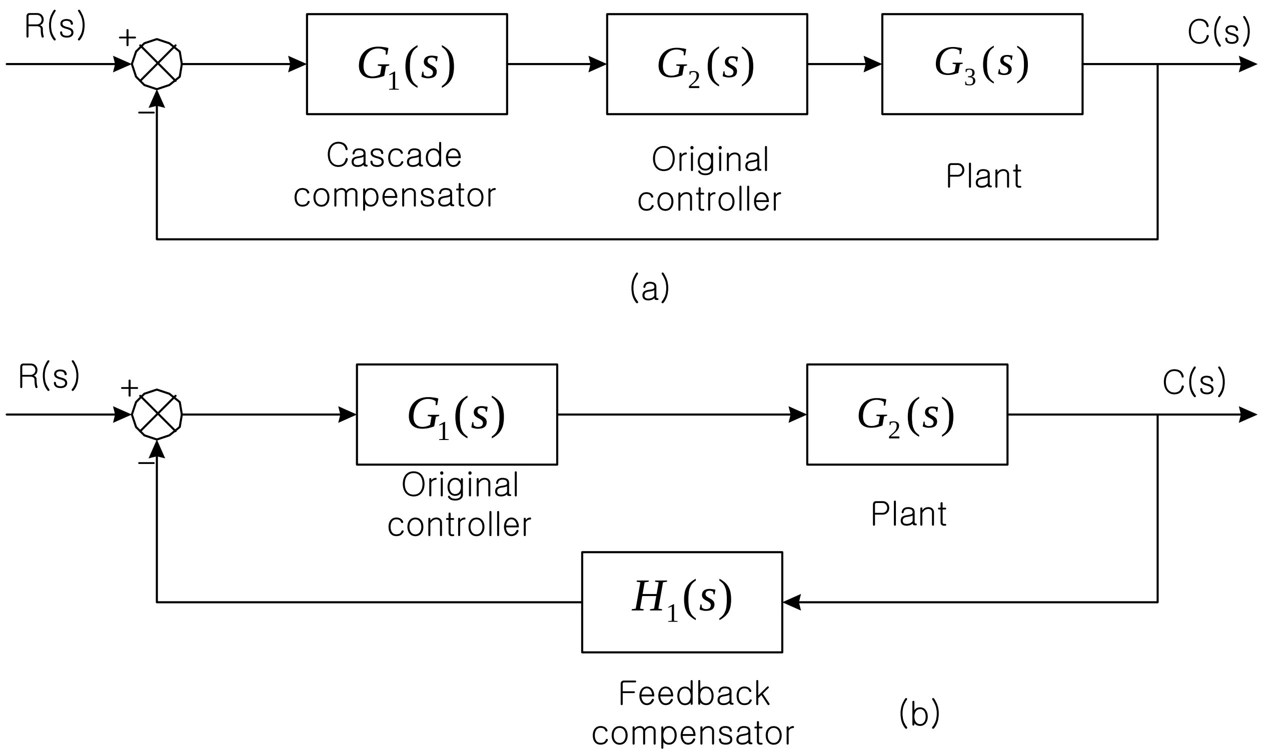
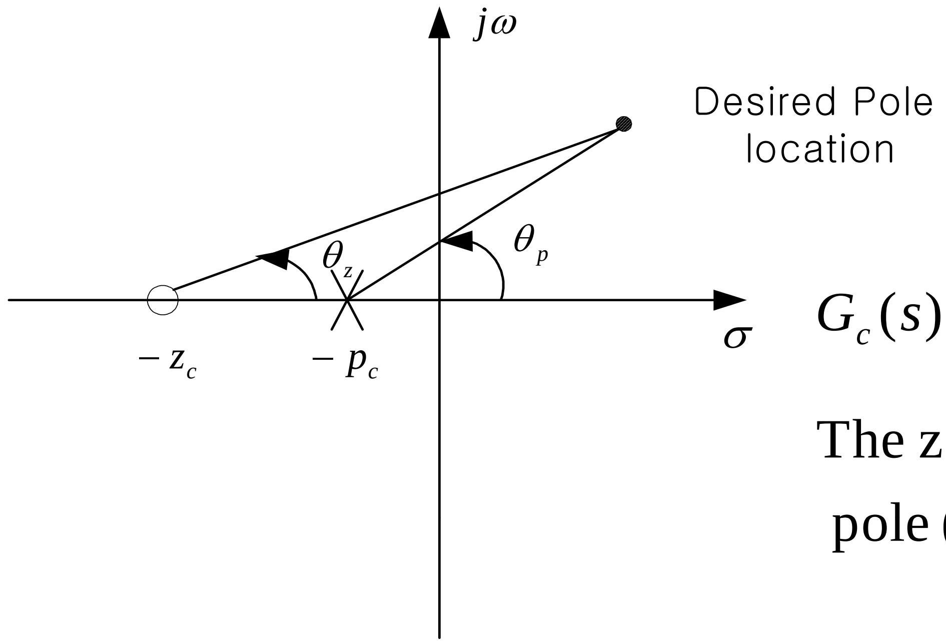


Figure 9.2
Compensation Technique
a. Cascade
b. Feedback

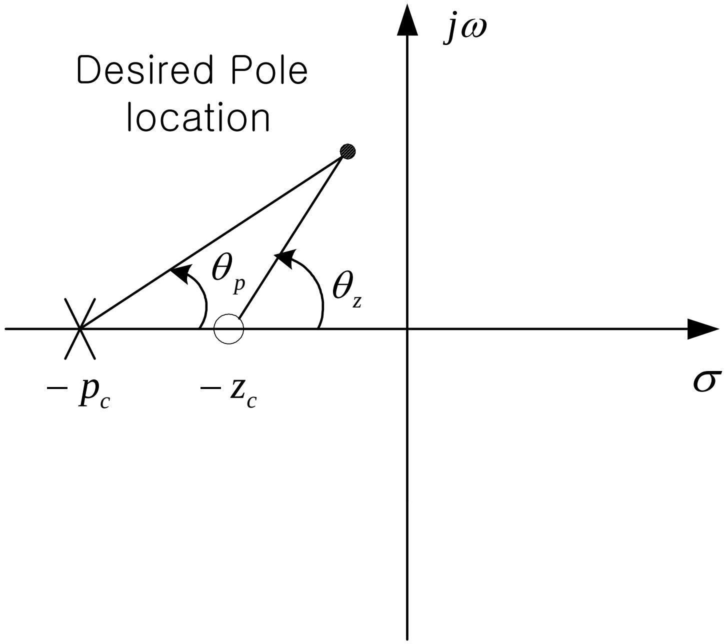
* Lag compensation



$$G_c(s) = \frac{s + z_c}{s + p_c}$$

The zero is on the left of the pole ($\theta_p > \theta_z$)

* Lead Compensation



$$G_c(s) = \frac{s + z_c}{s + p_c}$$

The zero is on the right of the pole ($\theta_p < \theta_z$)

* Classifications

1. Improving Steady - State Error via Cascade Compensation (chapter 9.2)
   - with Ideal Integral Compensators(PI controllers)
   - with Lag Compensation
2. Improving Transient Response via Cascade Compensation (chapter 9.3)
   - with Ideal Derivative Compensators(PD controllers)
   - with Lead Compensation
3. Improving Steady - State Error and Transient Response
   - with Ideal Integral and Derivative Compensators(PID controllers)
   - with Lag - Lead Compensation
4. Feedback Compensation
   - Via Rate Feedback
   - Via Minor - Loop Feedback
5. Physical Realization of Compensation

* Improving Steady-State Error via Cascade Compensation with Ideal Integral Compensators (PI Controllers)

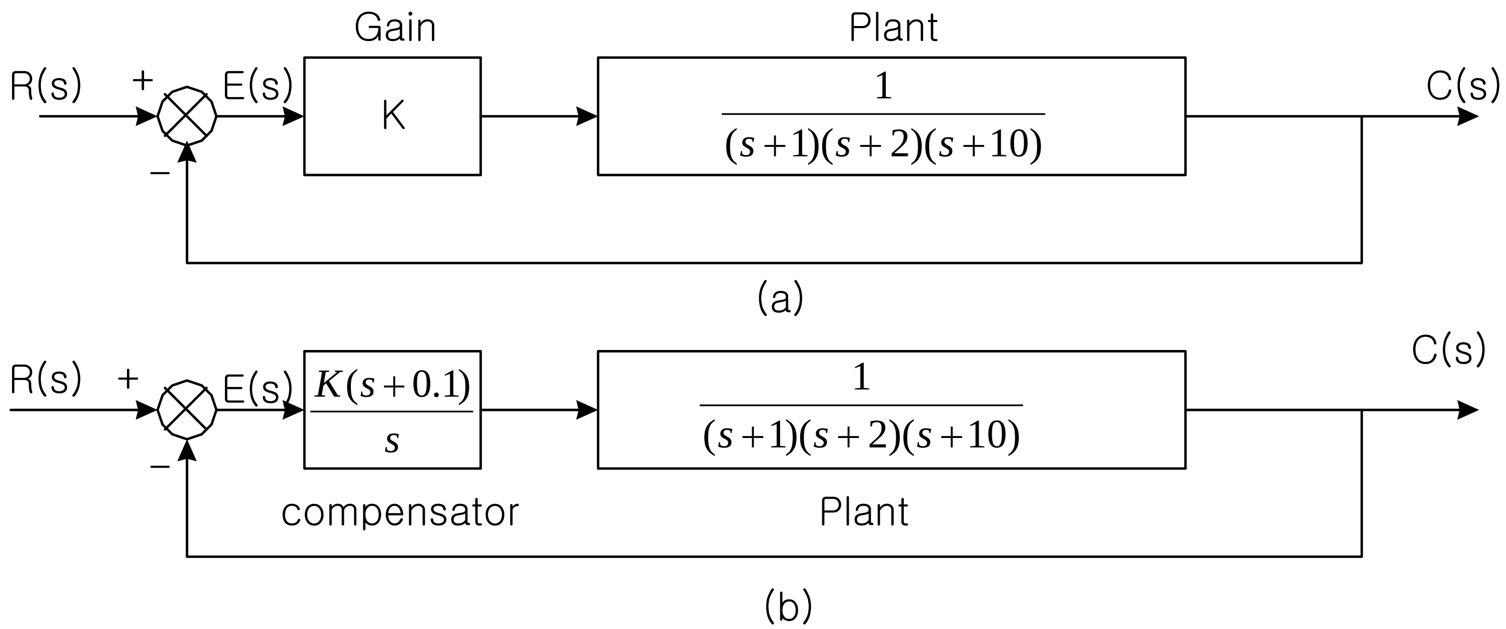
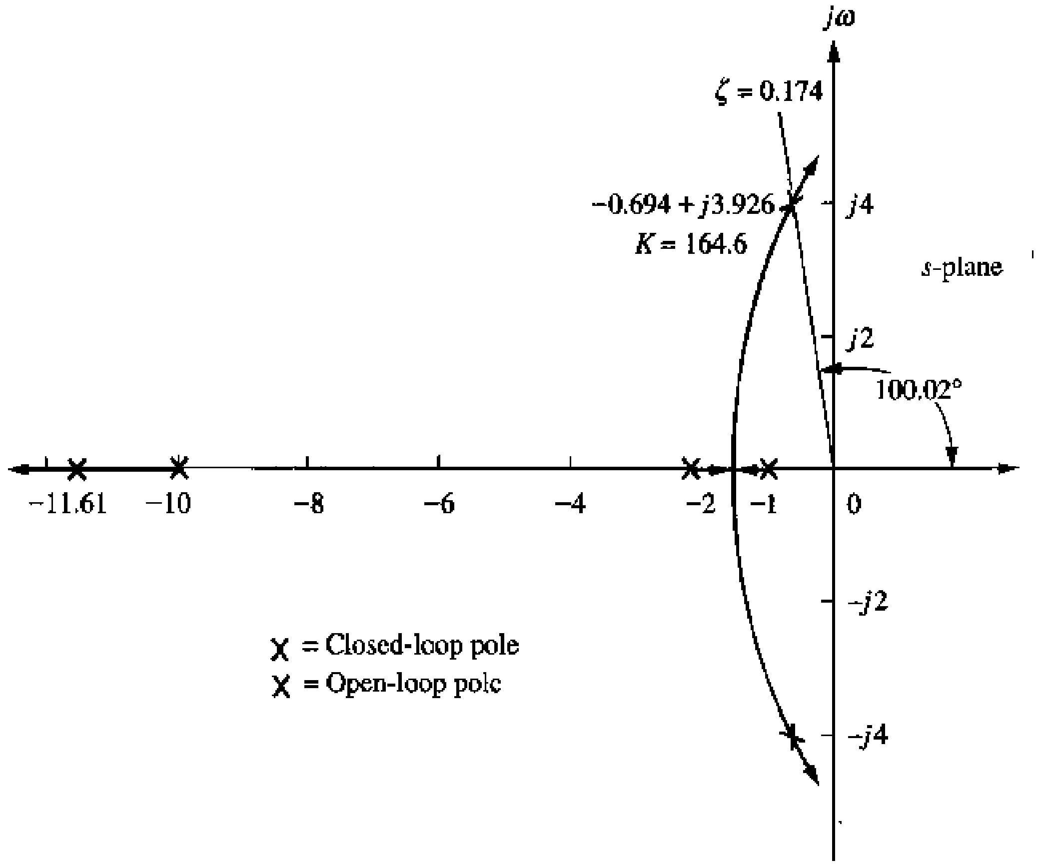


Figure 9.4

Closed-loop system for Example 9.1

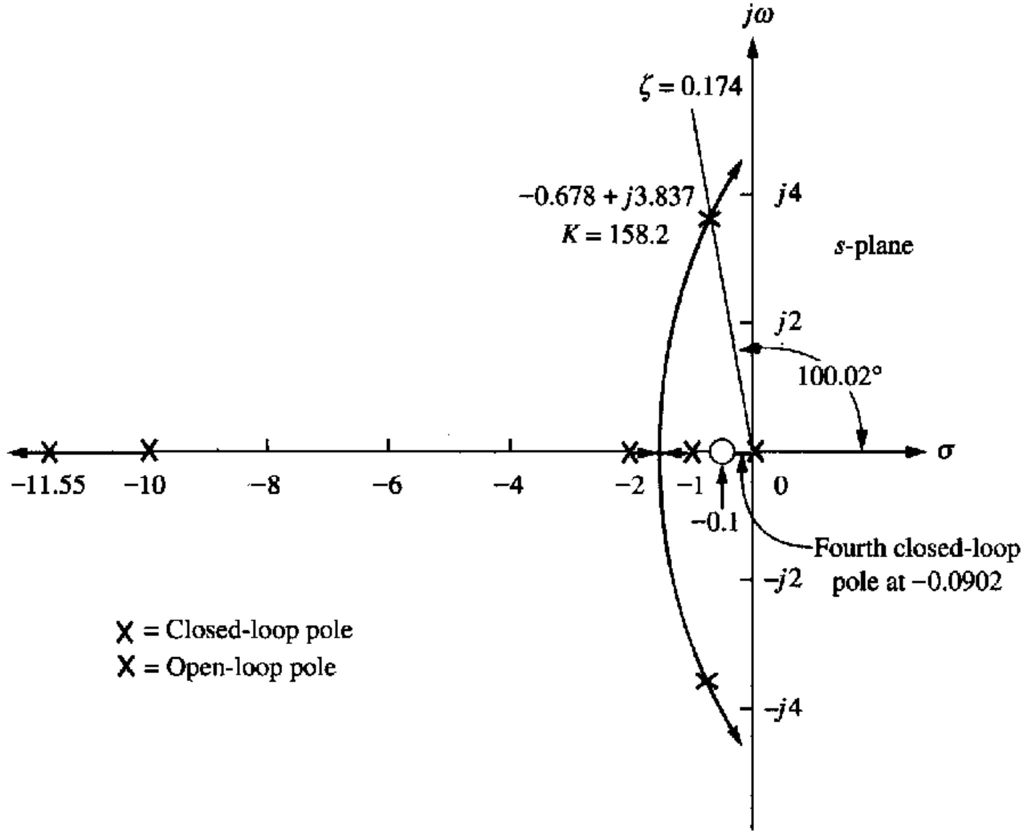
a. Before compensator

b. After ideal integral compensator



$$K_p = \lim_{s \to 0} KG(s) = \lim_{s \to 0} \frac{K}{(s+1)(s+2)(s+10)} = \frac{164.6}{20} = 8.23$$

$$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+8.23} = 0.108$$



Adding the zero at s = -0.1 and the pole at s = 0 yields approximately the same transient response as the uncompensated system and zero steady state error $e(\infty) = 0$ for a unit step input

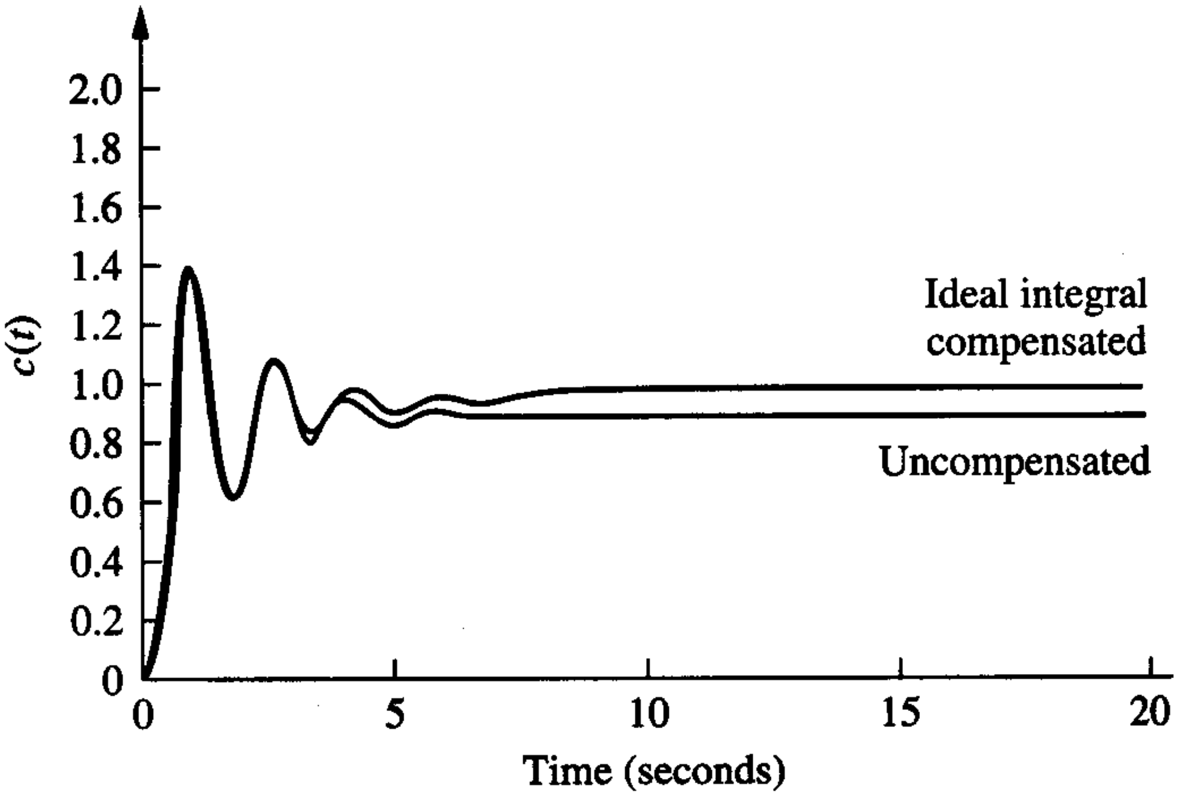
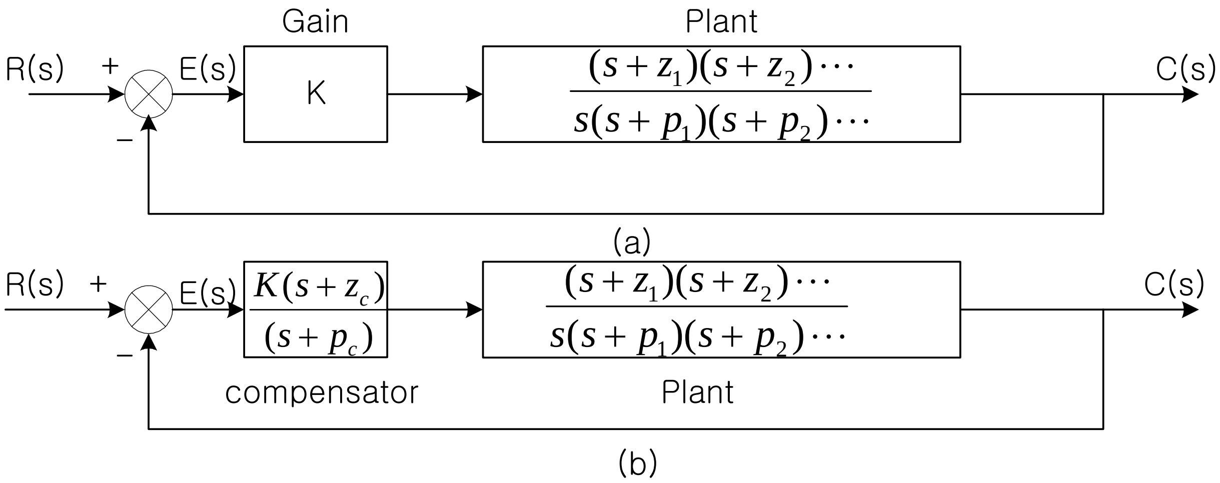


Figure 9.7

Ideal Integral compensated system response and the uncompensated system response of Example 9.1

* Improving Steady-State Error via Cascade Compensation with Lag Compensation



$$G_c(s) = \frac{K(s+z_c)}{(s+p_c)}$$

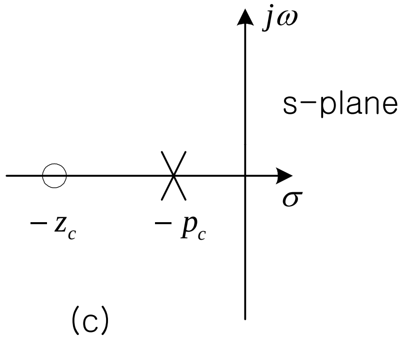


Figure 9.9
a. Type 1 uncompensated system ;
b. Type 1 compensated system ;
c. compensator pole-zero plot

To minimize the effect on the transient response, place the pole - zero pair close to the origin.

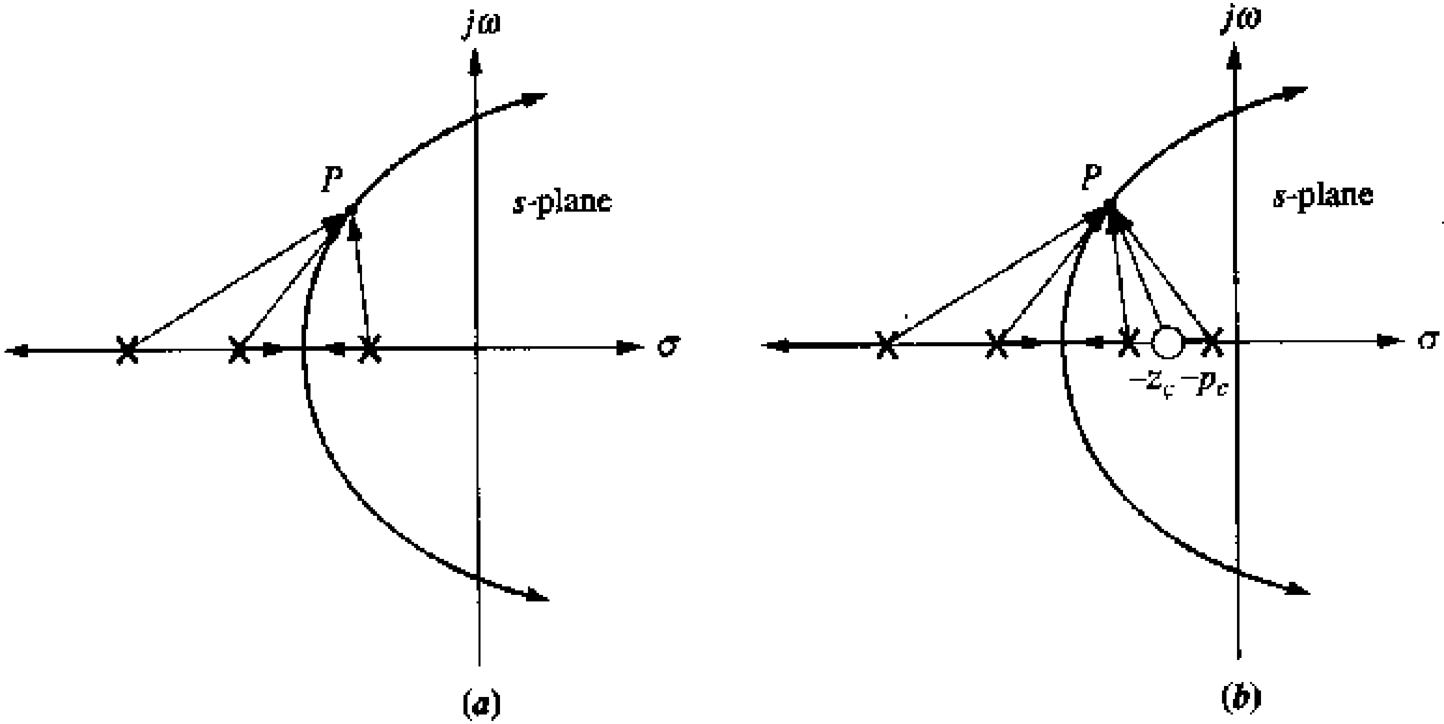


Figure 9.10
Root locus
a. before lag compensation
b. after lag compensation

To reduce the steady-state error, increase the new velocity constant K_{vN}

$$K_{vN} = \lim_{s \to 0} sKG(s) = K_{vo} \frac{z_c}{p_c} > K_{vo}$$

* Lag Compensator Design

(Problem) Improve the steady-state error by a factor of 10 if the system is operating with a damping ratio of 0.174

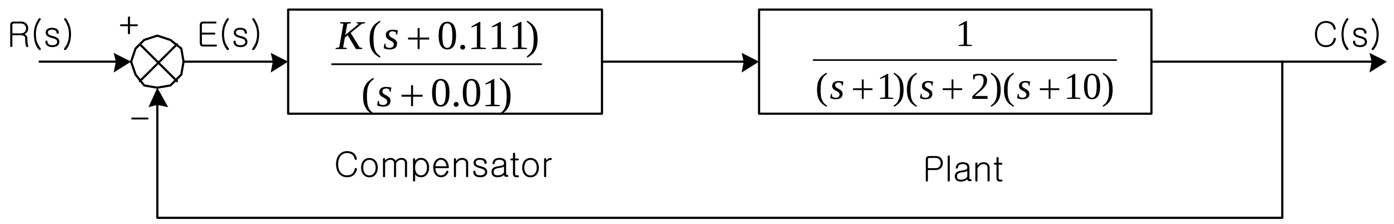


Figure 9.11
Compensated system for problem 9.2

For uncompensated system, $e(\infty) = 0.108$ with $K_{po} = 8.23$

A tenfold improvement means a steady state error of

$$e(\infty) = \frac{0.108}{10} = 0.0108$$

since $$e(\infty) = \frac{1}{1+K_p} = 0.0108$$

then $K_{pN} = 91.59.$

The compensator is $$\frac{z_c}{p_c} = \frac{K_{pN}}{K_{po}} = \frac{91.59}{8.23} = 11.13$$

If we arbitraily select $p_c = 0.01$ then $z_c = 11.13 \ p_c \cong 0.111$

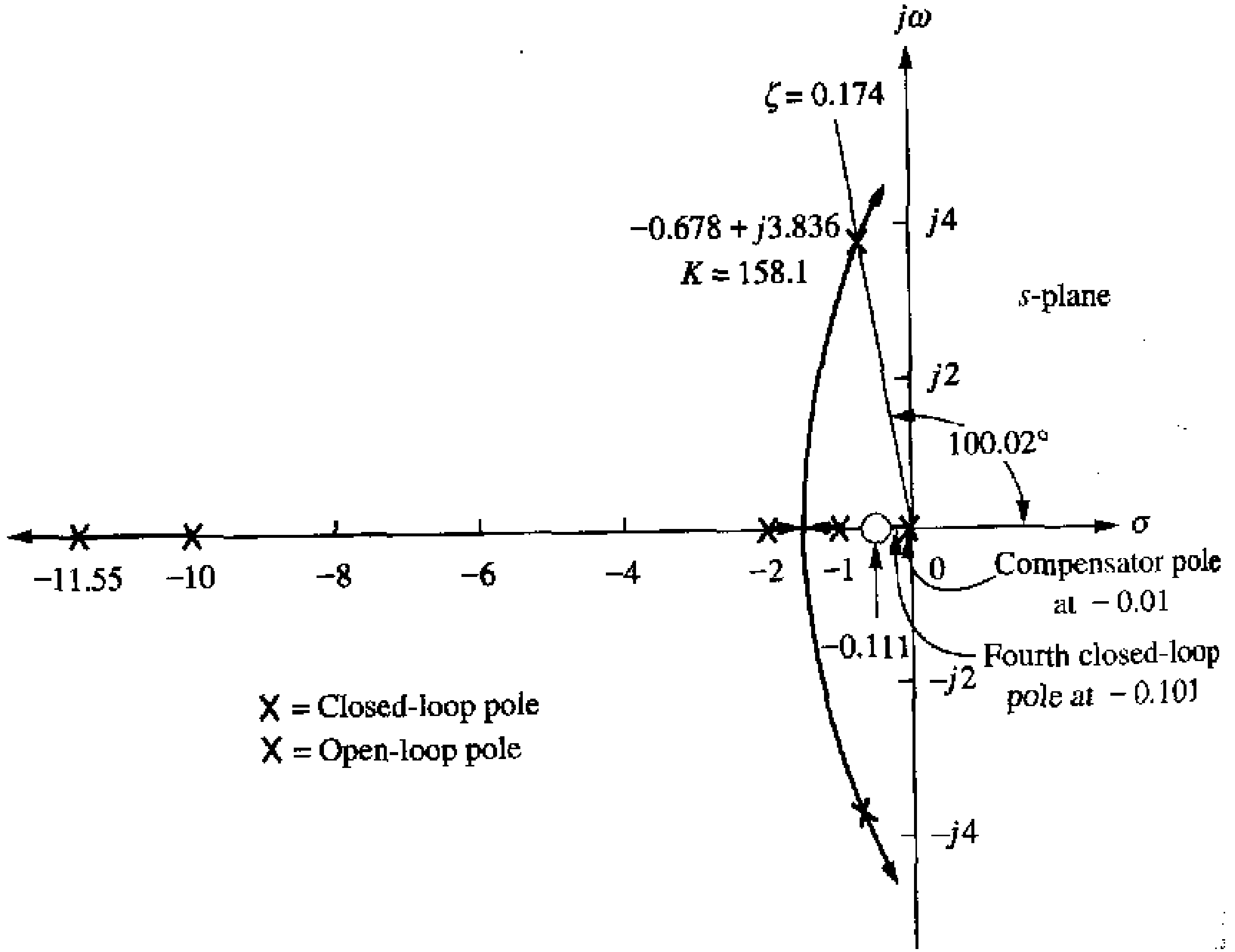


Figure 9.12

Root locus for compensated system of Figure 9.11

Table 9.1 Predicted characteristices of uncompensated and lag-compensated systems for Example 9.2

Parameter	uncompensated	Lag-compensated
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
K_p	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
zero	None	-0.111

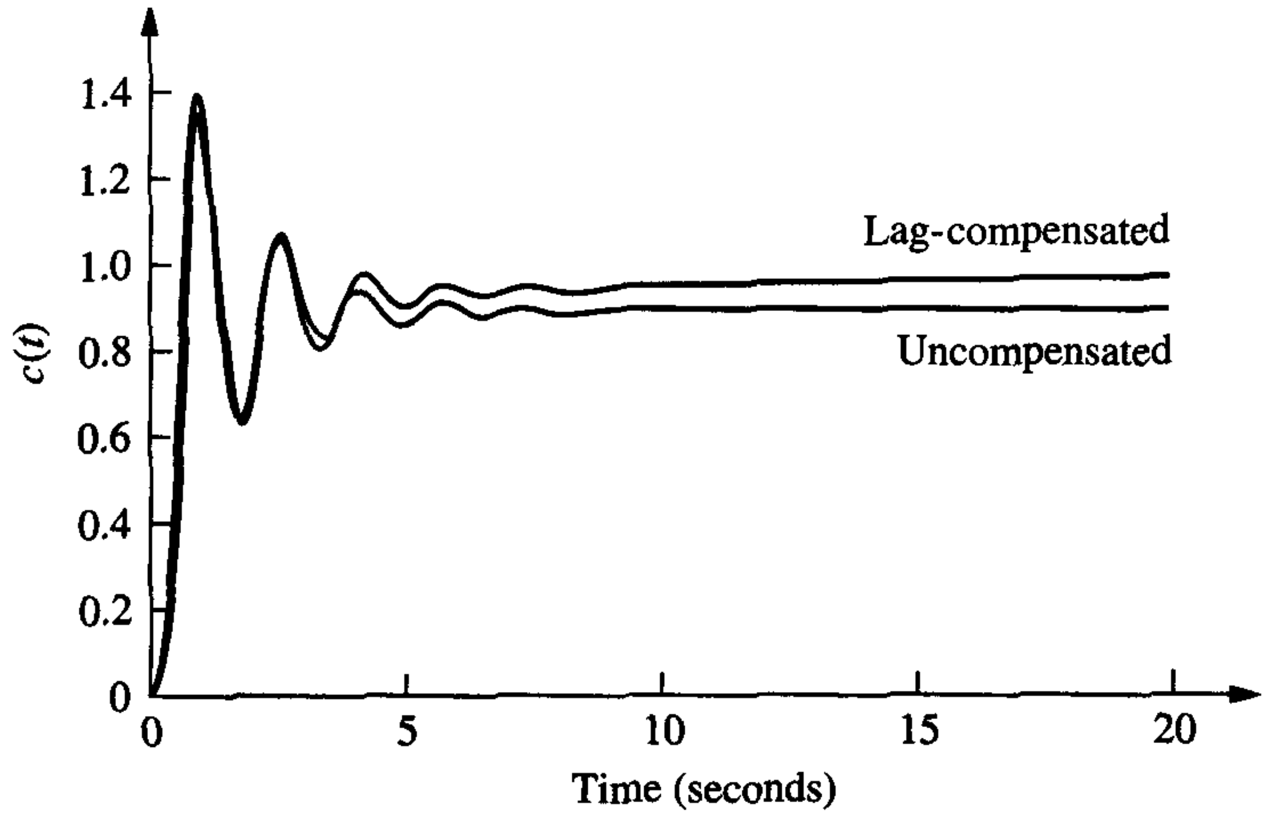


Figure 9.13

Step response of uncompensated and lag - compensated systems for Example 9.2

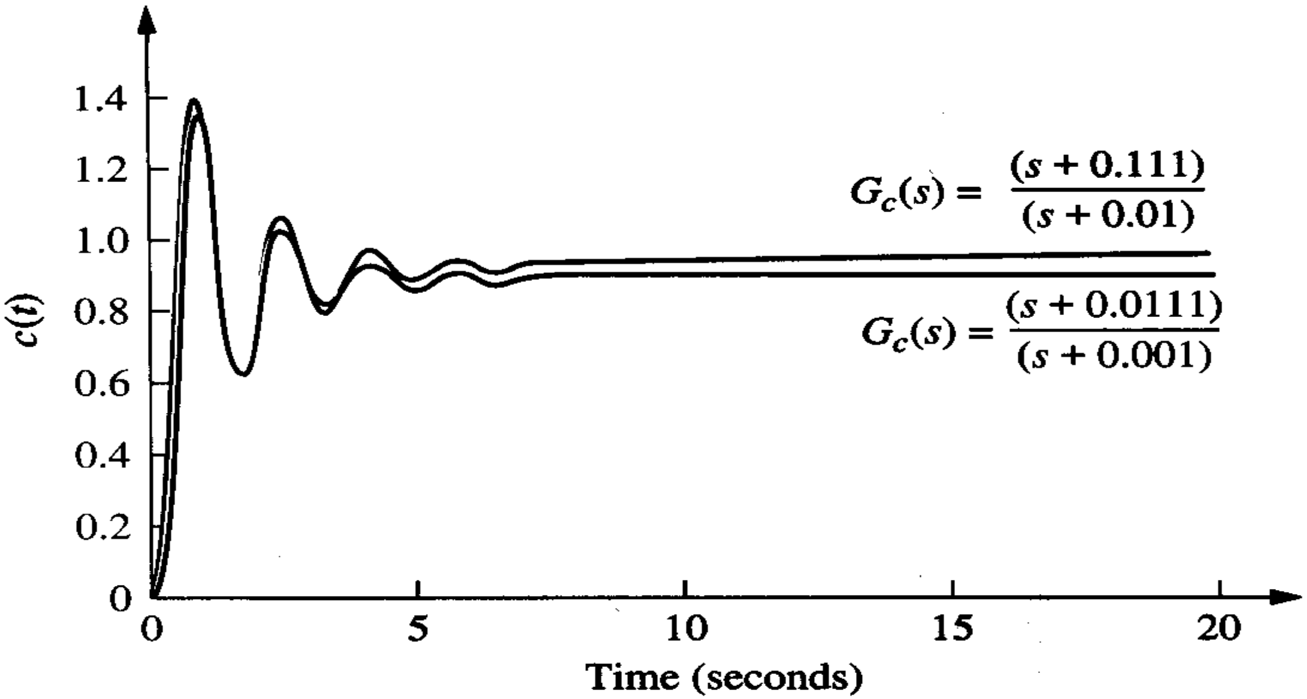
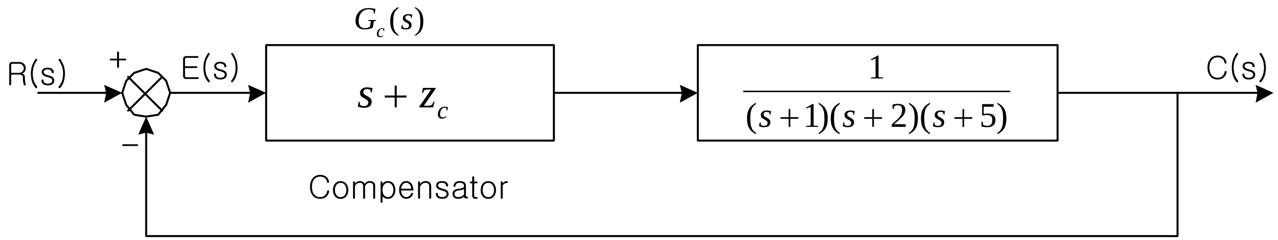


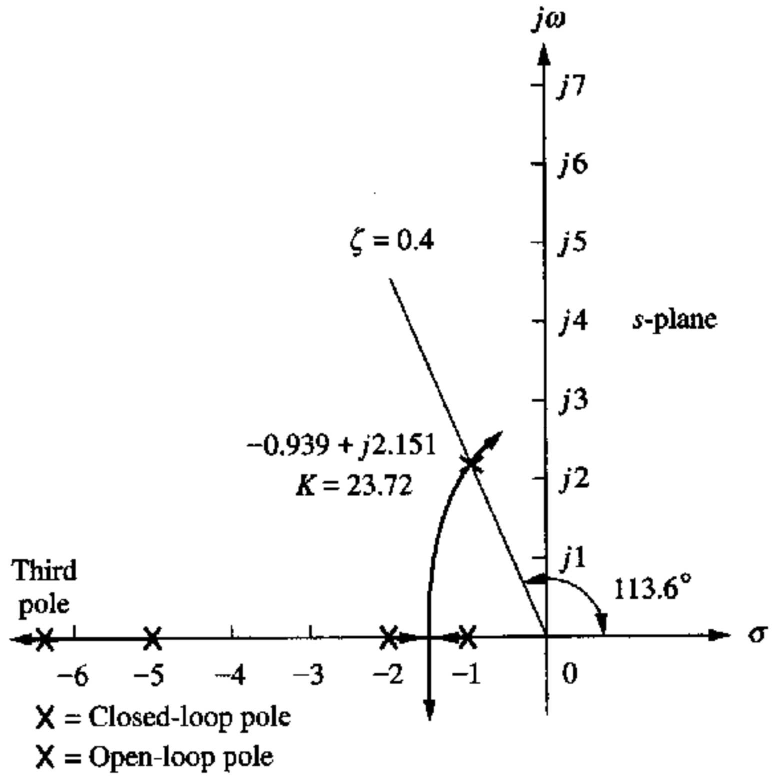
Figure 9.14

Step response of the system for Example 9.2 using different Lag compensators

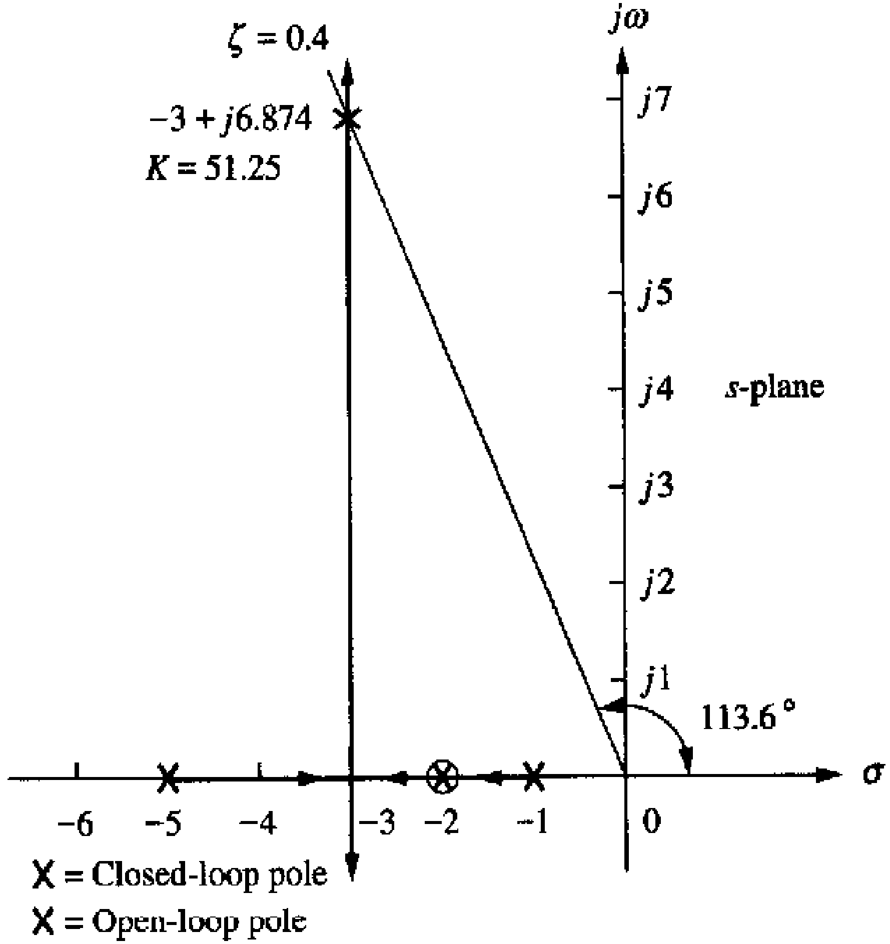
Since the new lag compensator has a closed-loop pole closer to the imaginary axis than the original lag compensator, the steady-state value will not be reached as quickly.

* Improving Transient Response via Cascade Compensation with Ideal Derivative Compensation (PD controller)

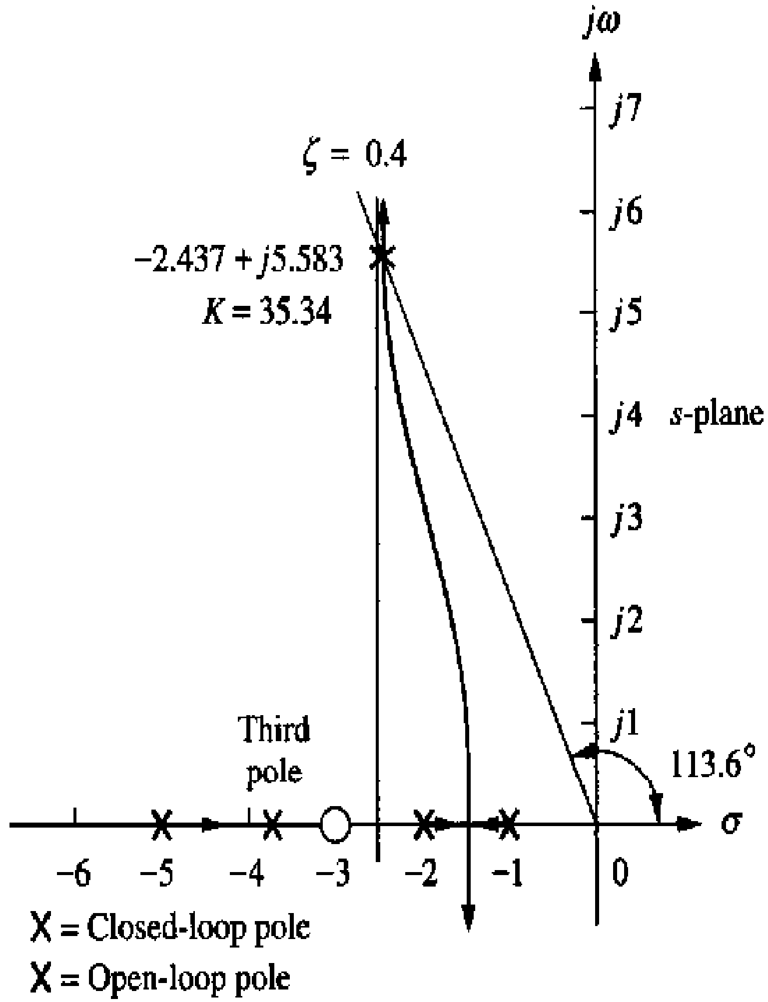




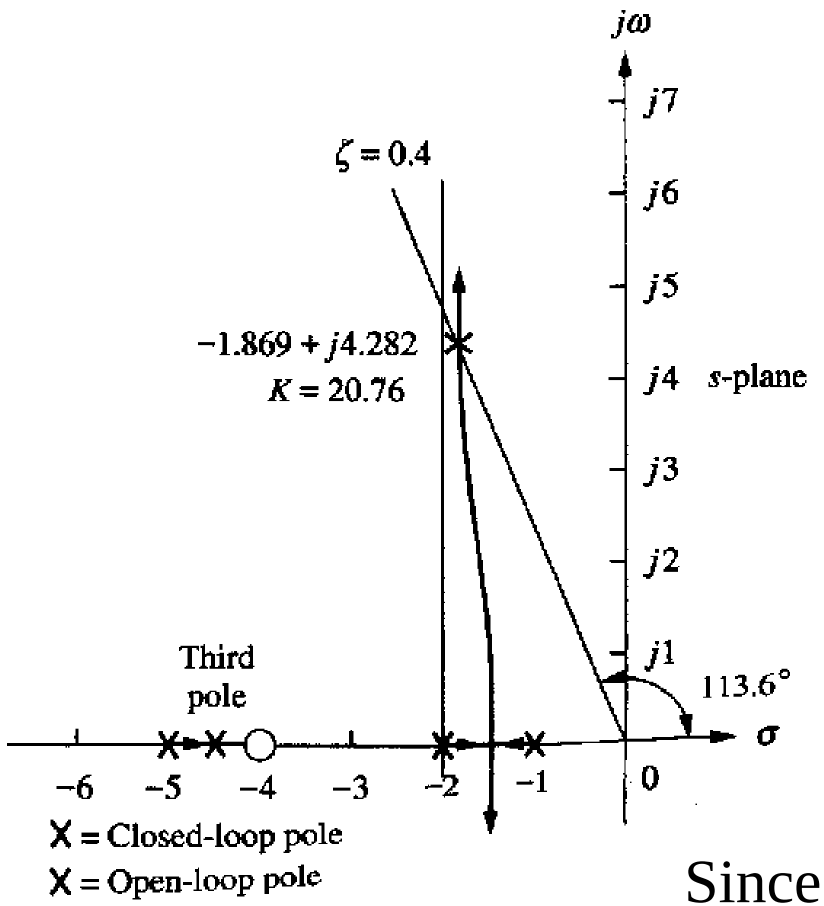
(a)


ζ = 0.4
−3 + j6.874
K = 51.25
jω
j7
j6
j5
j4
s-plane
j3
j2
j1
113.6°
σ
−6
−5
−4
−3
−2
−1
0
X = Closed-loop pole
X = Open-loop pole

(b)


jω
j7
ζ = 0.4
j6
−2.437 + j5.583
K = 35.34
j5
j4
s-plane
j3
j2
j1
Third
pole
113.6°
σ
−6
−5
−4
−3
−2
−1
0
X = Closed-loop pole
X = Open-loop pole

(c)



(d)

Since all systems are Type 0, some steady - state error is expected.

Table 9.2 Predicted characteristics for the system of Figure 9.15

	Uncompensated	compensation a	compensation b	compensation c
plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dominant poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
w_n	2.347	7.5	6.091	4.673
%OS	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	-6.123	None	-3.127	-4.262
zero	None	None	-3	-4
comments	second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

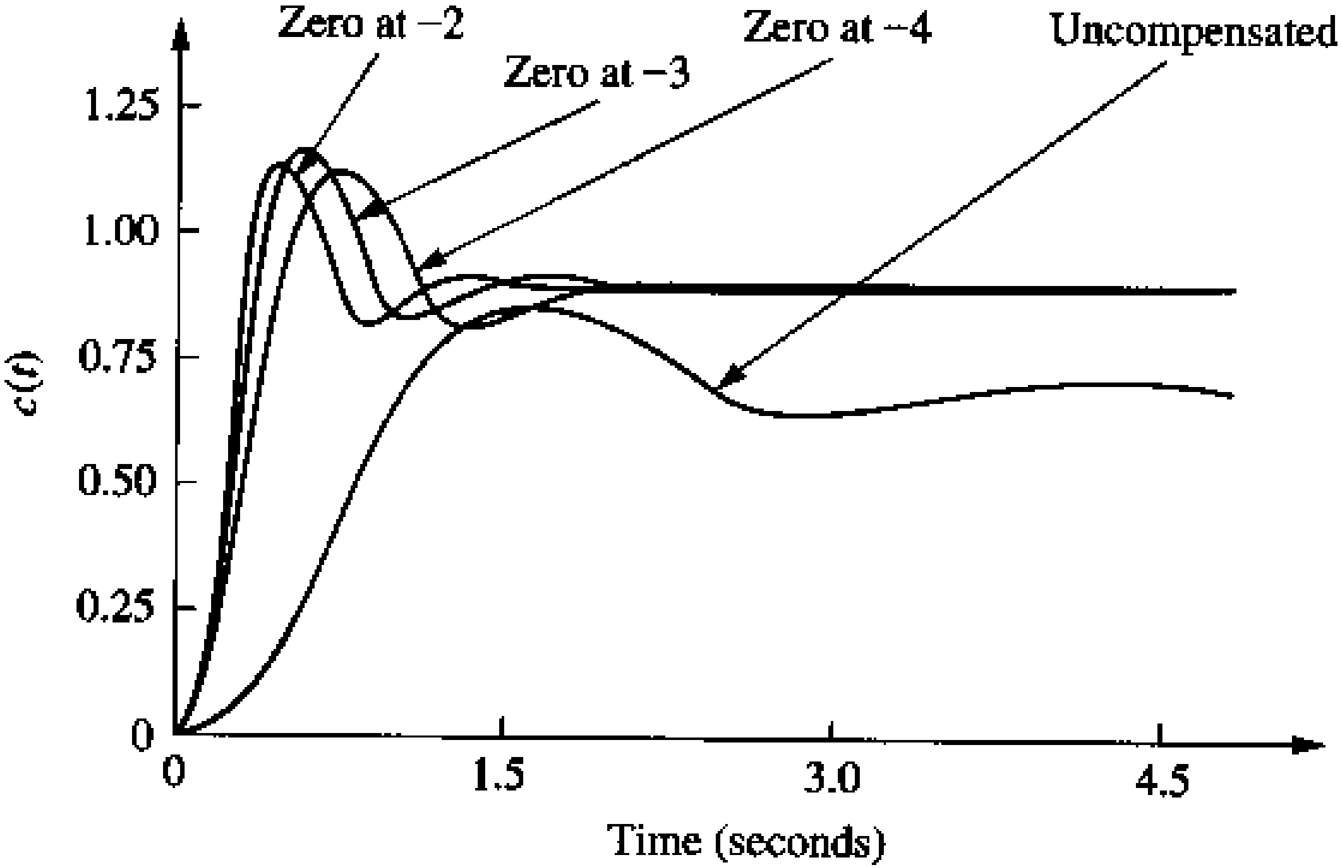
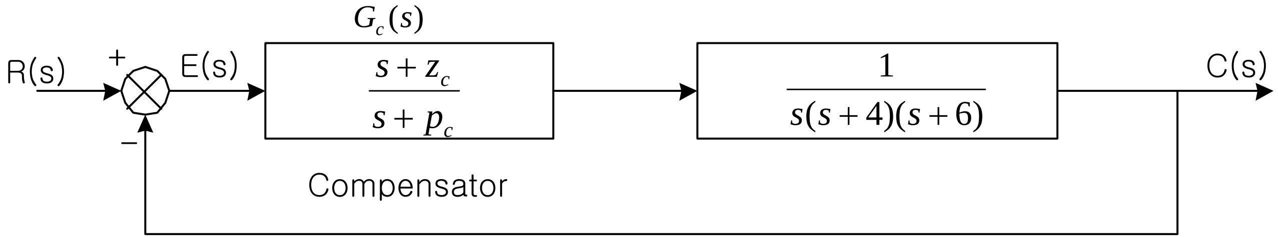


Figure 9.16

Uncompensated system and ideal derivative compensation solutions from Table 9.2

* Improving Transient Response via Cascade Compensation with Lead Compensation



(Problem) Design and compare three lead compensators that will reduce the settling time by a factor of 2 while maintaining 30% overshoot.

(solution) 30% overshoot is equivalent to a damping ratio of 0.358.

$$\%OS = 30 = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100 \rightarrow \zeta = 0.358$$

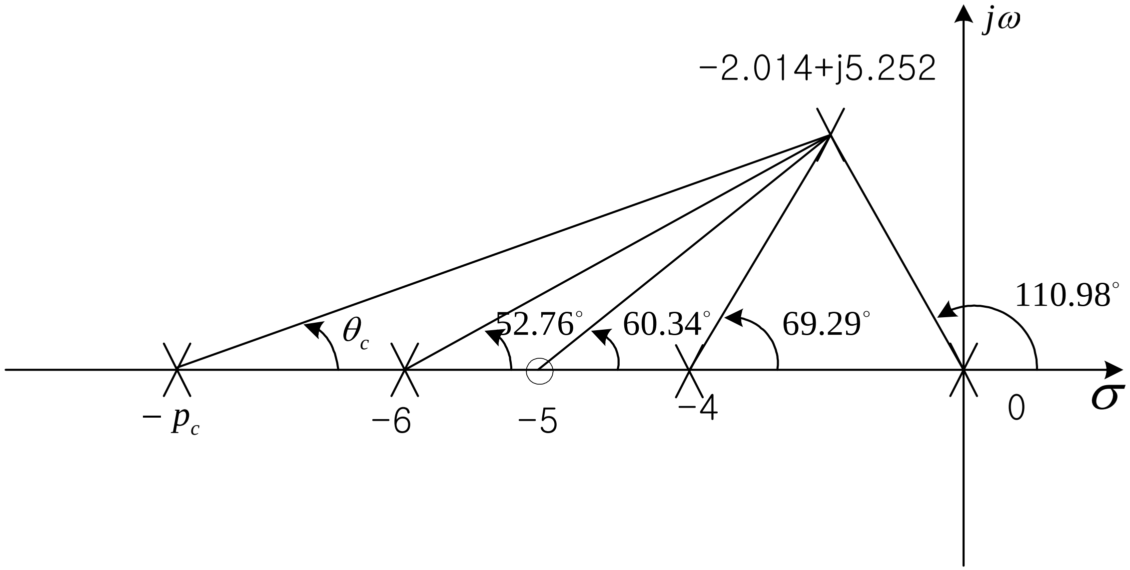
Uncompensated settling time $T_{so} = \dfrac{4}{\zeta w_n} = \dfrac{4}{1.007} = 3.792$ seconds

New compensated settling time $T_{sN} = \dfrac{3.972}{2} = 1.986$

$$-\zeta w_n = \frac{-4}{T_{sN}} = \frac{-4}{1.986} = -2.014$$

$$w_d = -2.014\tan(110.98^\circ) = 5.252$$

Arbitrarily select a compensator zero at -5 on the real axis



$$\sum zero\,angles - \sum pole\ angles = (2k+1)\times 180^\circ$$

$$60.34^\circ - (52.76^\circ + 69.29^\circ + 110.98^\circ) - \theta_c = -180^\circ\,(or\,180^\circ)$$

$$\therefore\ \theta_c = 7.31^\circ$$

$$\frac{5.252}{p_c - 2.014} = \tan 7.31^\circ \quad \rightarrow \quad p_c = 42.96$$

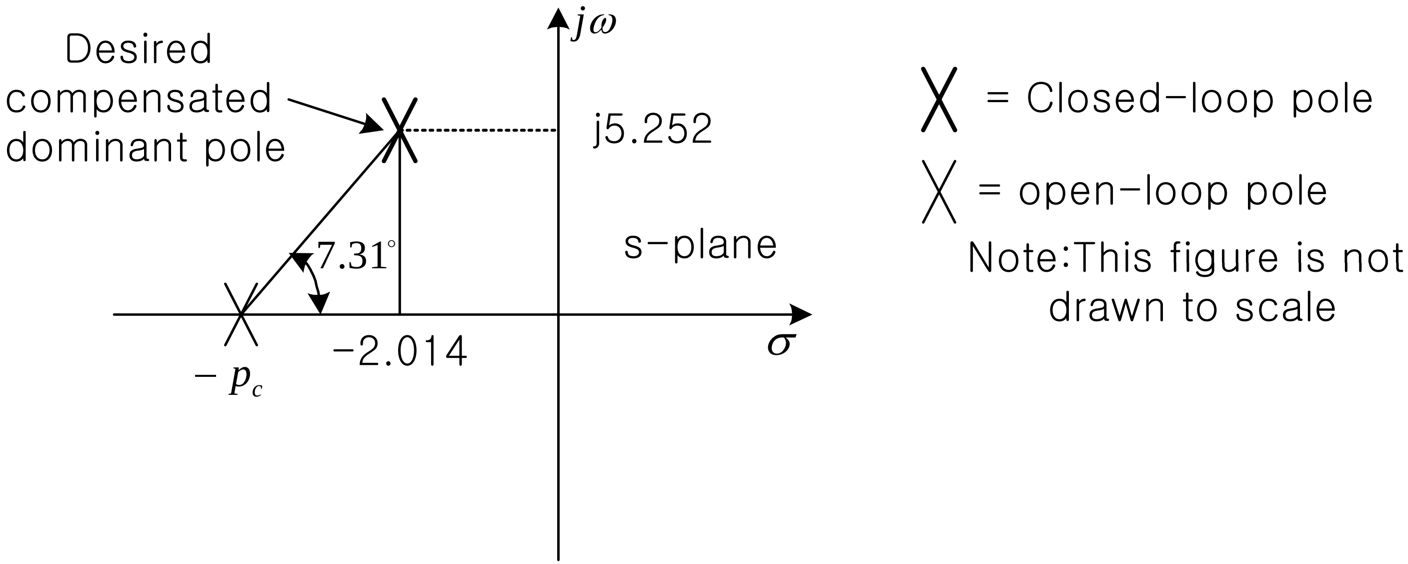


Figure 9.27

s - plane picture used to calculate the location of the compensator pole for Example 9.4

Table 9.4 Comparison of lead compensation designs for Example 9.4

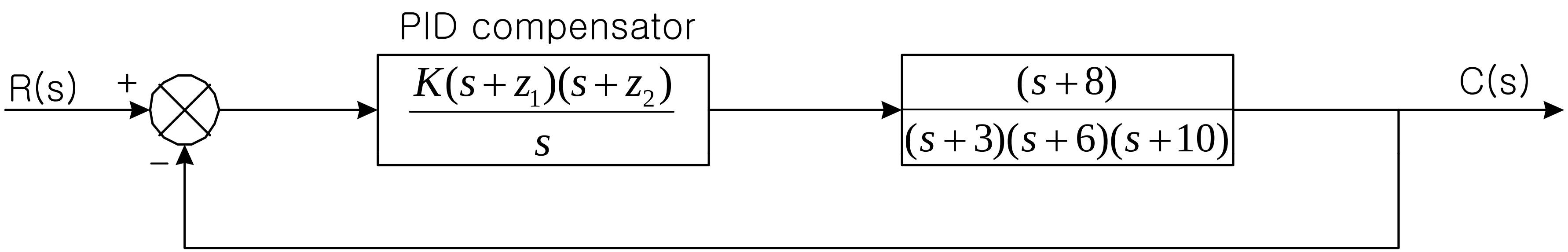
	Uncompensated	compensation a	compensation b	compensation c
plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
w_n	2.813	5.625	5.625	5.625
%OS	30(28)	30(30.7)	30(28.2)	30(14.5)
T_s	3.972(4)	1.986(2)	1.986(2)	1.986(1.7)
T_p	1.196(1.3)	0.598(0.6)	0.598(0.6)	0.598(0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
other poles	−7.986	−43.8, −5.134	−22.06	−13.3, −1.642
zero	None	−5	None	−2
comments	second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

* Improving Steady - State Error and Transient Response with PID controllers

- Design Procedures

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the PD controller to meet the transient response specifications. The design includes the zero location and loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Design the PI controller to yield the required steady - state error.
6. Determine the gains, $K_1, K_2,\ and\ K_3$ in Figure 9.30.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if simulation shows that requirements have not been met.

(Problem) Design a PID controller so that the system can operate with a peak time that is two-thirds that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input.



(solution)

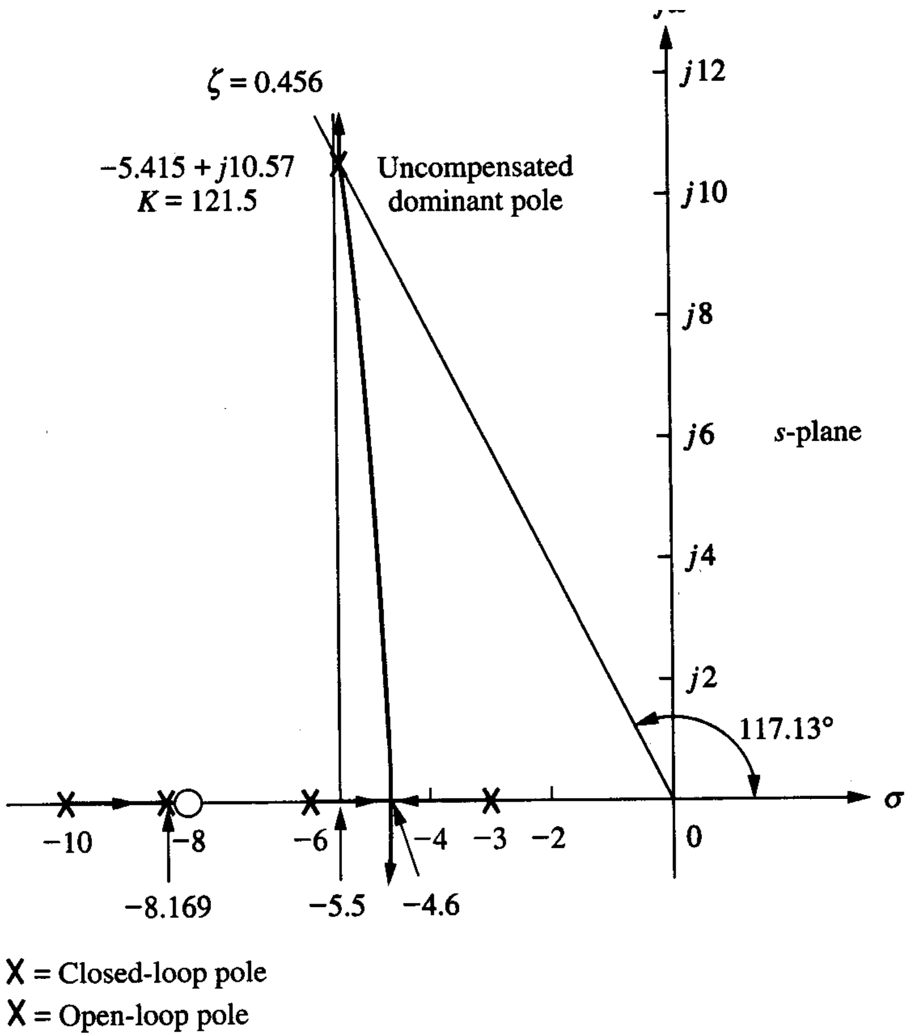


Figure 9.32

Root locus for the uncompensated system of Example 9.5

(step1) 20% overshoot $\Rightarrow \zeta = 0.456$

Dominant poles at s = -5.415 $\pm$ j10.57(K = 121.5)

The third pole at s = -8.169

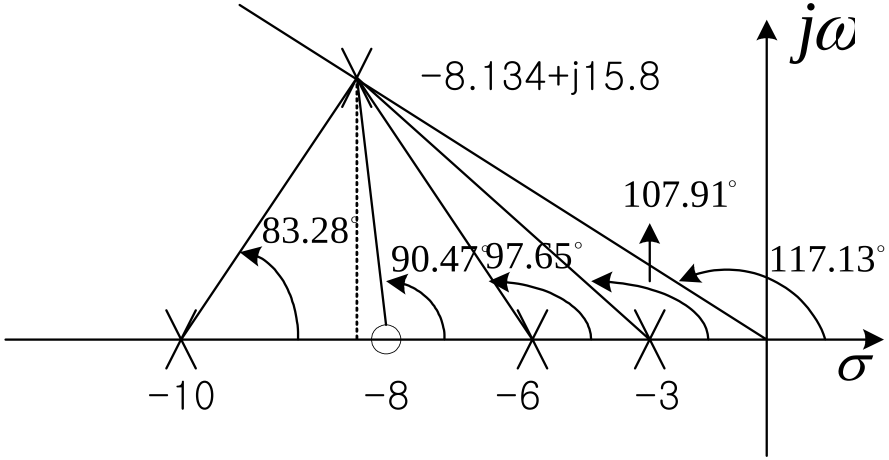
$$T_{po} = \frac{\pi}{\omega_d} = \frac{\pi}{10.57} = 0.297 \text{ seconds}$$

(step2) since $T_{pN} = \frac{2}{3}\ T_{po} = \frac{2}{3} \times (0.297) = 0.198$

The imaginary part of the compensated dominant pole

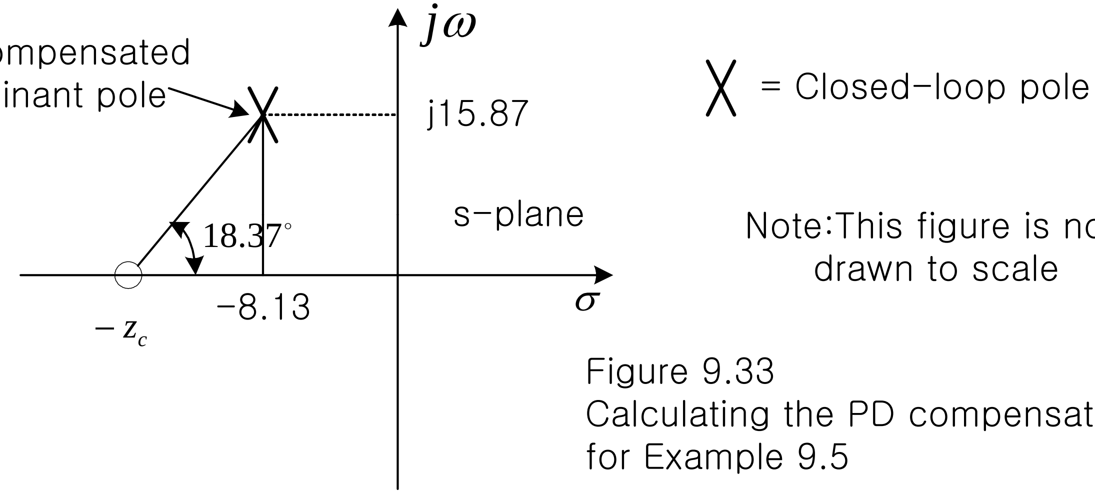
$$\omega_d = \frac{\pi}{T_{pN}} = \frac{\pi}{0.198} = 15.87$$

The real part of the compensated dominant pole is $\sigma = \dfrac{w_d}{\tan(117.13^\circ)} = -8.13$



$$90.47^\circ - (83.28^\circ + 97.65^\circ + 107.91^\circ) = -198.37^\circ$$

$$-198.37^\circ + \theta_z = -180^\circ \rightarrow \theta_z = 18.37^\circ$$



Note:This figure is not drawn to scale

Figure 9.33
Calculating the PD compensator zero for Example 9.5

$$\frac{15.87}{z_c - 8.13} = \tan 18.37^\circ = 0.3321 \rightarrow \; z_c = 55.92$$

Then, The PD controller is

$$G_{PD}(s) = s + z_c = s + 55.92$$

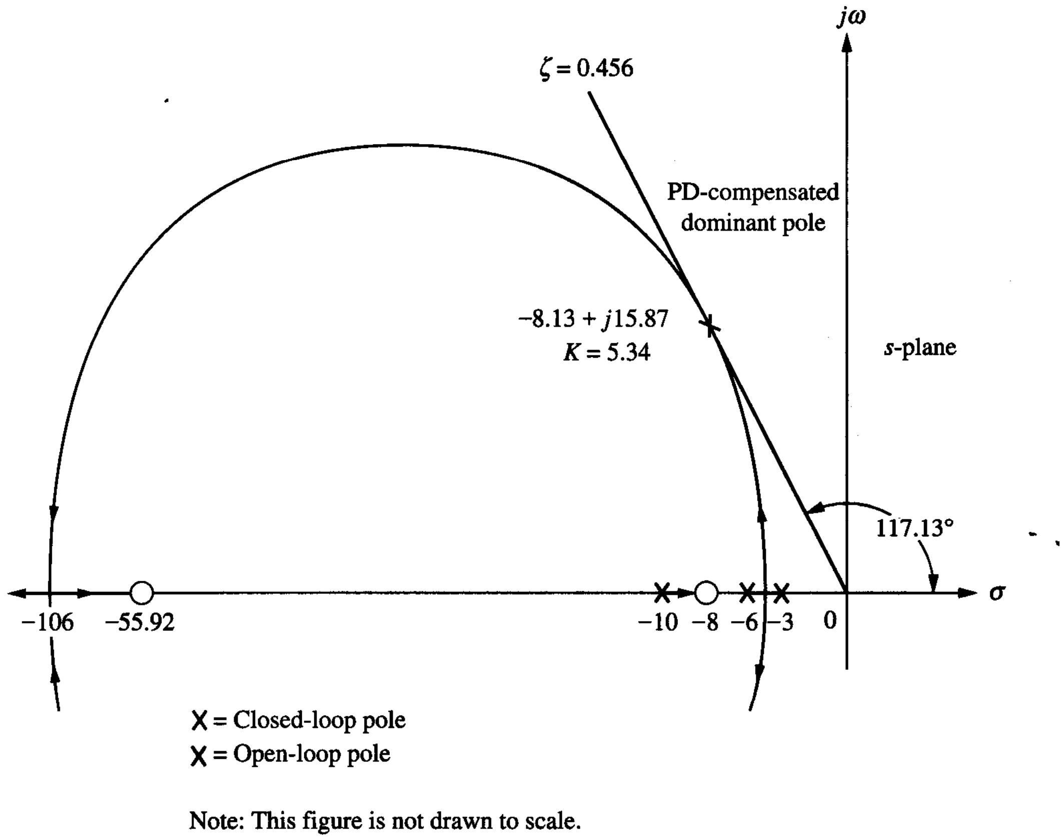


Figure 9.34

Root locus for PD - compensated system of Example 9.5

(step 3 and 4) The simulation is in Fig. 9.35

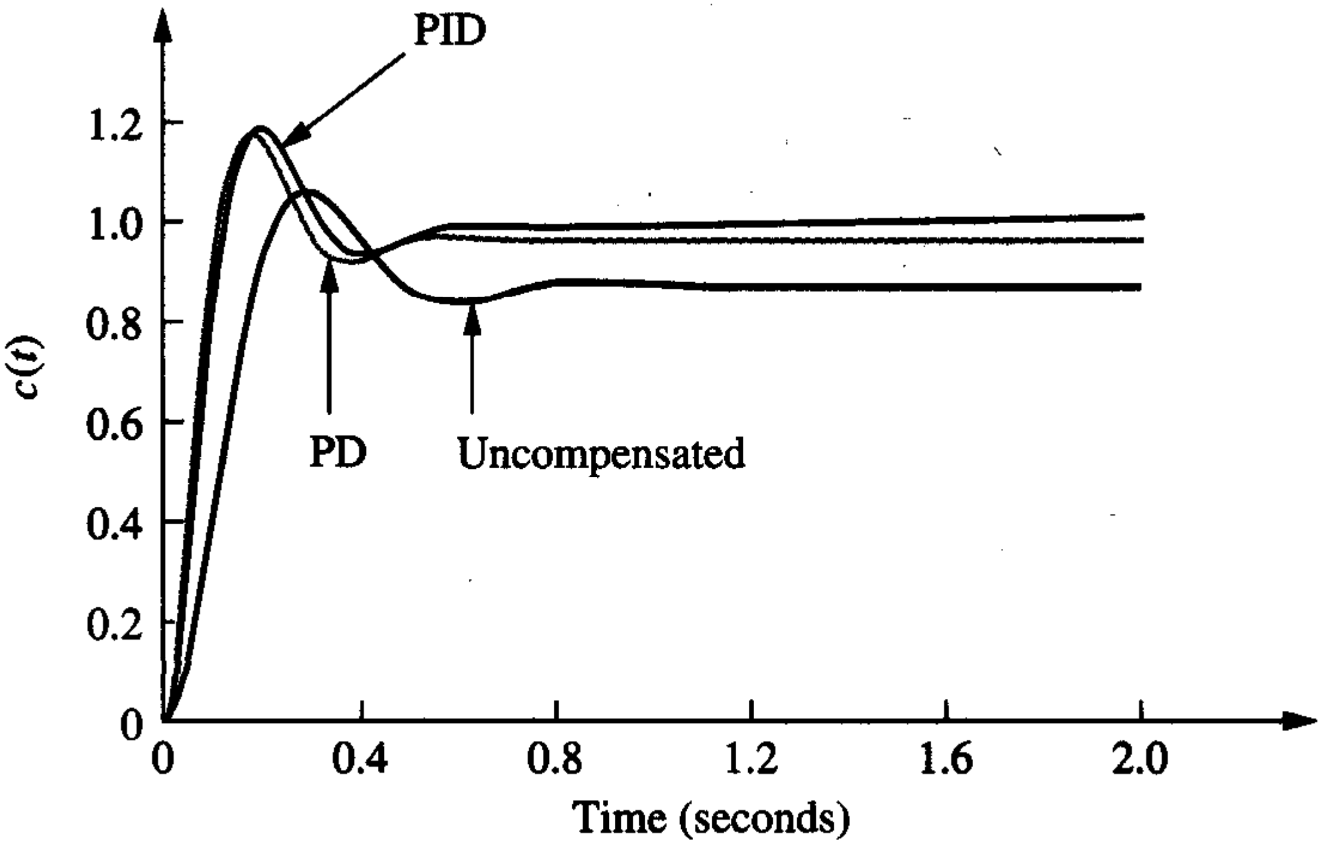


Figure 9.35

Step response for uncompensated, PD - compensated, and PID - compensated systems of Example 9.5

(step 5) Design a PI controller with placing the zero closer to the origin arbitrarily

$$G_{PI}(s) = \frac{s+0.5}{s}$$

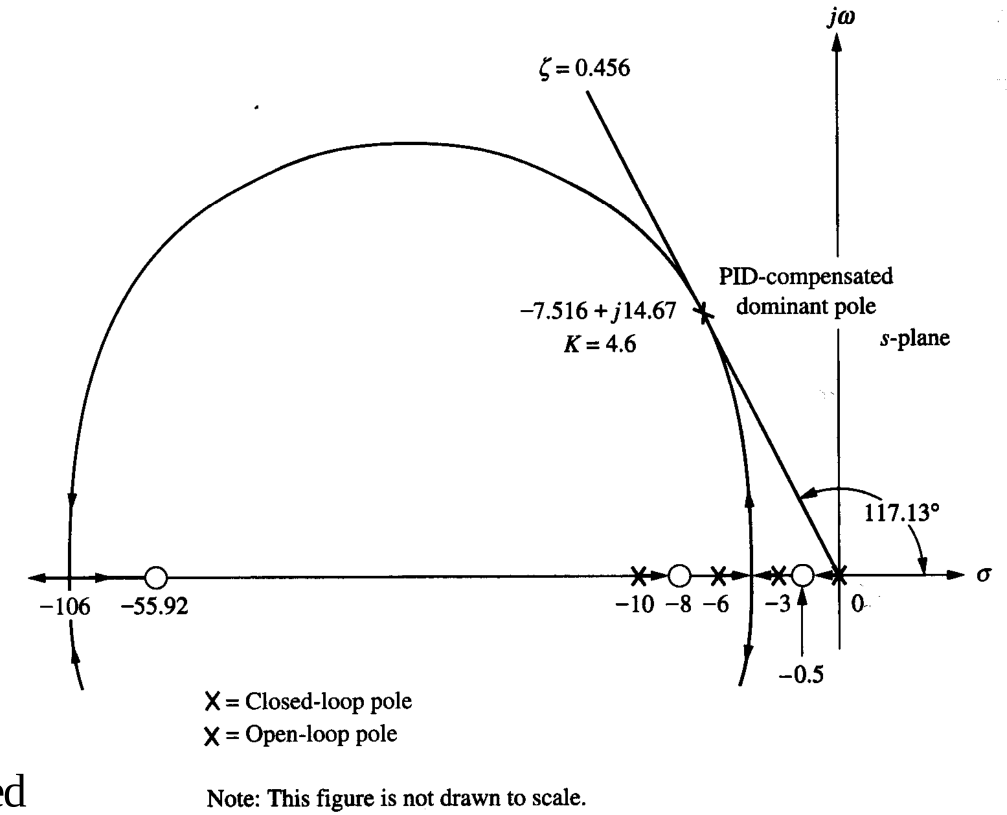


Figure 9.36
Root locus for PID-compensated system of Example 9.5

The dominant, second order poles are at $s = -7.516 \pm j14.67$ with $K = 4.6$

(step 6) Determine the gains K_1, K_2 *and* K_3.

$$G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s+55.92)(s+0.5)}{s}$$

$$= \frac{4.6(s^2+56.42s+27.96)}{s} \quad \text{..} \ (1)$$

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s} \ ..(2)$$

Matching (1) and (2) yields

$$K_1 = 259.5, \quad K_2 = 128.6, \ \ and \ \ K_3 = 4.6$$

(step 7 and 8) The complete PID controller improved the steady - state error without appreciably changing the transient response designed with the PD controller.

* Improving Steady-State Error and Transient Response with Lag-Lead compensator Design

- Design Procedure

1. Evaluate the performance of the uncompensated system to determine how much improvements in transient response is required.
2. Design the lead compensator to meet the transient response specifications. The design includes the zero location, pole location, and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Evaluate the steady-state error performance for the lead-compensated system to determine how much more improvements in steady-state error is required.
6. Design the lag compensator to yield the required steady-state error.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met.

(problem) see Example 9.6

This is very similar to the PID controller design

Table 9.7 Types of cascade compensators

Function	compensator	Transfer function	characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	1. Increase system type. 2. Error becomes zero. 3. Zero at $-z_c$ is small and negative. 4. Active circuits required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	1. Error improved but not driven to zero. 2. Pole at $-p_c$ is small and negative. 3. Zero at $-z_c$ is close and to the left of pole at $-p_c$ 4. Passive circuits required to implement.
Improve transient response	PD	$K(s+z_c)$	1. Zero at $-z_c$ is selected to put design point on root locus 2. Active circuits required to implement. 3. Can cause noise and saturation; implement with rate feedback or with a pole(lead)
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus 2. pole at $-p_c$ is more negative than zero at $-z_c$ 3. Passive circuits required to implement.

Table 9.7 Types of cascade compensators

Function	compensator	Transfer function	characteristics
Improve steady-state error and transient response	PID	$K\frac{(s+z_{lag})(s+z_{lead})}{s}$	1. Lag zero at $-z_{lag}$ and pole at origin improve steady-state error 2. Lead zero at $-z_{lead}$ improves transient response. 3. Lag zero at $-z_{lag}$ is close and to the left of the origin. 4. Lead zero at $-z_{lead}$ is selected to put design point on root locus. 5. Active circuits required to implements. 6. Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-Lead	$K\frac{(s+z_{lag})(s+z_{lead})}{(s+p_{lag})(s+p_{lead})}$	1. Lag pole at $-p_{lag}$ and lag zero at $-z_{lag}$ are used to improve steady-state error. 2. Lead pole at $-p_{lead}$ and lead zero at $-z_{lead}$ are used to improve transient response. 3. Lag pole at $-p_{lag}$ is small and negative. 4. Lag zero at $-z_{lag}$ is close and to the left of lag pole at $-p_{lag}$ 5. Lead zero at $-z_{lead}$ and lead pole at $-p_{lead}$ are selected to put design point on root locus. 6. Lead pole at $-p_{lead}$ is more negative than lead zero at $-z_{lead}$ 7. Passive circuits required to implements.

* Feedback Compensation

  See Rate Feedback.

* Physical Realization of compensation

  • Active-circuit Realization

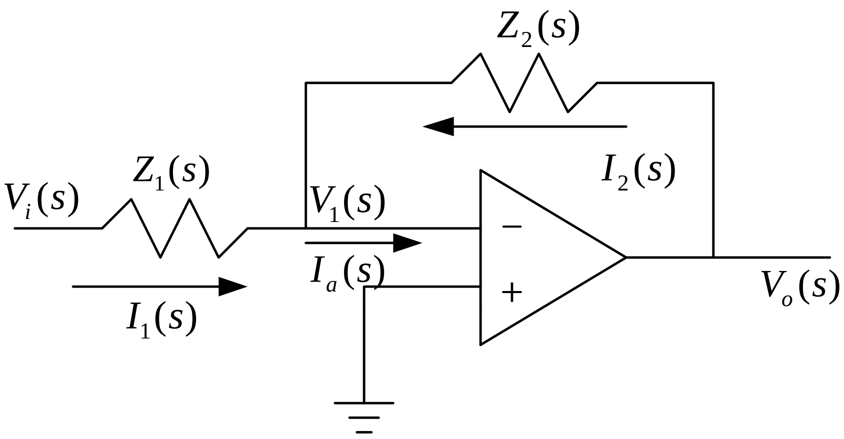


Figure 9.60
Operational amplifier configured for transfer function realization

$$\frac{V_o(s)}{V_i(s)} = -\frac{z_2(s)}{z_1(s)}$$

Table 9.10 Active realization of controllers and compensators, using an operational amplifier

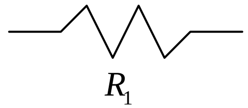
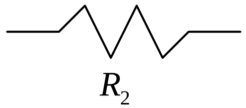
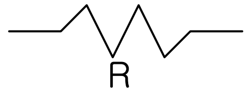
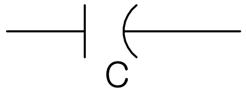
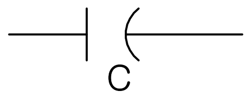
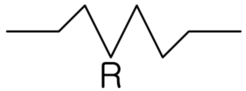
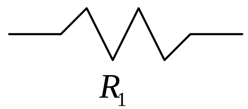
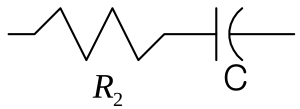
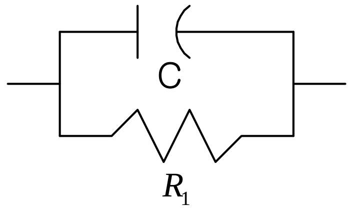
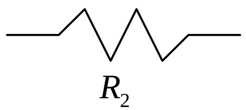
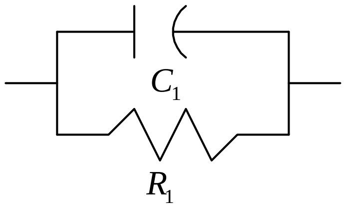
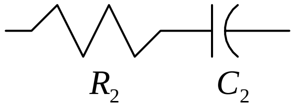
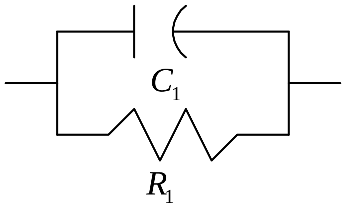
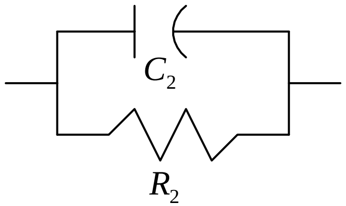
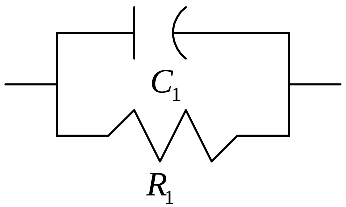
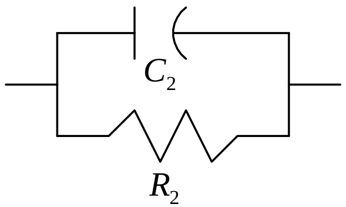
Function	$Z_1(s)$	$Z_2(s)$	$G_c(s)=\frac{Z_2(s)}{Z_1(s)}$
Gain	 R_1	 R_2	$-\frac{R_2}{R_1}$
Integration	 R	 C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	 C	 R	−RCs
PI controller	 R_1	 R_2 C	$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	 C R_1	 R_2	$-R_2C\left(s+\frac{1}{R_1C}\right)$

Table 9.10 Active realization of controllers and compensators, using an operational amplifier

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s)=\frac{Z_2(s)}{Z_1(s)}$
PID controller			$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$
Lag compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ *where* $R_2C_2>R_1C_1$
Lead compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ *where* $R_1C_1>R_2C_2$

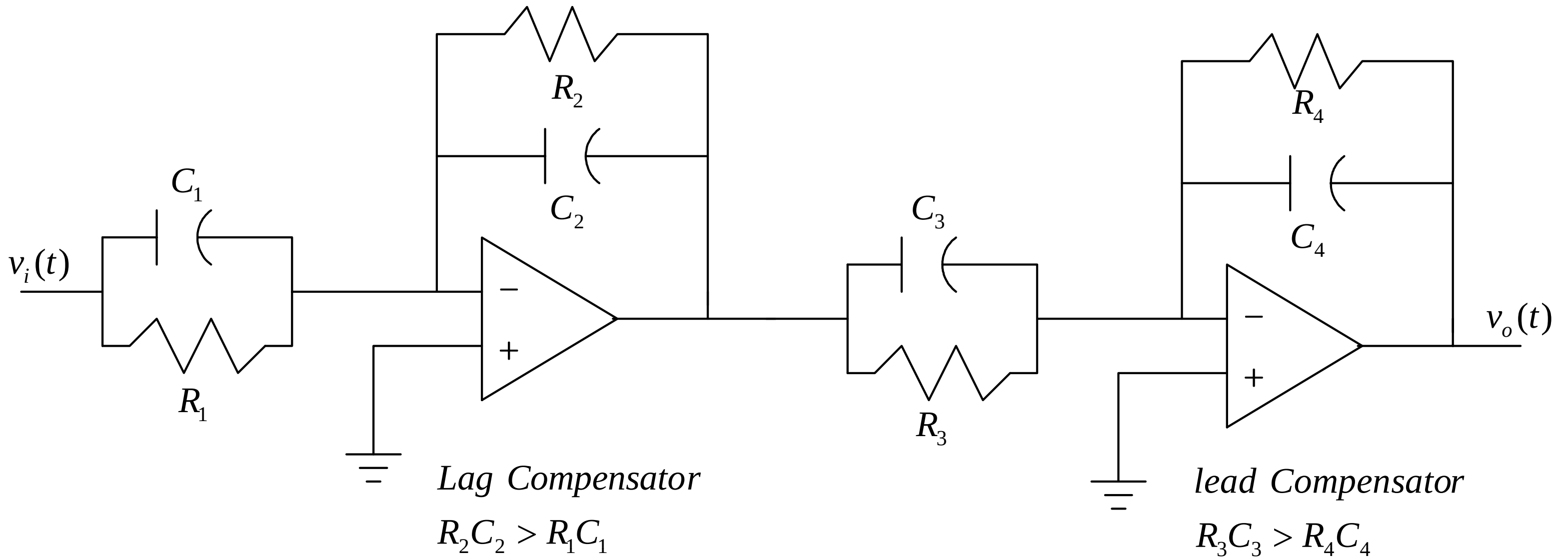


Figure 9.61
Lag-Lead compensator implemented with
operational amplifiers

* Implementing a PID controller

(Problem) Implement the PID controller

$$G_c(s) = \frac{(s+55.92)(s+0.5)}{s}$$

$$G_c(s) = \frac{s^2 + 56.42s + 27.96}{s} = s + 56.42 + \frac{27.96}{s} \text{.......(1)}$$

Comparing the PID controller in Table 9.10 with Eq(1) yields

$$R_2C_1 = 1, \quad \frac{R_2}{R_1} + \frac{C_1}{C_2} = 56.42, \quad \frac{1}{R_2C_2} = 27.96$$

Arbitrarily select $C_2 = 0.1uF$. The remaining value are $R_1 = 357.65k\Omega$, $R_2 = 178{,}891k\Omega \ and \ C_1 = 5.59uF$

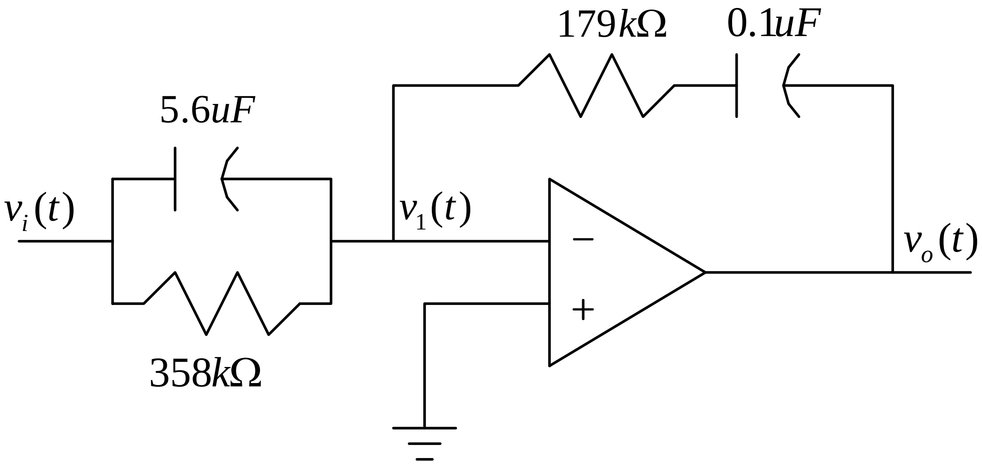


Figure 9.62
PID controller

$*$ Passive - circuit Realization similar to the active - circuit Realization

(Problem) see Example 9.10