Chapter 8. Root Locus Techniques

Root Locus

The closed-loop transfer function of a control system can be written as

$$T(s) = \frac{N(s)}{Q(s)}$$

The denominator polynomial Q(s) is the characteristic equation and can be written, in the form

$$Q(s) = 1 + F(s)$$

For a single-loop feedback system, $F(s) = G(s)H(s)$

It is the roots of Q(s) which determine the stability and transient response of the control system.

So, we look at

$$1 + F(s) = 0 \qquad \text{or} \qquad F(s) = -1$$

Thus, we have necessarily

$$|F(s)| = 1 \quad , \quad \angle F(s) = (1+2k)\times 180^\circ \ , \ k = 0, \pm 1, \pm 2, \cdots$$

In general, F(s) can be written in the form

$$\mathrm{F(s)} = \frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_{nz})}{(s+P_1)(s+P_2)\cdots(s+P_{np})}$$

The magnitude and angle requirements are then written as

$$\left|F(s)\right| = \frac{K\left|s+Z_1\right|\left|s+Z_2\right|\cdots\left|s+Z_{nz}\right|}{\left|s+P_1\right|\left|s+P_2\right|\cdots\left|s+P_{np}\right|} = 1$$

and

$$\angle F(s) = \angle(s+Z_1) + \angle(s+Z_2) + \cdots - (\angle(s+P_1) + \angle(s+P_2) + \cdots)$$

$$= (2k+1)\times 180^\circ, \quad k = 0, \pm 1, \pm 2, \cdots$$

After writing the characteristic equation

$$1 + \mathrm{F(s)} = 0,$$

We rearrange it such that the particular parameter of interest K appears in the form

$$1 + \mathrm{KP(s)} = 0$$

Then we factor P(s) to obtain the form

$$1 + K\frac{\Pi_{i=1}^{N_z}(s + Z_i)}{\Pi_{j=1}^{N_p}(s + P_j)} = 0$$

The root locus will show us the location of the poles of the characteristic equation as K varies from 0 to ∞

Root loci - for $0 \le K < \infty$

Complementary root loci - for $-\infty < K \le 0$

Complete root loci = Root loci + Complementary root loci

* Rules for Developing a Root Locus Plot

Rule 1. The Starting Points of the root loci (K = 0)

The root loci start at the poles (finite and infinite) of P(s)

$$P(s) = \frac{-1}{K} = \frac{\Pi_{i=1}^{N_z}(s+Z_i)}{\Pi_{j=1}^{N_p}(s+P_j)}$$

Rule 2. The Ending Points of the root loci ($K = \infty$)

The root loci end at the zeros (finite and infinite) of P(s)

Remark > The magnitude condition implies

$$\frac{\Pi_{i=1}^{N_z}\left|s+Z_i\right|}{\Pi_{j=1}^{N_p}\left|s+P_j\right|} = \frac{1}{K} \rightarrow 0 \quad as \quad K \rightarrow \infty$$

Note that if $N_z < N_p$, then there are $N_p - N_z$ value of s with magnitudes equal to infinity s.t. P(s) = 0

Example

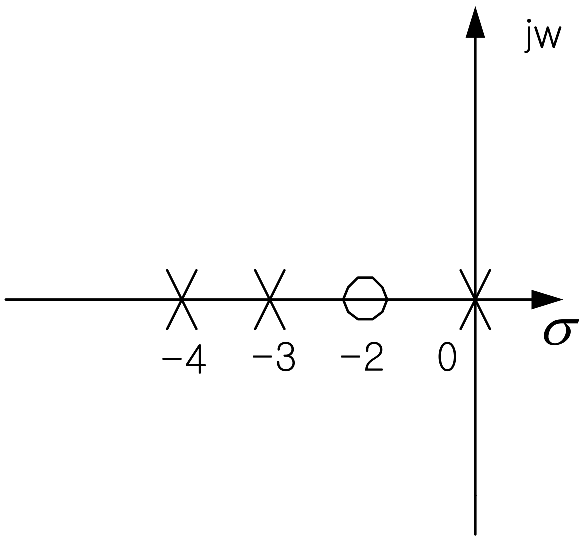
$$s(s+3)(s+4)+K(s+2)=0$$

$$if \quad K=0$$

$$roots \ are \ s=0,-3, \ and \ -4$$

$$1+G(s)H(s)=1+\frac{K(s+2)}{s(s+3)(s+4)}=0$$

$$G(s)H(s)=\frac{K(s+2)}{s(s+3)(s+4)}$$



Rules 3. Number of separate Branches of conventional Root Locus

If N is the number of separate branches of conventional Root Locus, then

$$N = N_p \quad if \quad N_p > N_z$$

$$N = N_z \quad if \quad N_z > N_p$$

Rules 4. Symmetry of the Root Loci

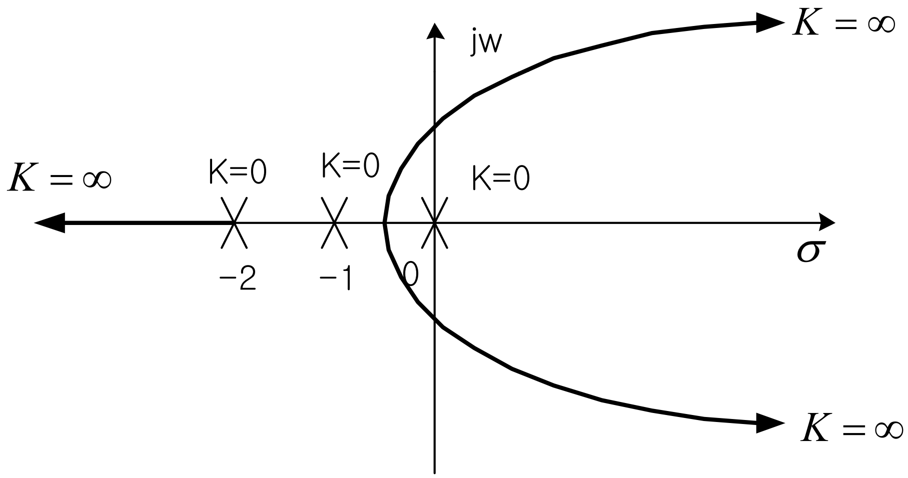
The branches of the conventional root locus must be symmetrical with respect to the horizontal real axis.

Since all the coefficients of P(s) are real, the roots of P(s) must be real or conjugate pairs.

Ex)

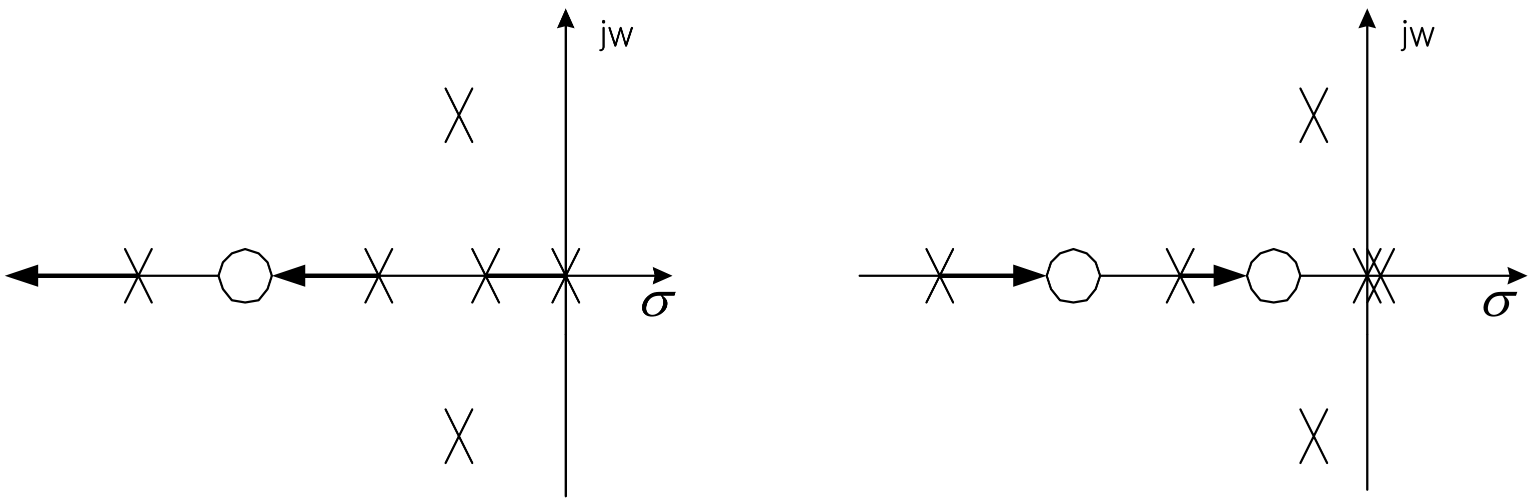
$$s(s+1)(s+2) + K = 0$$

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$



Rules 5. Root loci on the Real Axis.

The branches of the conventional root locus can exist at a point on the real axis if and only if the total number of poles and zeros of P(s) to the right of the point is odd.



Rule 6. Asymptotes of the root loci

If $N_z < N_p$, then $\mathrm{N} = N_p - N_z$ branches of the conventional root locus must terminate ($\mathrm{K} \to \infty$) at the zero of P(s) that lie on the infinite circle. These branches are asymptotic to $N_p - N_z$ straight lines that intersect the real axis at

$$\sigma_{\mathrm{A}} = \frac{\sum_{j=1}^{N_p}(-P_j) - \sum_{i=1}^{N_z}(-Z_i)}{N_p - N_z}$$

and form angles (measured ccw) with respect to the positive real axis that are given by

$$\mathrm{k} > 0 \qquad \Phi_A = \frac{(2k+1)\times 180^\circ}{N_p - N_z}, \qquad \mathrm{k} = 0,1,2,\cdots,N_p - N_z - 1$$

$$\mathrm{k} \le 0 \qquad \theta_A = \frac{2k\pi}{N_p - N_z}, \qquad \mathrm{k} = 0,1,2,\cdots,N_p - N_z - 1$$

Ex) Given the characteristic equation

$$s(s+4)(s^2+2s+2)+K(s+1)=0$$

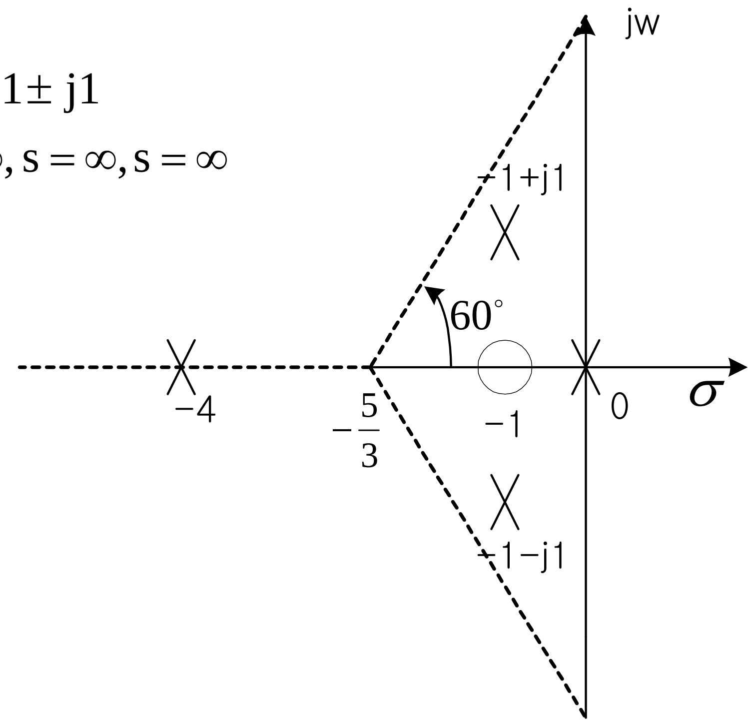
$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+2)}=0$$

1. $\mathrm{K}=0$: poles $\mathrm{s}=0,-4,\ \mathrm{s}=-1\pm \mathrm{j}1$
2. $\mathrm{K}=\pm\infty$: zeros $\mathrm{s}=-1, \mathrm{s}=\infty, \mathrm{s}=\infty, \mathrm{s}=\infty$
3. 4 complement root loci
4. Root loci $(\mathrm{K}\geq 0)$

$$K=0 \qquad \theta_0=\frac{180^\circ}{3}=60^\circ$$

$$K=1 \qquad \theta_1=\frac{540^\circ}{3}=180^\circ$$

$$K=2 \qquad \theta_2=\frac{900^\circ}{3}=300^\circ$$



complementary root loci $(\mathrm{K}<0)$

$$K=0 \qquad \theta_0=\frac{0^\circ}{3}=0^\circ$$

$$K=1 \qquad \theta_1=\frac{360^\circ}{3}=120^\circ$$

$$K=2 \qquad \theta_2=\frac{720^\circ}{3}=240^\circ$$

5. $$\sigma_1=\frac{(0-4-1-j1-1-j1)-(-1)}{4-1}=-\frac{5}{3}$$

Rule 7. Intersection of the branches of the Root Locus with Imaginary Axis.

The root loci cross the imaginary axis at $s_n = \pm j\omega_n$, where S_n are the roots of the auxiliary equation formed when a complete row of zeros appears in the Routh-Hurwitz array.

Example.

Suppose $1 + G(s)H(s) = 1 + \dfrac{K}{s(s+2)(s+3)} = 0$

Rewrite to get

$$\frac{s(s+2)(s+3)+K}{s(s+2)(s+3)} = 0 \quad or \quad s^3 + 5s^2 + 6s + K = 0$$

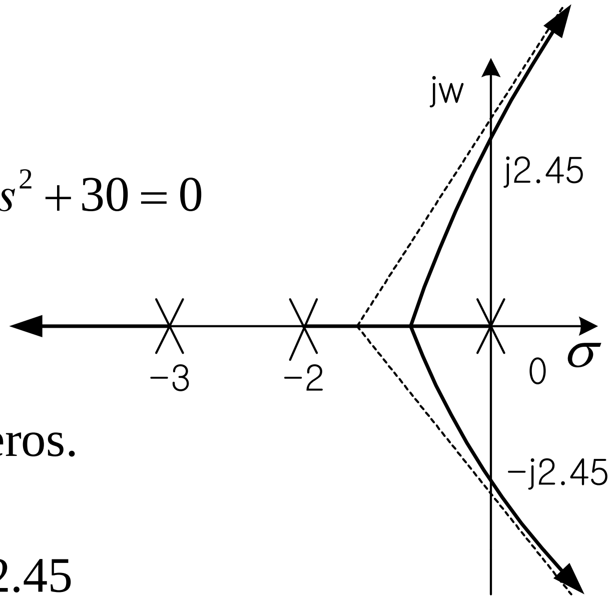
The Routh array is

$$\begin{array}{lll} s^3 & 1 & 6 \\ s^2 & 5 & K \leftarrow Auxiliary \;\; P(s) = 5s^2 + 30 = 0 \\ s^1 & \frac{30-K}{5} \leftarrow If \;\; K = 30 \; then \; 0 & \\ s^0 & K & \end{array}$$

If $K = 30$, we have a complete row of zeros.

The auxiliary polynomial is

$$5s^2 + 30 = 0 \;\; \rightarrow \;\; s^2 = -6 \;\; \rightarrow s = \pm j2.45$$



Rule 8. Angles of Departure from poles and Arrival to Zeros.

The angle of departure of the root locus from a pole (or the angle of arrival at a zero) of P(s) can be determined by assuming a point s_1 which is very close to the pole (or zero) and which is on the branch associated with the pole (or zero). Then apply

$$\angle P(s) = \sum_{i=1}^{N_z} \angle(s_1 + Z_i) - \sum_{j=1}^{N_p} \angle(s_1 + P_j) = (2k+1)\times 180^\circ$$

Example

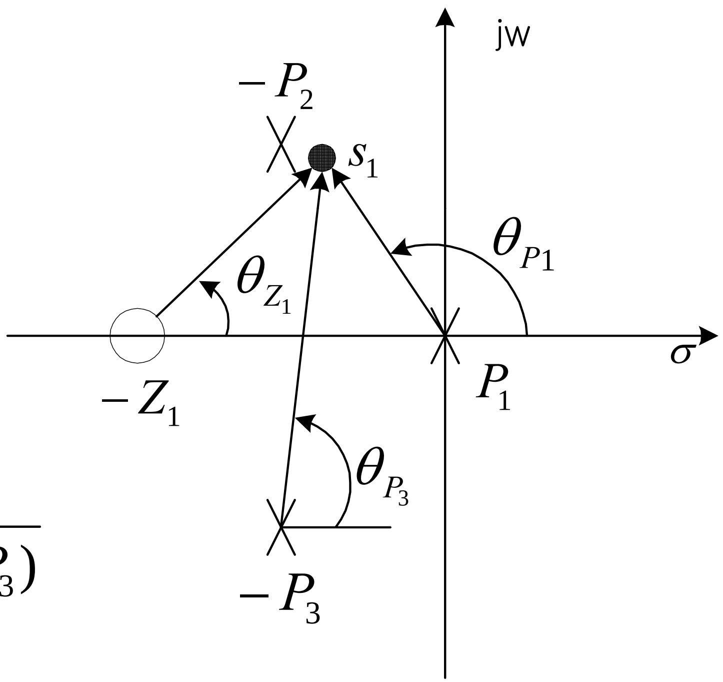
<Ref>

$$1 + KG(s)H(s) = 0$$

$$1 + KP(s) = 0$$

$$P(s) = \frac{-1}{K} = \frac{\Pi_{i=1}^{N_Z}(s+Z_i)}{\Pi_{j=1}^{N_P}(s+P_j)}$$

$$G(s)H(s) = \frac{K(s+Z_1)}{s(s+P_2)(s+P_3)}$$



Rule 9. Breakaway Points

All breakaway point on the root loci can be determined by finding the roots of $\frac{dK}{ds} = 0$, where K = f(s) is the closed-loop characteristic equation.

Breakpoints occur either on the real axis or in complex-conjugate pairs

Example

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

characteristic equation is :

$$1 + G(s)H(s) = 1 + \frac{K}{s(s+4)(s^2+4s+20)} = 0$$

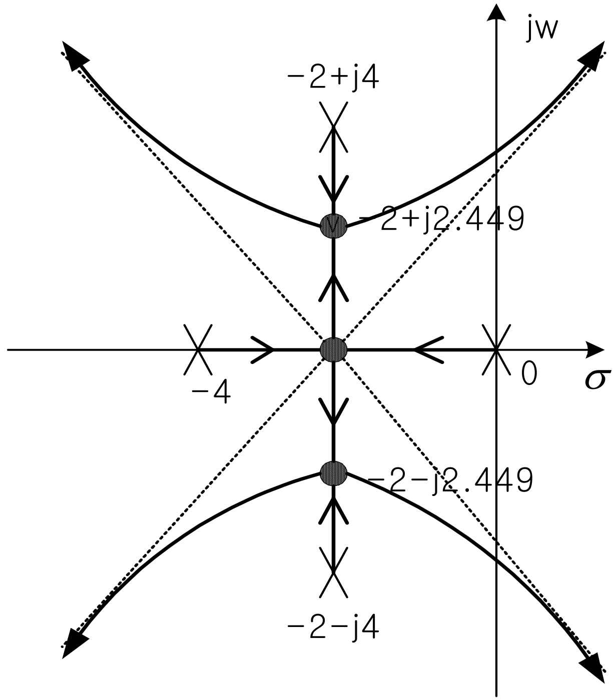
or

$$K = -s(s+4)(s^2+4s+20) = -s^4 - 8s^3 - 36s^2 - 80s$$

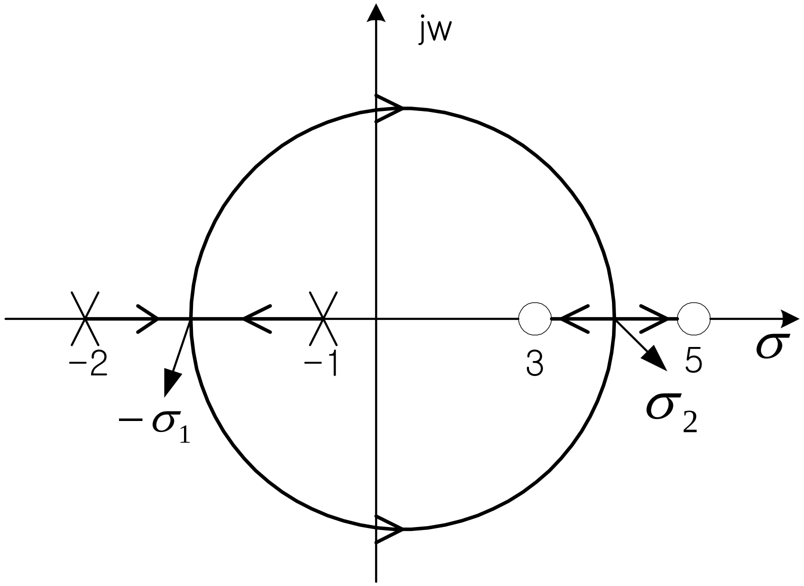
$$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80 = 0$$

solving, find the breakaway point to be

$$s_{1,2,3} = -2, -2 \pm j2.449$$



* Real axis Breakaway (BA) and Break-in (BI) points



$-\sigma_1$: breakaway point

σ_2 : breakin point

1. Via Differentiation

$$KG(s)H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)} = \frac{K(s^2-8s+15)}{s^2+3s+2}$$

since KG(s)H(s) = -1 and s = σ on the real axis

$$\frac{K(\sigma^2-8\sigma+15)}{\sigma^2+3\sigma+2} = -1$$

Sloving for K

$$K = \frac{-(\sigma^2+3\sigma+2)}{(\sigma^2-8\sigma+15)}$$

$$\frac{dK}{d\sigma} = \frac{11\sigma^2-26\sigma-61}{(\sigma^2-8\sigma+15)^2} = 0 \rightarrow \sigma = -1.45 \ and \ \sigma = 3.82$$

BA BI

2. Without Differentiation relationship

$$\sum_{i=1}^{m} \frac{1}{\sigma + Z_i} = \sum_{j=1}^{n} \frac{1}{\sigma + P_j}$$

where Z_i and P_j are the negative of the zero and pole values, respectively, of G(s)H(s)

$$\frac{1}{\sigma - 3} + \frac{1}{\sigma - 5} = \frac{1}{\sigma + 1} + \frac{1}{\sigma + 2}$$

$$\frac{2\sigma - 8}{\sigma^2 - 8\sigma + 15} = \frac{2\sigma + 3}{\sigma^2 + 3\sigma + 2}$$

$$11\sigma^2 - 26\sigma - 61 = 0$$

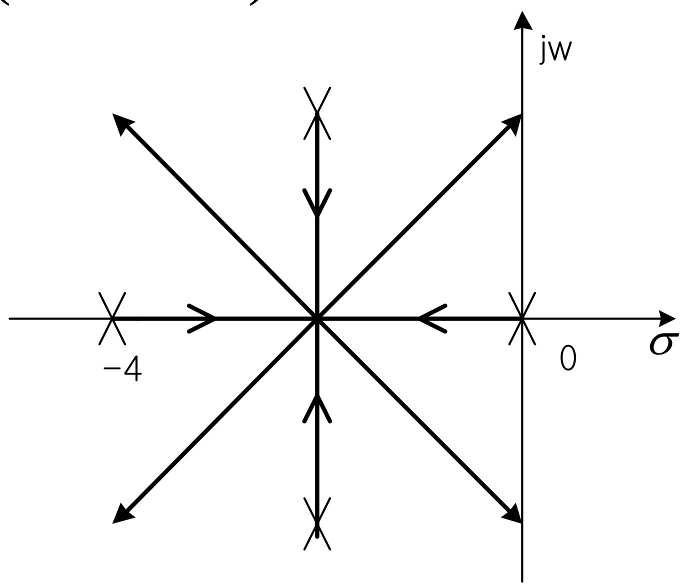
$$\sigma = -1.45\ (BA) \quad and \quad \sigma = 3.82\ \ (BI)$$

$\rightarrow$ The same result.

Rule 10. Angle of Departure and Arrival at Breakaway Points

Straight-line tangents to the branches of the conventional root locus at the breakaway points divide the surrounding region into pie-shaped areas with internal angles equal to $\dfrac{180^\circ}{m}$ where m = number of distance arriving branches. The arrival and departing branches alternate.

$$Ex)\ \ 1+G(s)=1+\frac{K}{s(s+4)(s^2+4s+8)}=0$$



Rule 11. Angle of Departure and angles of arrival of RL

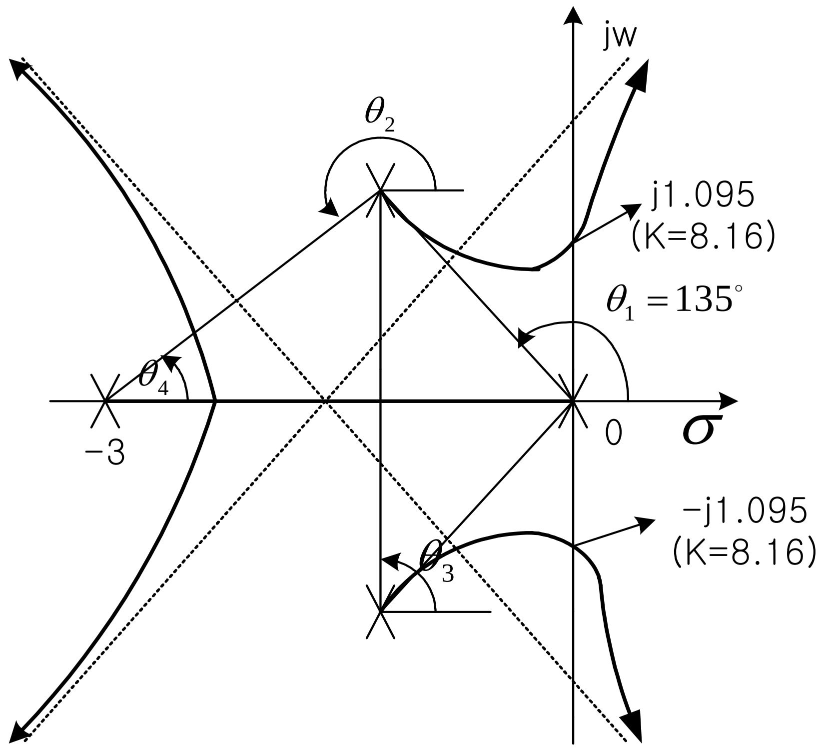
The angle of departure or arrival of a RL at a pole or zero, respectively, of P(s) denotes the angle of the tangent to the locus near the point.

$$KG(s)H(s) = \frac{K}{s(s+3)(s^2+2s+2)}$$

$$pole\ at\ s = 0, -3, -1 \pm j1$$

$$\begin{aligned}\angle(KG(s_1)H(s_1)) &= -(\theta_1+\theta_2+\theta_3+\theta_4) \\ &= (2i+1)\times 180^\circ \\ &= -(135^\circ+\theta_2+90^\circ+26.6^\circ) \\ &= (2i+1)\times 180^\circ\end{aligned}$$

$$\theta_2 = -71.6^\circ \quad or \quad \theta_2 = 288.4^\circ$$



Rule 12. Calculation of K on the Root Loci

The value of K at any point s_1 on the root loci is determined graphically from the equation.

$$\mathrm{K} = \frac{1}{\mathrm{P(s)}} = \frac{\text{product of all vector lengths from the poles of P(s) to } s_1}{\text{product of all vector lengths from the zeros of P(s) to } s_1}$$

$$= \frac{\Pi\ poles\ lengths}{\Pi\ zeros\ lengths}$$

$Ex)\ \ s^2 + 2s + 2 + K(s+2) = 0$

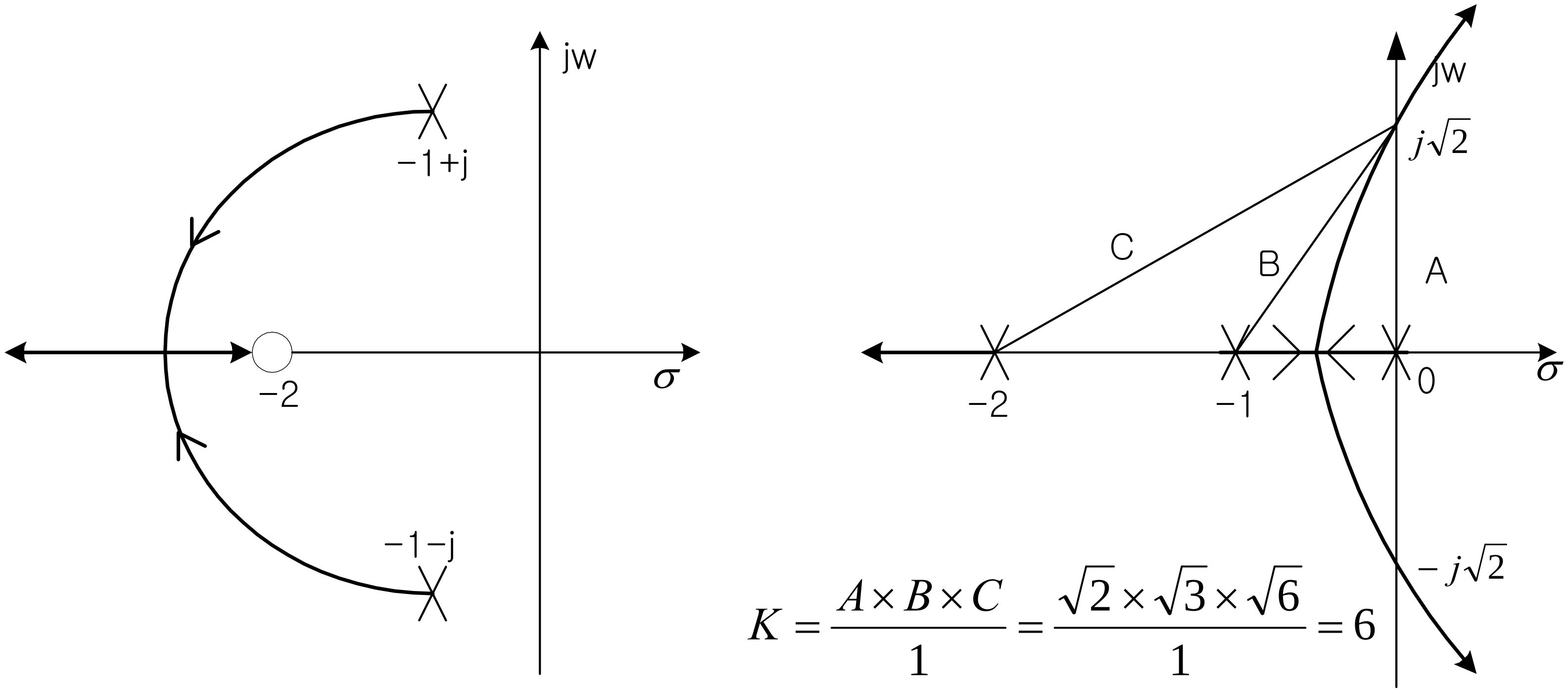
$$1 + \frac{K(s+2)}{(s^2+2s+2)} = 0$$

If there are no zeros, then denominator is 1

$Ex)\ s(s+1)(s+2) + K = 0\ \ or\ \ G(s)H(s) = \dfrac{K}{s(s+1)(s+2)}$

$$s^3 + 3s^2 + 2s + K = 0$$

s^3	1	2	$If\ \ K = 6$
s^2	3	K	$3s^2 + 6 = 0$
s^1	$\dfrac{6-K}{3}$	0	$s^2 = -2\ \ or\ \ s = \pm j\sqrt{2}$



$$K=\frac{A\times B\times C}{1}=\frac{\sqrt{2}\times\sqrt{3}\times\sqrt{6}}{1}=6$$

Example < problem1 >

G(s)H(s) has poles at 0,-1, -3, -5 : no finite zeros.

Sketch the root locus

1. Characteristic equation : The characteristic equation is

$$1+G(s)H(s)=0 \quad \text{or} \quad s(s+1)(s+3)(s+5)+K=0$$

$$\text{or} \quad s^4+9s^3+23s^2+15s+K=0$$

2. Beginning and ending points :

There are $N_p = 4$ loop poles (corresponding to K = 0)

There are $N_z = 0$ loop zeros (corresponding to $K = \infty$)

The RL starts at loop poles, ending at loop zeros.

The number of branches is $\max(N_p, N_z) = 4$.

The loop poles are at 0, -1, -3, and -5.

3. Root loci on real axis :

The root loci lie on the real axis wherever there is an odd number of poles and zeors to its right.

In this case, they lie on the interval [-5,-3] and [-1,0].

4. Asymptotes:

$\left|N_p - N_z\right| = 4$ branches must begin or end at infinity, so there must be that many asymptotes.

Their intersection with the real axis is at

$$\sigma_A = \frac{\sum(-P_i) - \sum(-Z_i)}{\left|N_p - N_z\right|}$$

or,

$$\sigma_A = \frac{0 + (-1) + (-3) + (-5) - 0}{\left|4 - 0\right|} = -2.25$$

The angles formed are

$$\Phi_A = \frac{(2k+1)\times 180^\circ}{N_p - N_z}, \qquad k = 0, 1, \cdots, \left|N_p - N_z\right| - 1$$

$$= \frac{(2k+1)\times 180^\circ}{4} = (2k+1)\times 45^\circ = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

5. Breakpoints:

Necessary condition for a breakpoint to occur is

$$\frac{dK}{ds} = 0$$

The characteristic equation, rearranged, is

$$K = -s^4 - 9s^3 - 23s^2 - 15s$$

Then,

$$\frac{dK}{ds} = -4s^3 - 27s^2 - 46s - 15 = 0$$

Solving for the breakpoints yields them as

$-0.4258, -2.070$, and -4.254

6. jw-axis crossing:

The limiting vlaue of K for stability can be found from the Routh array

s^4	1	23	K
s^3	9	15	
s^2	21.33	K	← auxiliary equation
s^1	15 - 0.422K		
s^0	K		

The s^{-1} – row becomes all-zero for $15 - 0.422K = 0$; i.e.,

when $K = 35.56$

The auxiliary equation is then

$$21.33s^2 + K = 0 \quad or \quad 21.33s^2 + 35.56 = 0$$

$$or \quad s = \pm j1.291$$

Thus, two branches of the root locus cross the $j\omega$-axis, one at $s = j1.291$ and the other at $s = -j1.291$

7. Roots for various values of K:

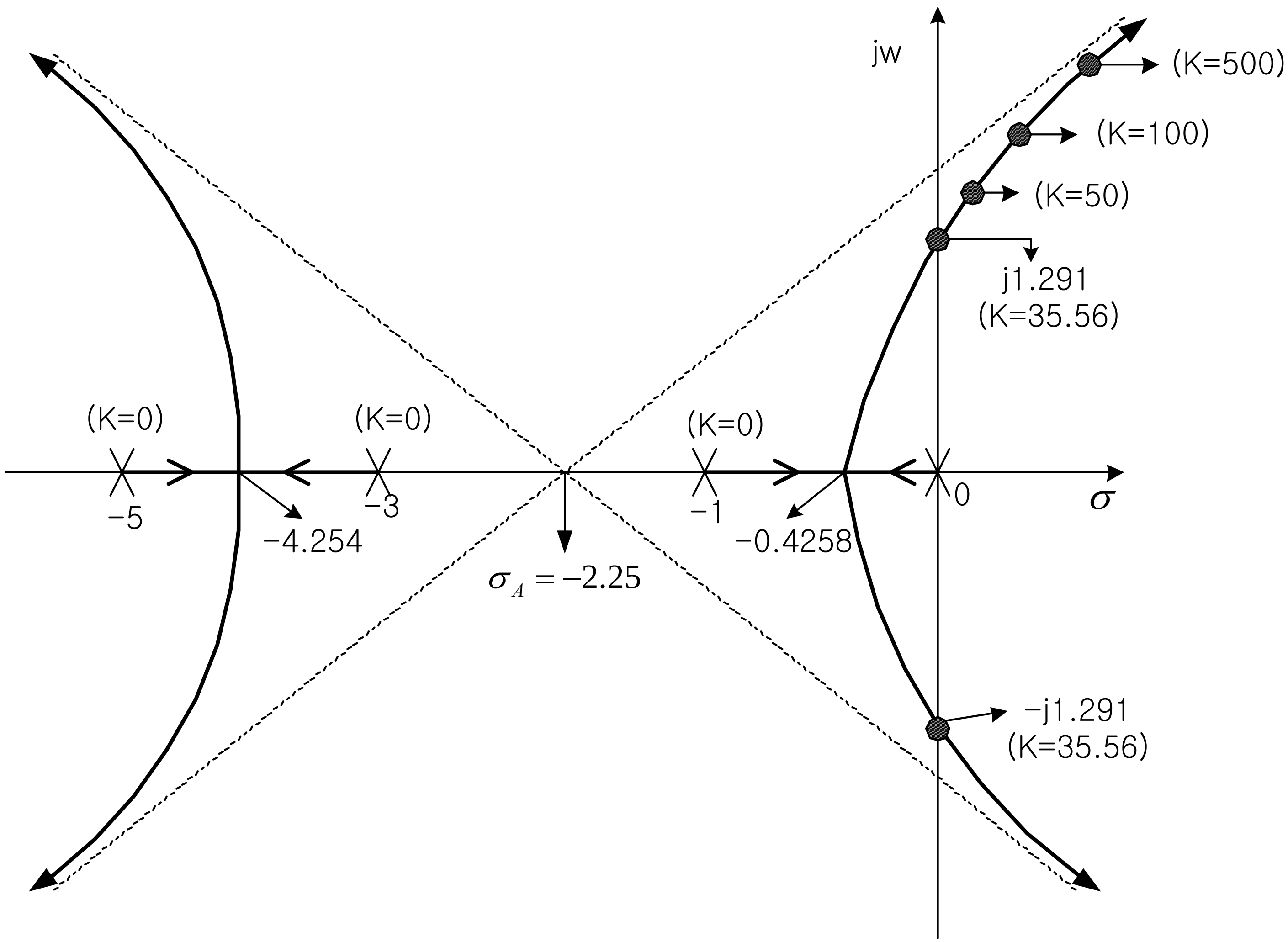
$K = 35.56$; $s = \pm 1.291$; $-4.50 \pm j1.041$

$K = 50$; $s = 0.1125 \pm j1.474$; $-4.612 \pm j1.271$

$K = 100$; $s = 0.3935 \pm j1.884$; $-4.894 \pm j1.747$

$K = 500$; $s = 1.368 \pm j3.096$; $-5.868 \pm j3.036$

obtained by Muller's Method.


jw
(K=500)
(K=100)
(K=50)
j1.291
(K=35.56)
(K=0)
(K=0)
(K=0)
−5
−3
−1
0
σ
−4.254
−0.4258
$\sigma_A = -2.25$
−j1.291
(K=35.56)

Example < Problem 2 >

G(s)H(s) has poles at 0, 0, 0, 1; zeros at -1, -2, -3

Sketch the root locus

$$G(s)H(s) = \frac{K(s+1)(s+2)(s+3)}{s^3(s-1)}$$

Characteristic equation :

$$1 + G(s)H(s) = 0 \quad or \quad s^3(s-1) + K(s+1)(s+2)(s+3) = 0$$

$$or \quad s^4 + (K-1)s^3 + 6Ks^2 + 11Ks + 6K = 0$$

Beginning and ending points :

The root loci lie on the real axis on the intervals [-∞,3],[-2,-1], and [0,1].

Asymptotes :

There is $\left|N_p - N_z\right| = |4-3| = 1$ asymptote.

Its angle $\Phi_A = \frac{(2k+1)\times 180^\circ}{1},\ k = 0$

$= 180^\circ$

Breakpoints:

Solve $\frac{dK}{ds} = 0$ for possible breakpoints.

The characteristic equation is

$$K = \frac{-s^3(s-1)}{(s+1)(s+2)(s+3)} = \frac{-s^4+s^3}{s^3+6s^2+11s+6}$$

$$\frac{dK}{ds} = \frac{s^6+12s^5+27s^4+2s^3-18s^2}{(s^3+6s^2+11s+6)^2} = 0 \quad or \quad s^6+12s^5+27s^4+2s^3-18s^2 = 0$$

Solving get the breakaway points as

-9.072, -2.400, -1.210, 0, 0, 0.6830

jw-axis crossing: use the Routh array.

s^4	1	$6K$	$6K$
s^3	$K-1$	$11K$	
s^2	$6K-\frac{11K}{K-1}$	$6K$ ← auxiliary equation	
s^1	$11K-\frac{6K(K-1)}{6K-\frac{11K}{K-1}}$		
s^0	$6K$		

The s^1 – row becomes all - zero for

$$11K - \frac{6K(K-1)}{6K - \frac{11K}{K-1}} = 0 \quad or \quad 10K^2 - \frac{175}{6}K - 1 = 0$$

solving for K gives K = -0.0389, 2.951

The auxiliary equation is

$$\left[6K - \frac{11K}{K-1}\right]s^2 + 6K = 0 \quad K = 2.951$$

$$or \;\; 1.068s^2 + 17.71 = 0 \qquad or \quad s = \pm 4.072$$

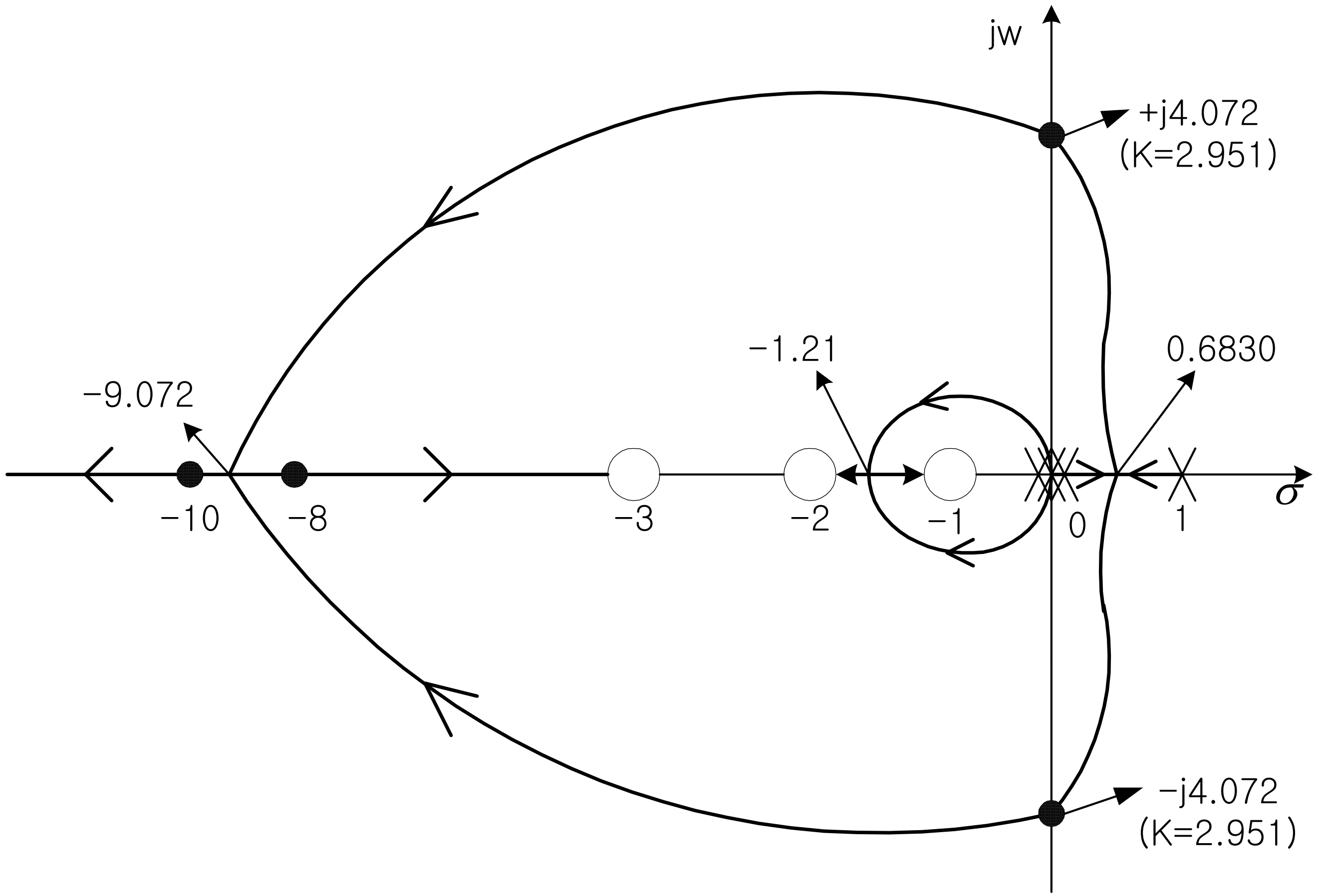
For

K = 1 : s = -0.7912 ± j0.3878 : 0.7912 ± j2.664

K = 5 : s = -1.063 ± j0.2821 : - 0.9371 ± j4.891

K = 20 : s = -1.460 : -1.071 : -8.235 ± j2.991

K = 50 : s = -42.20 : - 4.077 : -1.706 : -1.022


jw
+j4.072
(K=2.951)
−1.21
0.6830
−9.072
−10
−8
−3
−2
−1
0
1
σ
−j4.072
(K=2.951)

The effect of adding poles and zeros

1. Adding poles in L.H.P. causes RL to be moved toward R.H.P.

ex) A) $G(s)H(s) = \dfrac{K}{s(s+a)}, \quad a > 0$

B) Adding a real pole

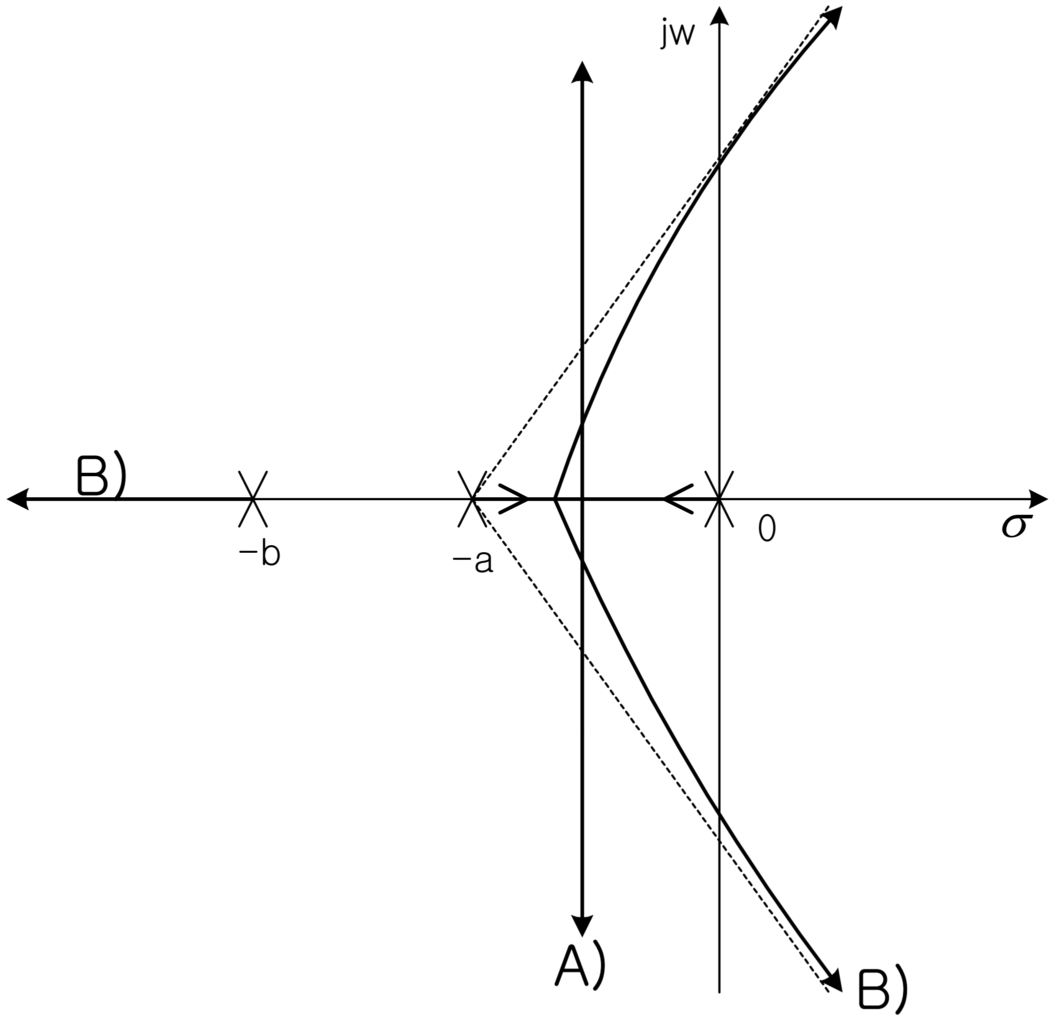
$$G(s)H(s) = \frac{K}{s(s+a)(s+b)}, \quad b > a > 0$$

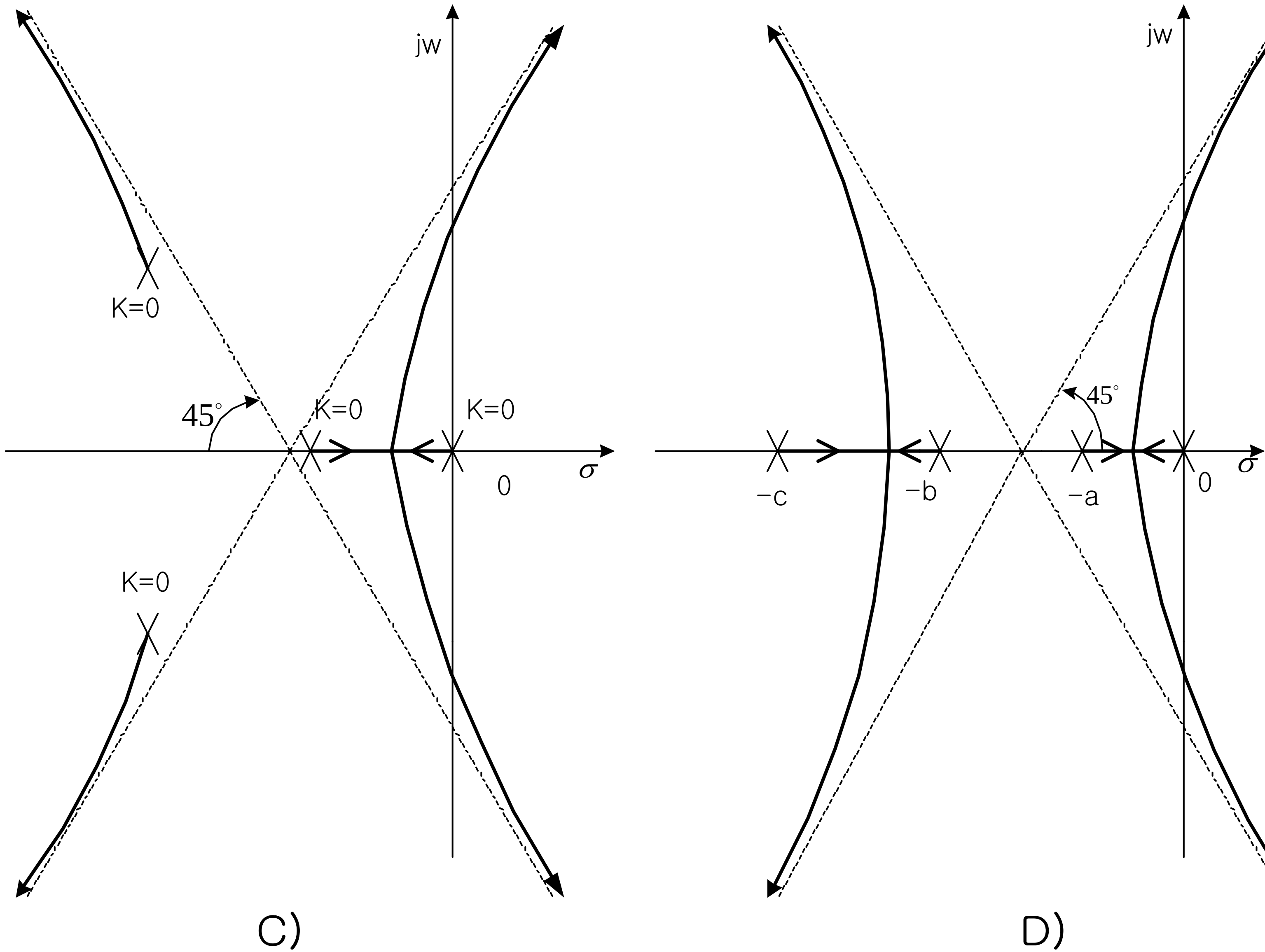
C) Adding complex poles

$$G(s)H(s) = \frac{K}{s(s+a)(s+c+jd)(s+c-jd)}, \quad c > a > 0$$

D) Adding another pole

$$G(s)H(s) = \frac{K}{s(s+a)(s+b)(s+c)}, \quad c > b > a > 0$$


jw
B)
−b
−a
0
σ
A)
B)


jw
K=0
45°
K=0
K=0
0
σ
K=0
C)
jw
45°
σ
−c
−b
−a
0
D)

2. Adding zeros in L.H.P. causes RL to be moved toward L.H.P.

A) $G(s)H(s) = \dfrac{K}{s(s+a)}, \quad a > 0$

B) Adding a real zero, $b > a > 0$

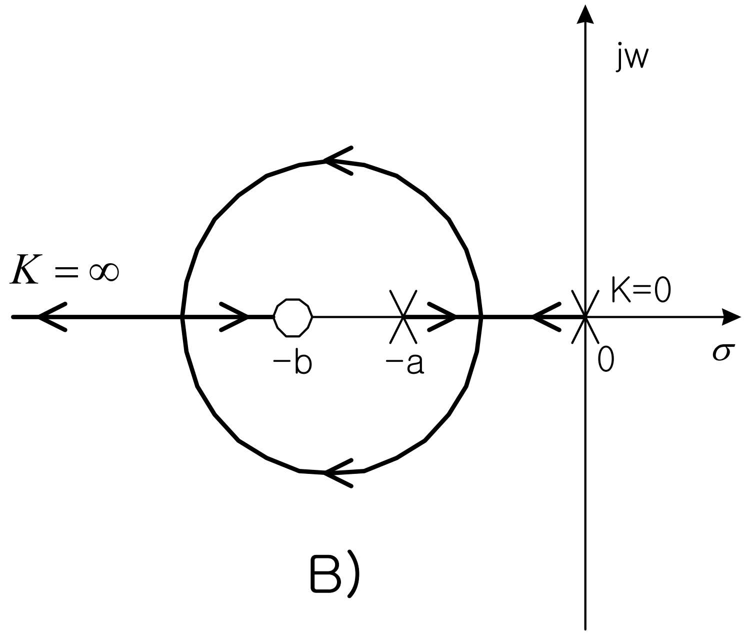
$$G(s)H(s) = \frac{K(s+b)}{s(s+a)}$$

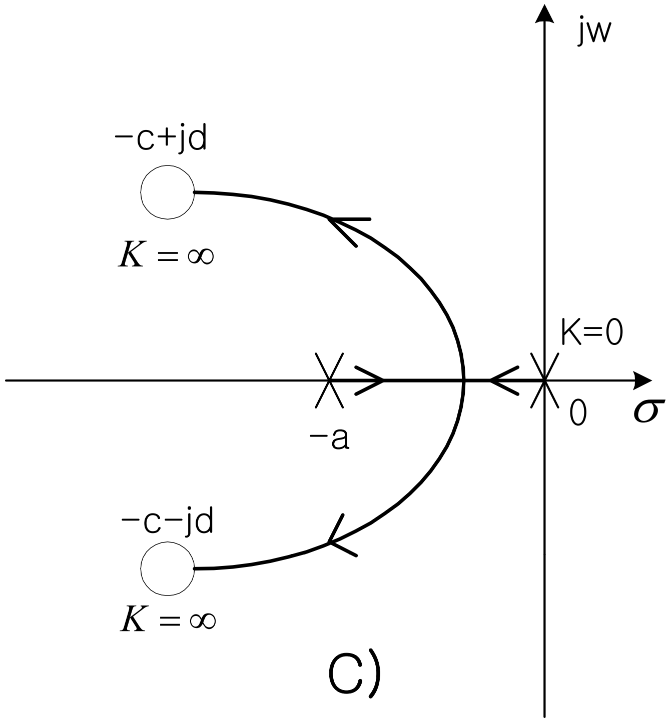
C) Adding complex zeros, $c > a > 0$

$$G(s)H(s) = \frac{K(s+c+jd)(s+c-jd)}{s(s+a)}$$

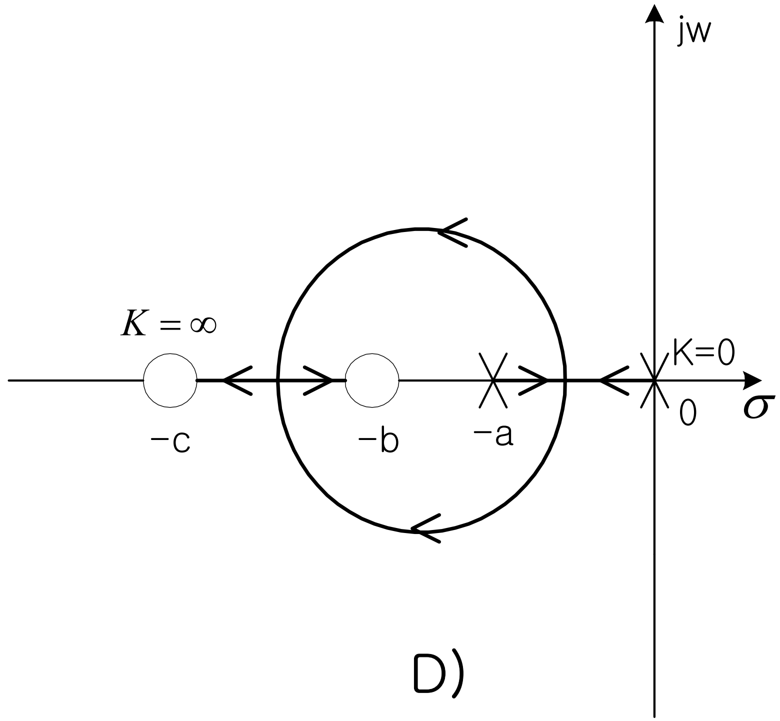
D) Adding two zeros, $c > b > a > 0$

$$G(s)H(s) = \frac{K(s+b)(s+c)}{s(s+a)}$$





C)

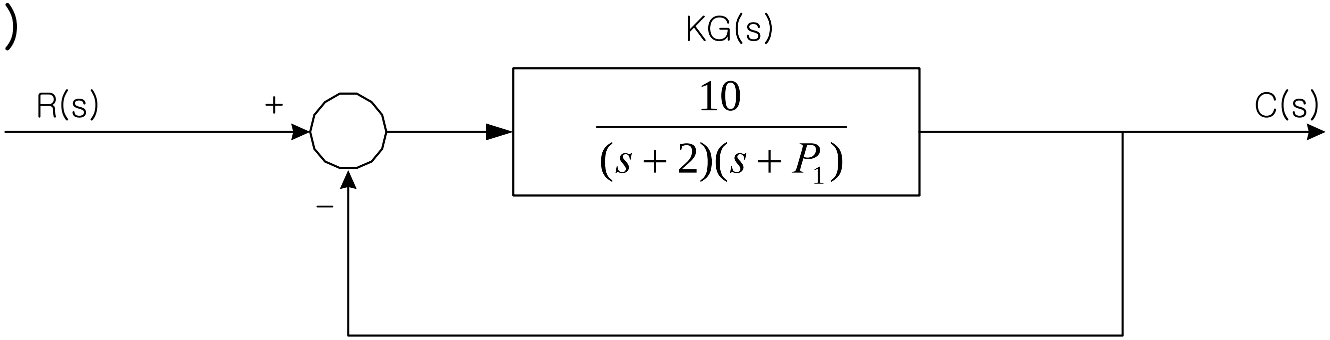


D)

Generalized Root Locus

Root Locus for variations of the open-loop pole value of P_1

Ex)



$$KG(s) = \frac{10}{(s+2)(s+P_1)} \quad H(s) = 1$$

$$KG(s)H(s) = \frac{10}{(s+2)(s+P_1)}$$

$$T(s) = \frac{KG(s)}{1+KG(s)H(s)} = \frac{\frac{10}{(s+2)(s+P_1)}}{1+\frac{10}{(s+2)(s+P_1)}} = \frac{10}{s^2+(P_1+2)s+2P_1+10}$$

$$= \frac{10}{s^2+2s+10+P_1(s+2)}$$

$$T(s) = \frac{\dfrac{10}{s^2+2s+10}}{1+\dfrac{P_1(s+2)}{s^2+2s+10}}, \quad KG(s) = \frac{10}{s^2+2s+10}$$

$$KG(s)H(s) = \frac{P_1(s+2)}{s^2+2s+10}$$

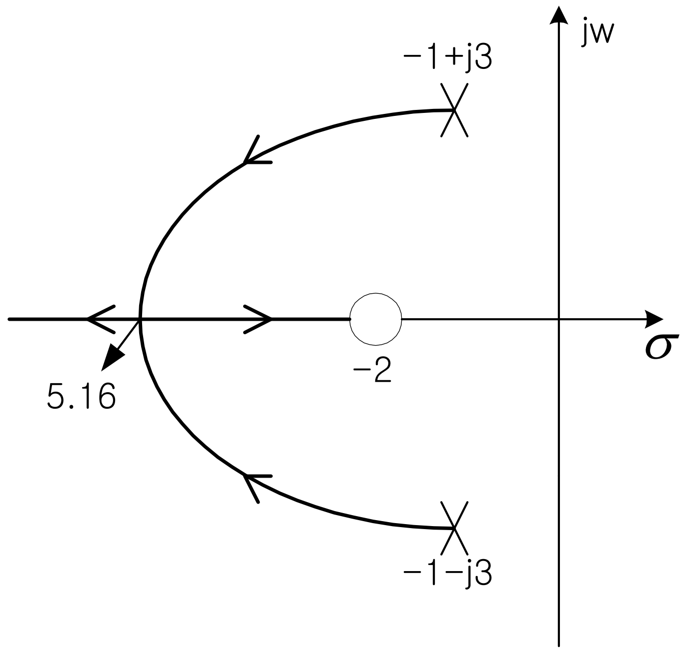
$$P_1 = \frac{-(s^2+2s+10)}{s+2}$$

$$\frac{dP_1}{ds} = \frac{-s^2-4s+6}{(s+2)^2} = 0$$

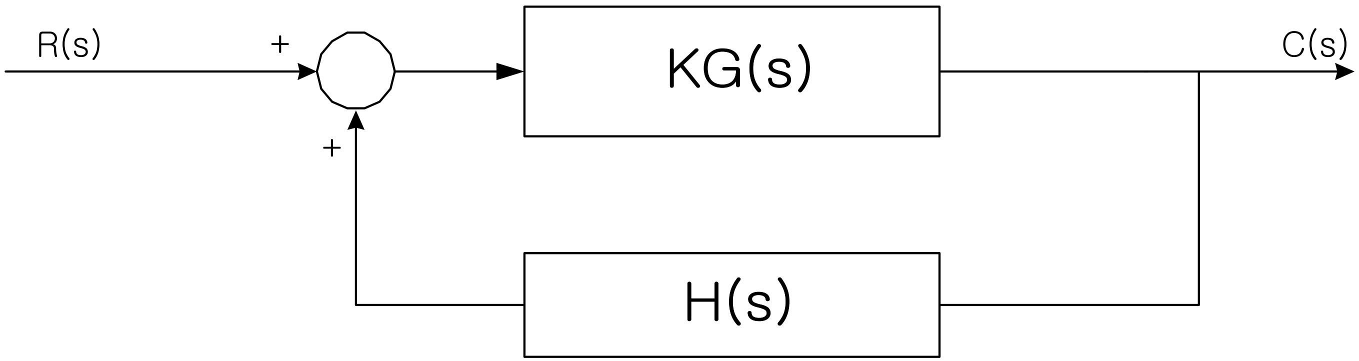
$$s^2+4s-6=0$$

$$s=-2\pm 3.16$$

$$s=-5.16 \quad and \quad s=1.16$$



Root Locus for Positive-Feedback systems or (complementary root locus)



$$T(s) = \frac{KG(s)}{1 - KG(s)H(s)}$$

$$KG(s)H(s) = 1 = 1\angle(k360^\circ), \quad k = 0, \pm 1, \pm 2, \pm 3$$

Negative-feedback $\rightarrow$ Positive feedback

1. No change

   Beginning and ending points

   Asymptotes intersection σ_A

   Number of branches $\max(N_P, N_z)$

   Symmetry

2. Change

Real axis segments (or RL on the real axis)

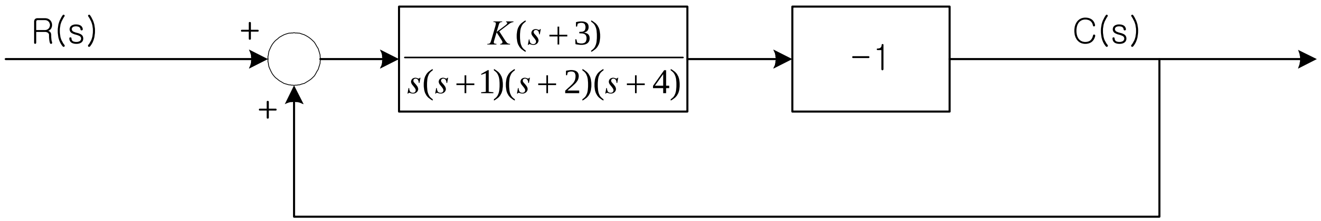
The root loci lie on the real axis wherever there is an <u>even</u> number of poles and end zeros to its right (0 is considered even number)

Asymptotes angle

$$\Phi_A = \frac{k \times 360^\circ}{\left|N_p - N_z\right|} \qquad k = 0, 1, \cdots, \left|N_p - N_z\right| - 1$$

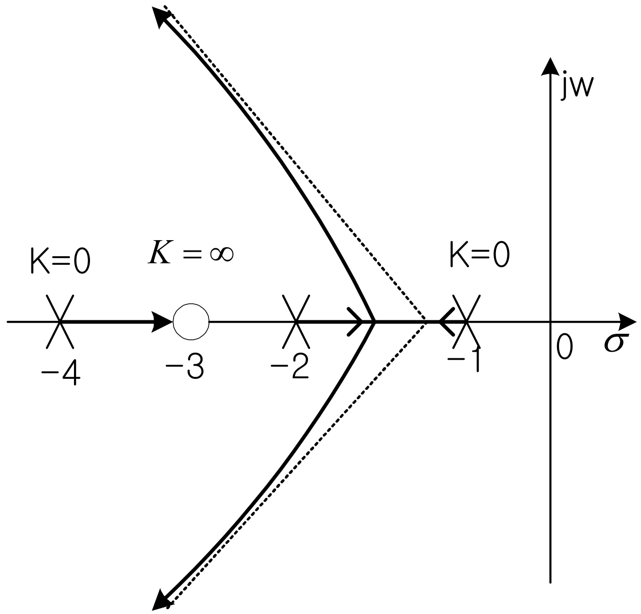
Breakaway point

Ex. Sketch the root locus as a function of negative gain, K, for the following system.



$$\sigma_A = \frac{(-1-2-4)-(-3)}{|4-1|} = -\frac{4}{3}$$

$$\Phi_A = \frac{k \times 360^\circ}{|N_p - N_z|} \Rightarrow 120^\circ, 240^\circ, 360^\circ$$



* Matlab

Problem 1.

1) Problem

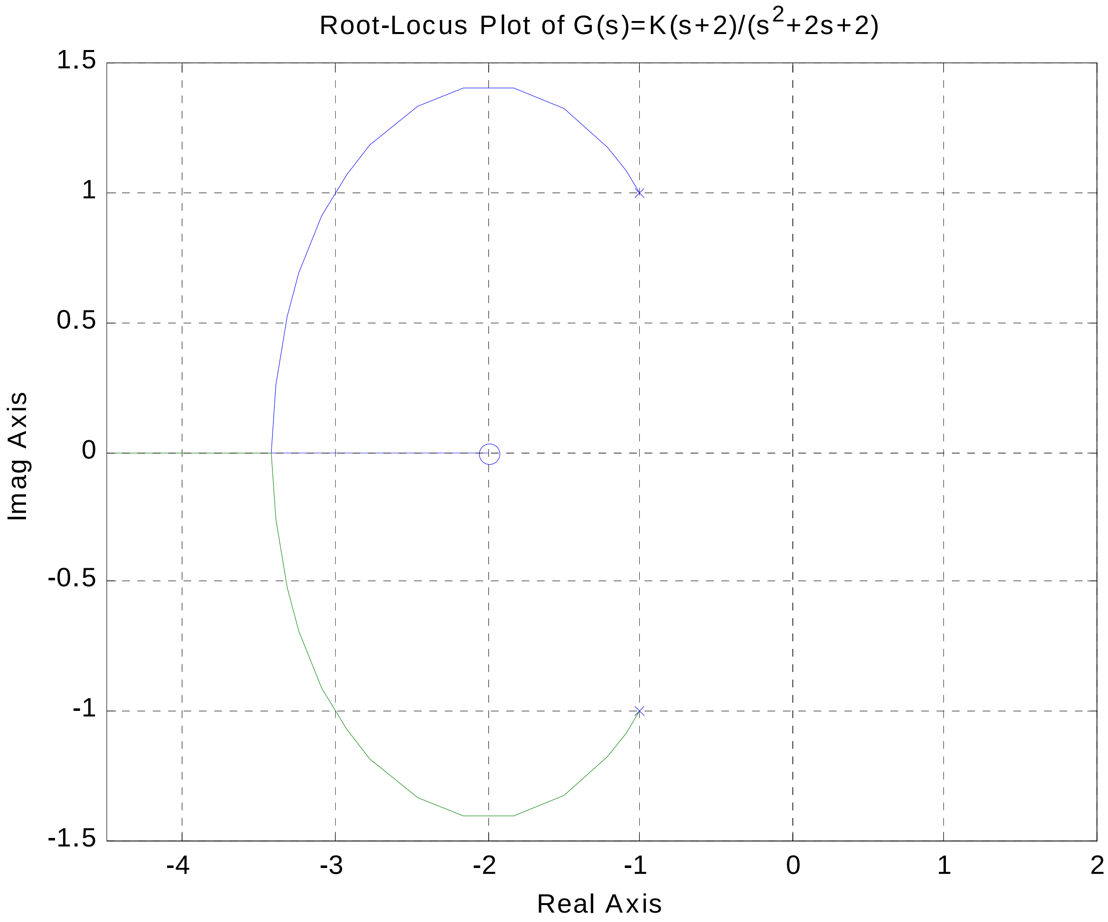
Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K(s+2)}{s^2+2s+2} \rightarrow \text{Poles at s = -1} \pm \text{j1, Zeros at s = -2}$$

2) Source code

```
num = [1 2];
den = [1 2 2];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s+2)/(s^2+2s+2)')
```

3) Plot Printout



Problem 2.

1) Problem

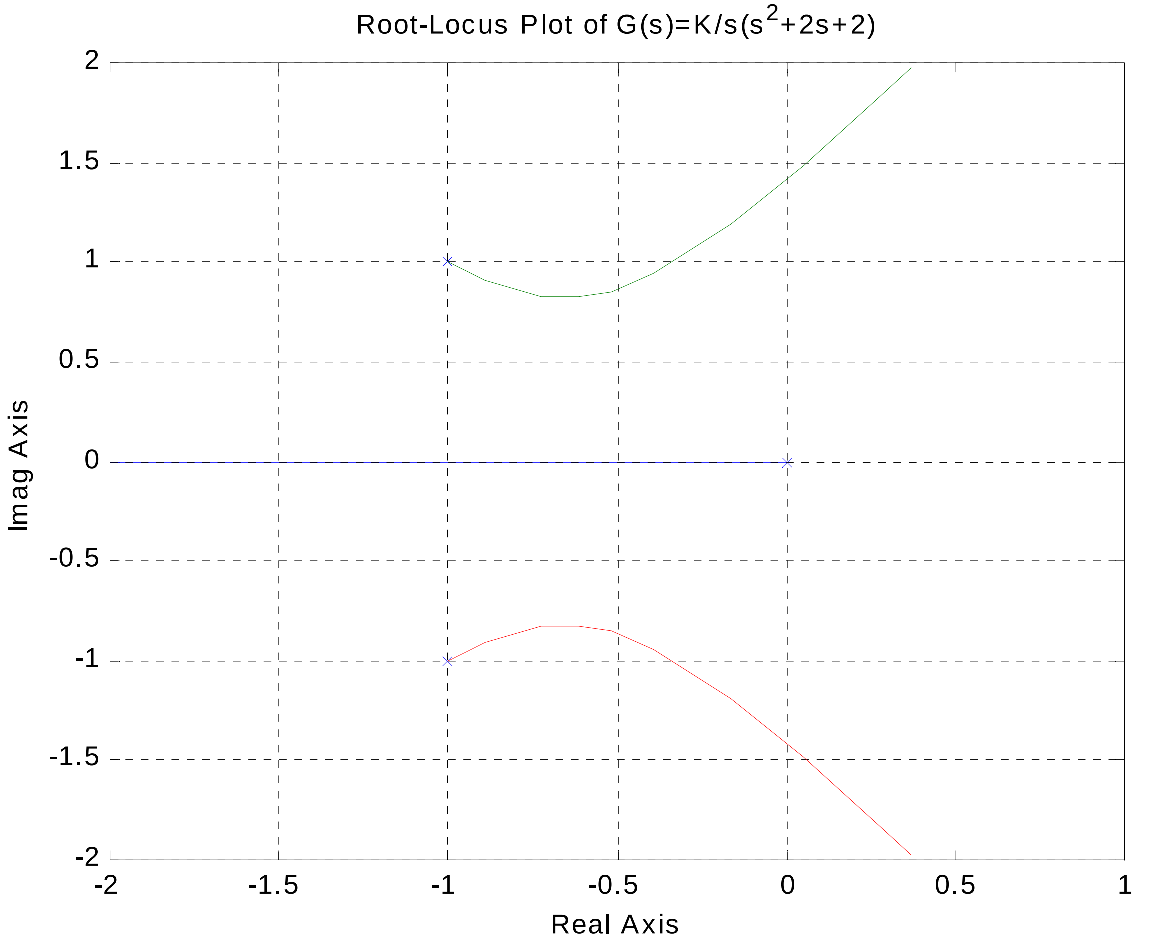
Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K}{s(s^2 + 2s + 2)} \rightarrow \text{Poles at s} = 0,\ -1 \pm j1,\ \text{No Zero}$$

2) Source code

```
num = [1];
den = [1 2 2 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K/s(s^2 + 2s + 2)')
```

3) Plot Printout



Problem 3.

1) Problem

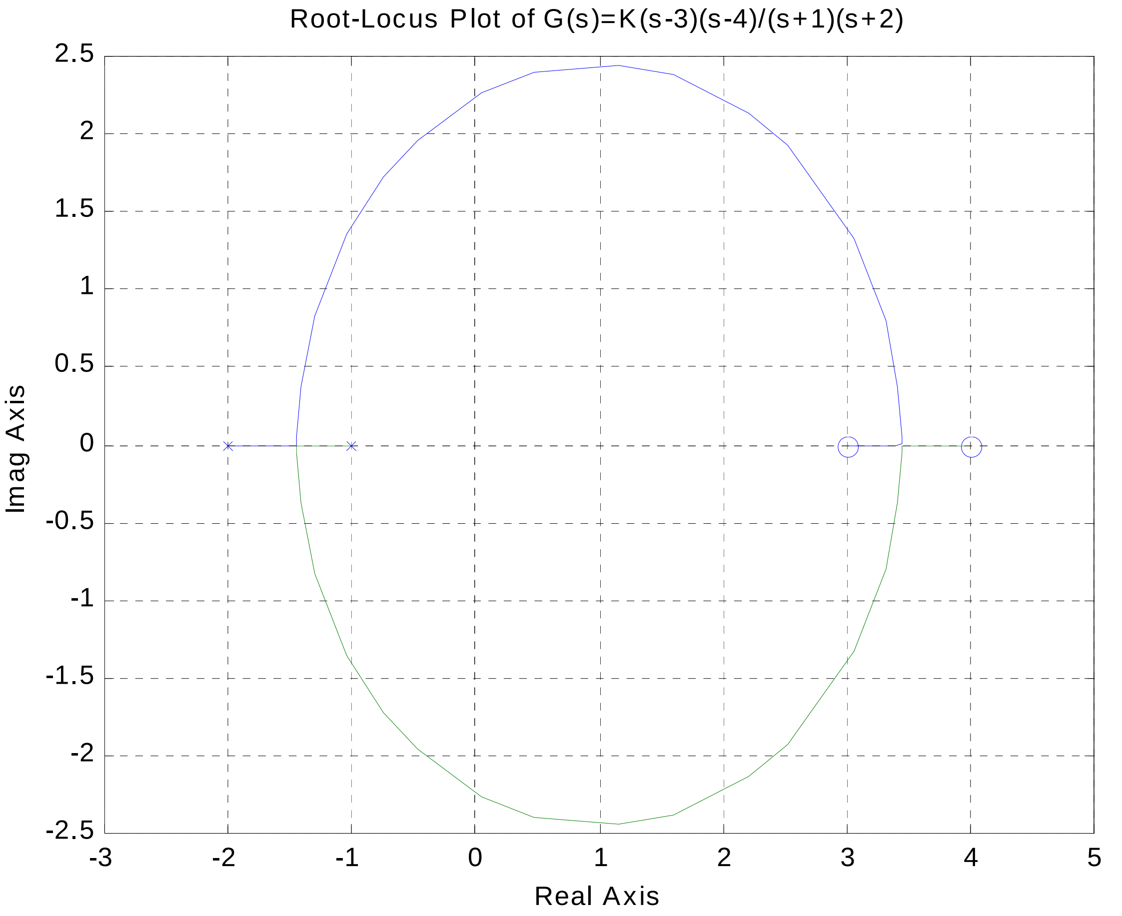
Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K(s-3)(s-4)}{(s+1)(s+2)} \rightarrow \text{Poles at s = -1,-2}, \ \text{Zeros at s = 3,4}$$

2) Source code

```
num = [1 -7 12];
den = [1 3 2];
rlocus(num, den)
grid
title('Root-Locus Plot of G(s) = K(s-3)(s-4)/(s+1)(s+2)')
```

3) Plot Printout



Problem 4.

1) Problem

Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K}{s(s+a)(s^2+4s+20)} \rightarrow \text{Poles at s} = 0, -a, -2 \pm j4\text{, No Zero}$$

a) a = 5 b) a = 4.2 c) a = 4 d) a = 3.8 e) a = 3

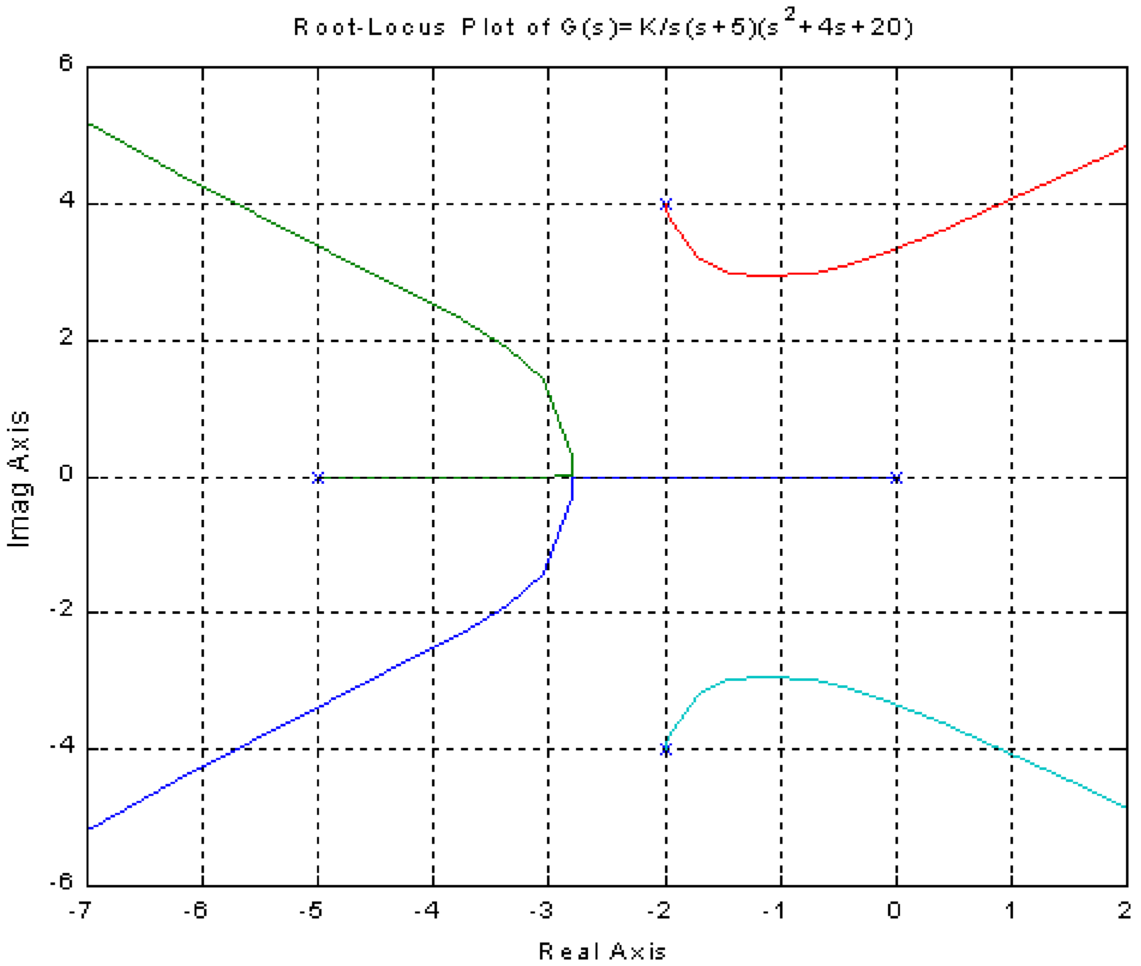
2) Source code

a) a = 5

```
num = [1];
den = [1 9 40 100 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K/s(s+5)(s^2+4s+20)')
```

3) Plot Printout

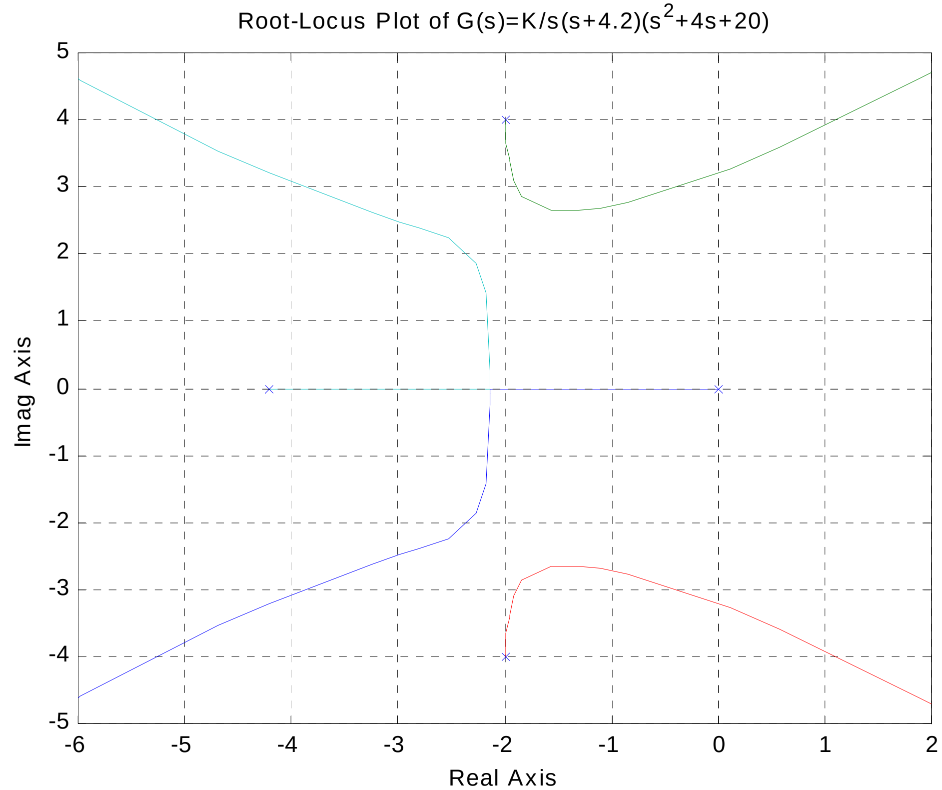
a) a=5일 경우



2) Source code

b) a = 4.2

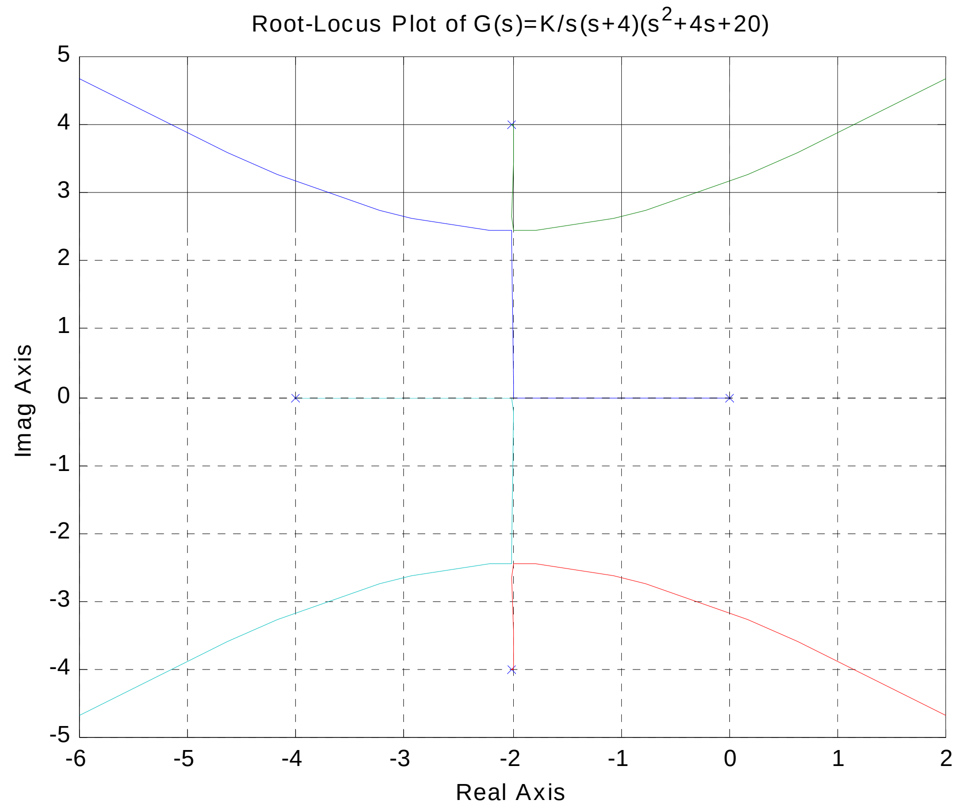
```
num = [1];
den = [1 8.2 36.8 84 0];
rlocus(num, den)
grid
title('Root-Locus Plot of G(s) = K/s(s + 4.2)(s^2 + 4s + 20)')
```



2) Source code

c) a = 4

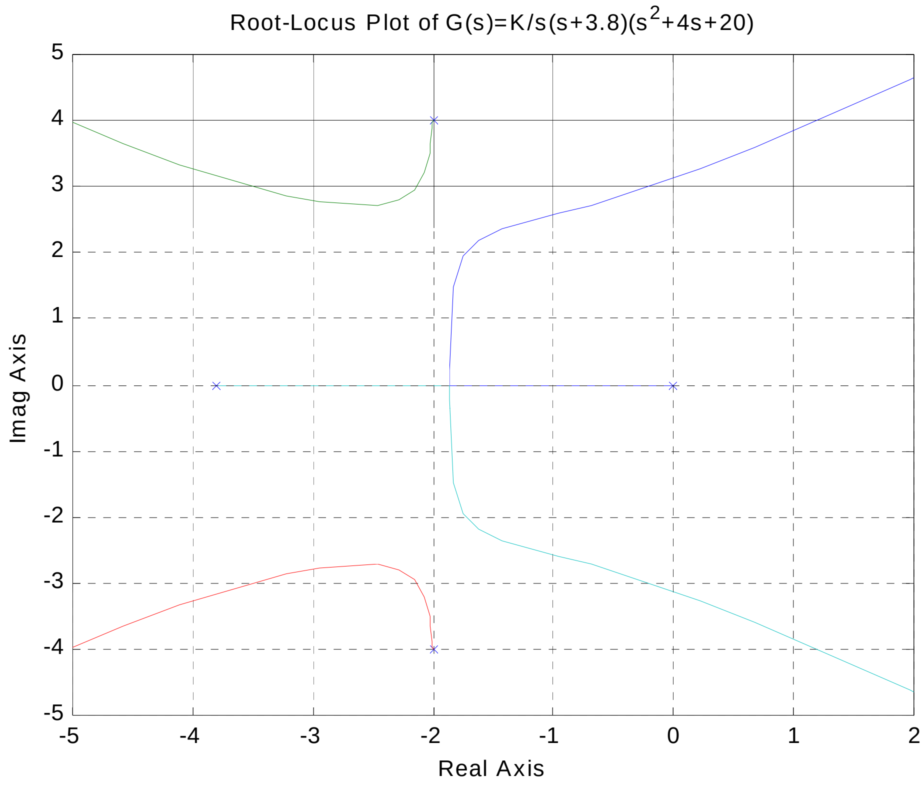
```
num = [1];
den = [1 8 36 80 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K/s(s + 4)(s^2 + 4s + 20)')
```



2) Source code

d) a = 3.8

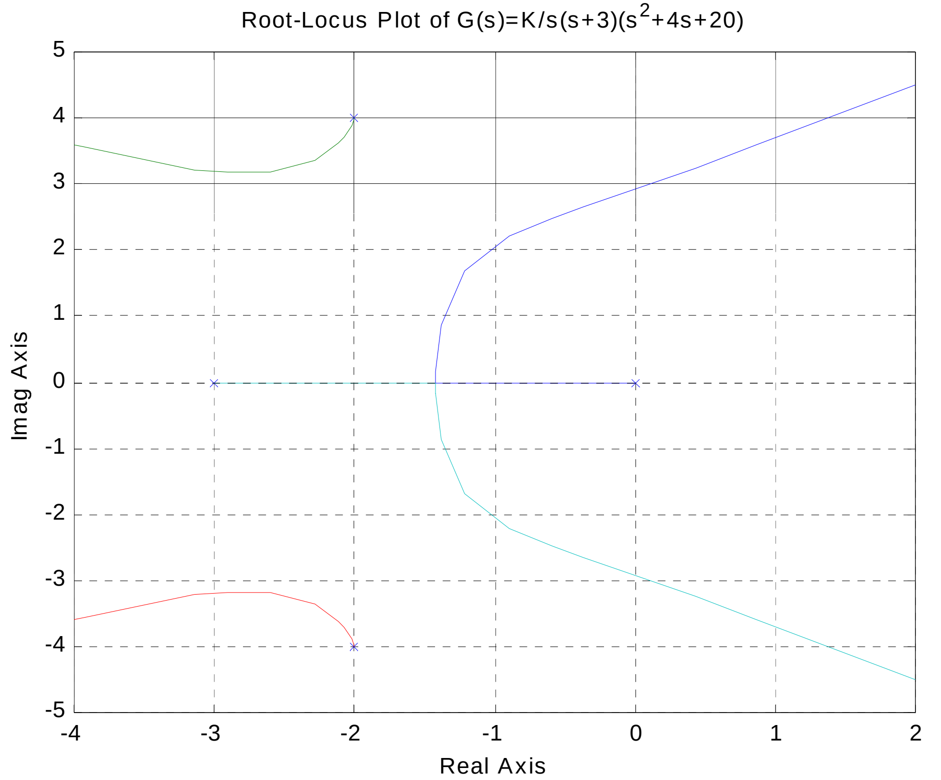
```
num = [1];
den = [1 7.8 35.2 76 0];
rlocus(num, den)
grid
title('Root-Locus Plot of G(s) = K/s(s+3.8)(s^2+4s+20)')
```



2) Source code

e) $a = 3$

```
num = [1];
den = [1 7 32 60 0];
rlocus(num, den)
grid
title('Root-Locus Plot of G(s) = K/s(s+3)(s^2+4s+20)')
```



Problem 5.

1) Problem

Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)} \rightarrow \text{Poles at s = 0, -1, -2, -4, Zeros at s = -3}$$

a) $0 < K < \infty$: Root Locus

b) $-\infty < K < 0$: Complementary Root Locus

2) Source code

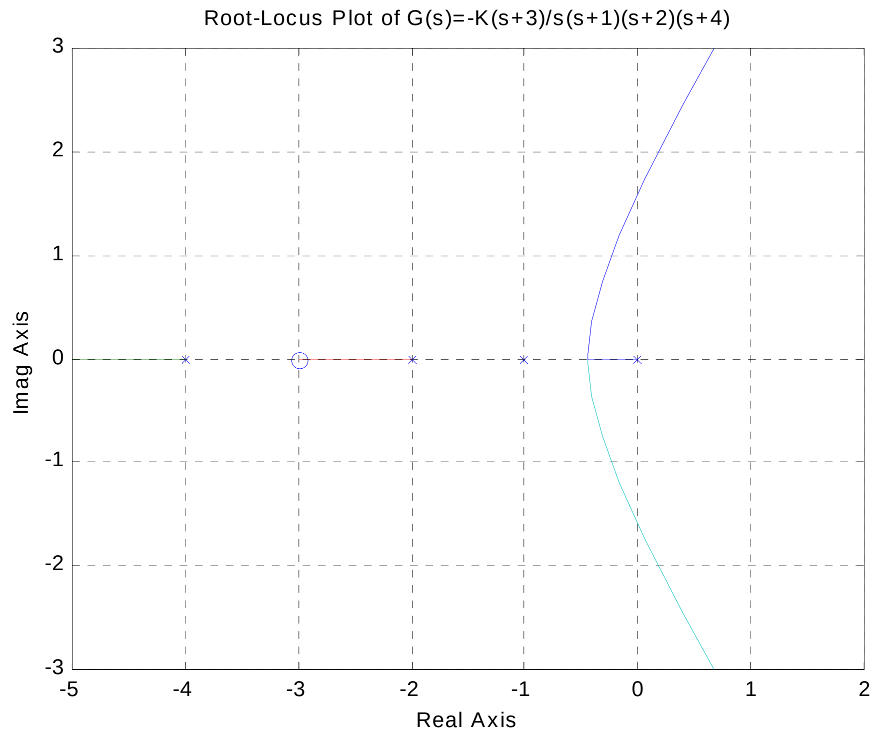
a) $0 < K < \infty$: Root Locus

```
num = [1 3];
den = [1 7 14 8 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s + 3)/s(s + 1)(s + 2)(s + 4)')
```

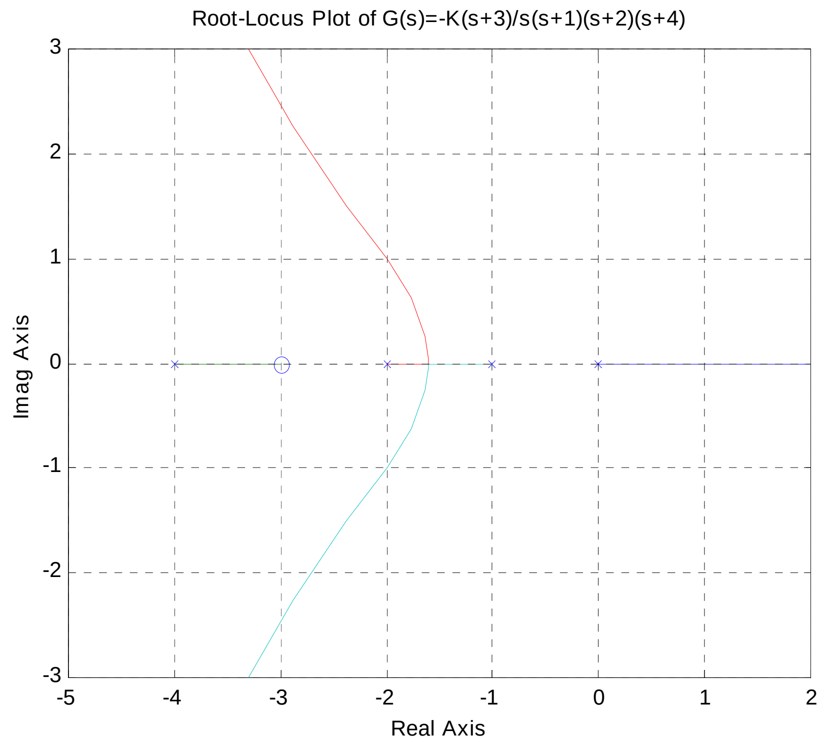
b) $-\infty < K < 0$: Complementary Root Locus

```
num = [-1 -3];
den = [1 7 14 8 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = -K(s + 3)/s(s + 1)(s + 2)(s + 4)')
```

a)Root Locus



b) Complementary Root Locus



Problem 6.

1) Problem

Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K(s+a)}{s^2(s+2)(s+4)} \rightarrow \text{Poles at s = 0(multi pole), -2, -4 Zeros at s = -a}$$

a) a = 1 b) a = 1.8 c) a = 2.2

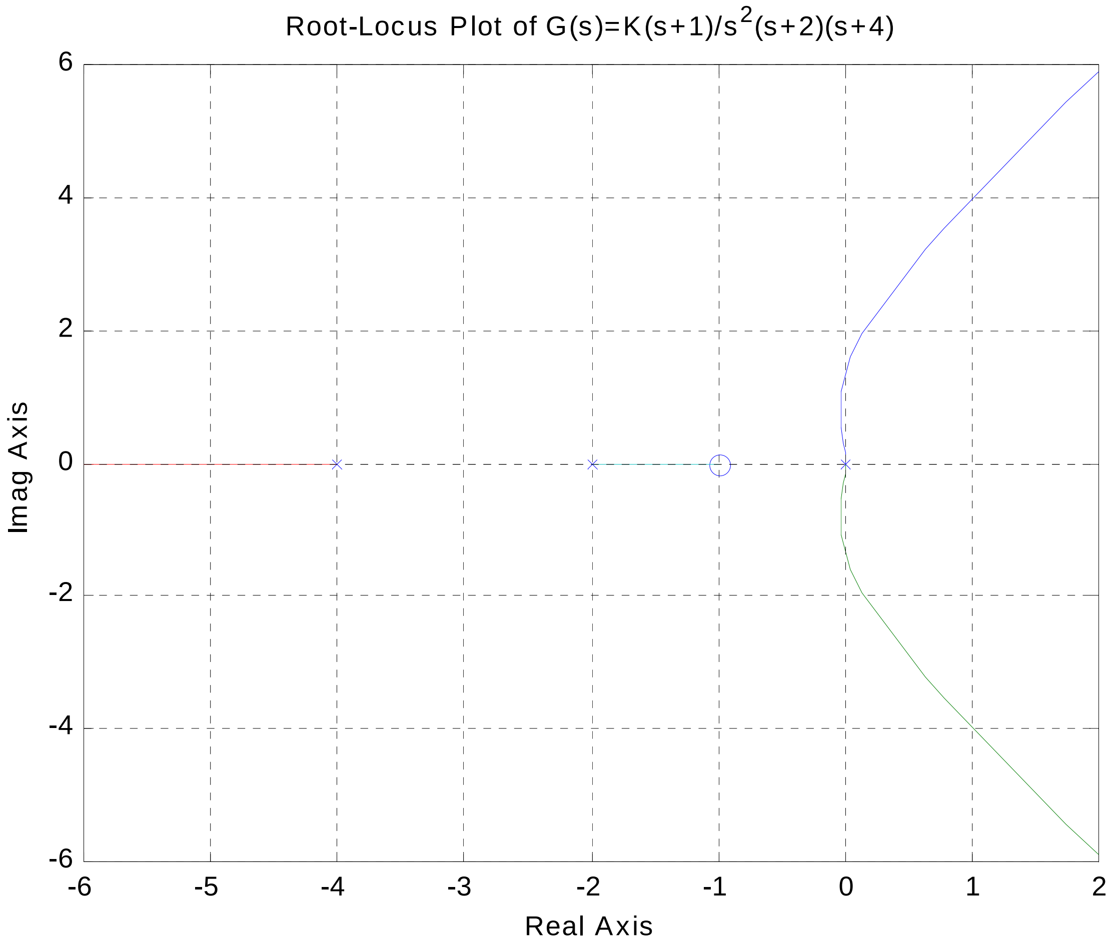
2) Source code

a) a = 1

```
num = [1 1];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s+1)/s^2(s+2)(s+4)')
```

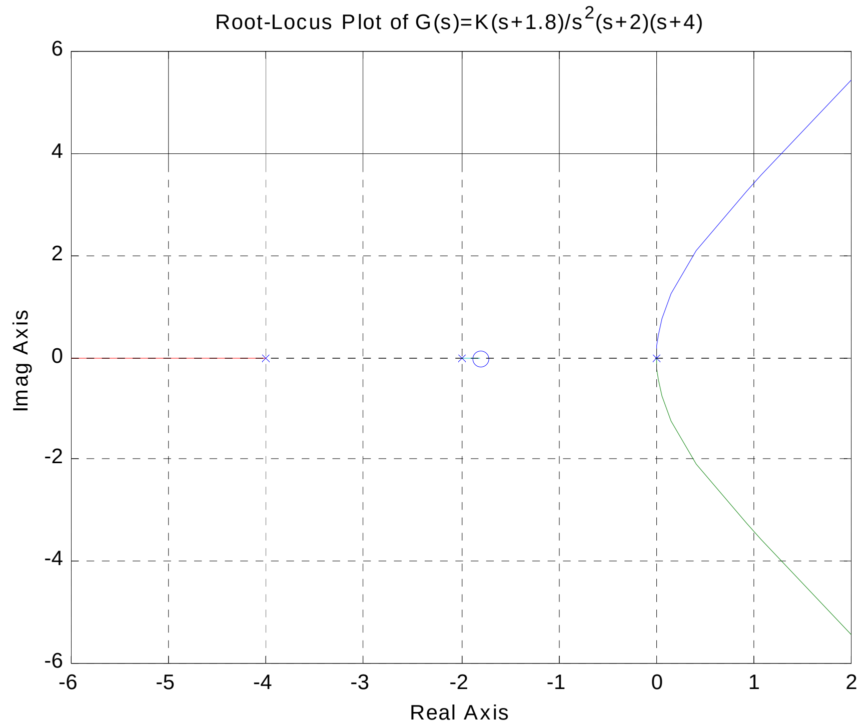
3) Plot Printout

a) a=1일 경우



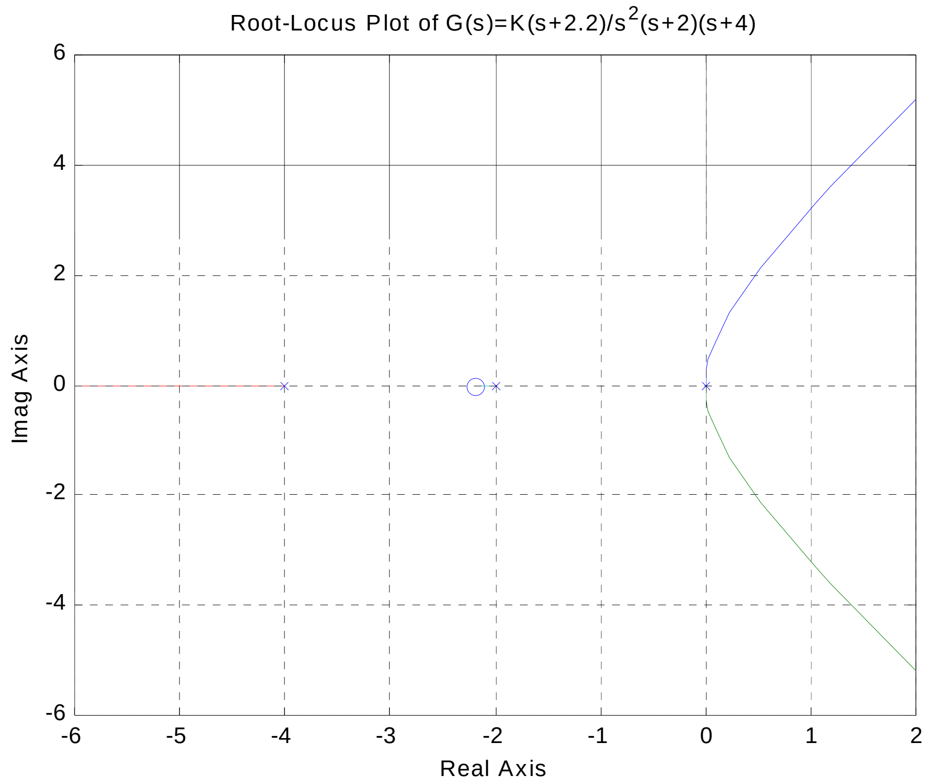
b) a=1.8일 경우

```
num = [1 1.8];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s + 1.8)/s^2(s + 2)(s + 4)')
```



c) a=2.2일 경우

```
num = [1 2.2];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s + 2.2)/s^2(s + 2)(s + 4)')
```



Problem 7.

1) Problem

Plot the Root - Locus for untiy feedback

$$KG(s) = \frac{K(s+1)(s+a)}{s^2(s+2)(s+4)} \rightarrow \text{Poles at s = 0(multi pole),-2, - 4, Zeros at s = -1, - a}$$

a) a = 1.5 b) a = 1.7 c) a = 1.9 d) a = 2 e) a = 2.1 f) a = 2.7 g) a = 3

2) Source code

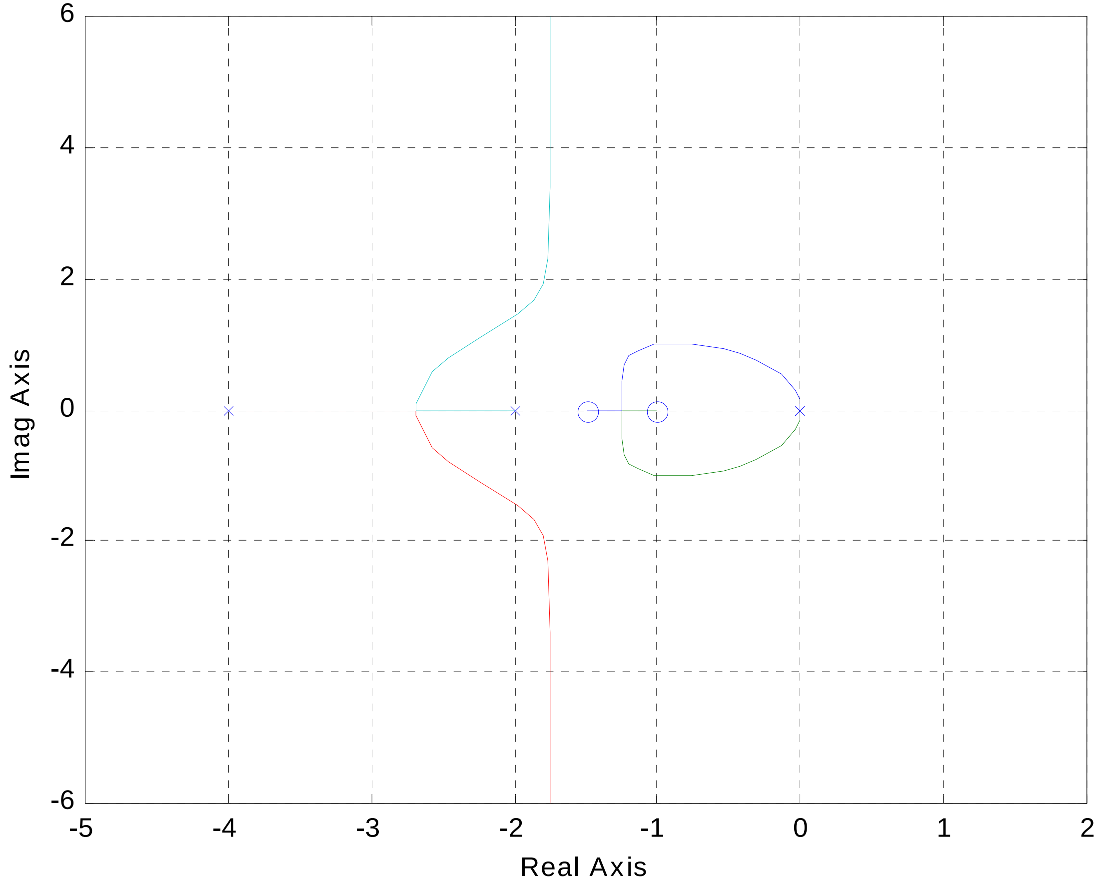
a) a = 1.5

```
num = [1 2.5 1.5];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s+1)(s+1.5)/s^2(s+2)(s+4)')
```

3) Plot Printout

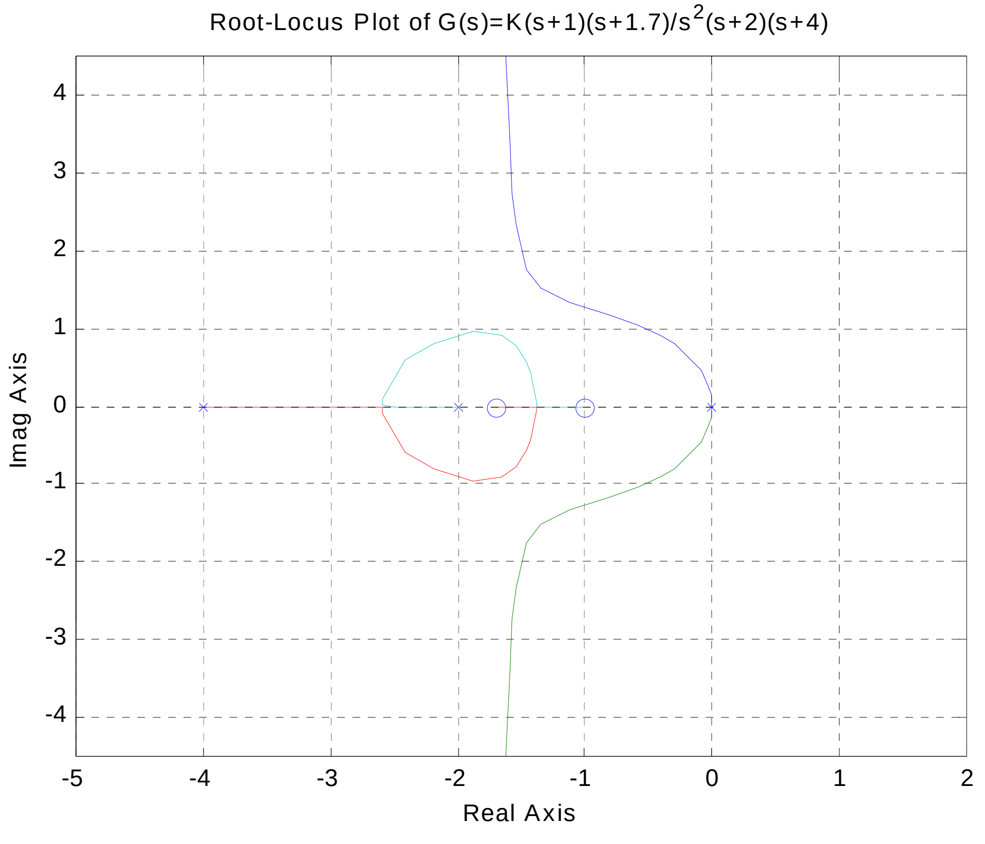
a) a=1.5일 경우

Root-Locus Plot of G(s)=K(s+1)(s+1.5)/s^2(s+2)(s+4)



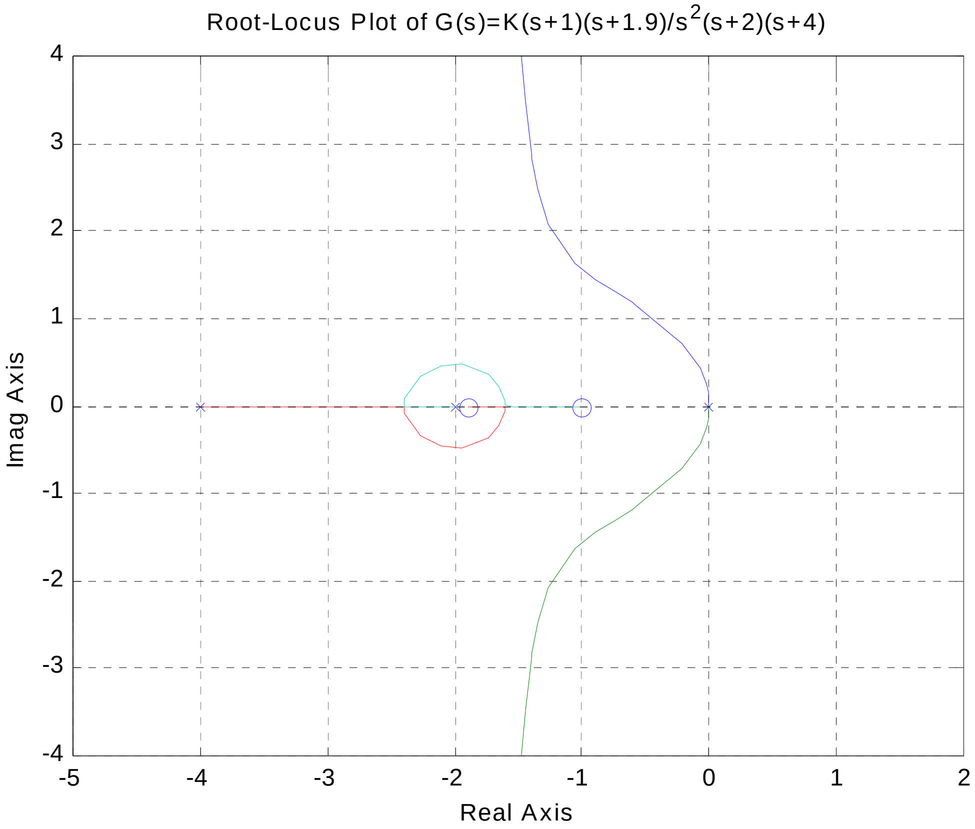
b) a = 1.7

```
num = [1 2.7 1.7];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s + 1)(s + 1.7)/s^2(s + 2)(s + 4)')
```



c) $a = 1.9$

```
num = [1 2.9 1.9];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s+1)(s+1.9)/s^2(s+2)(s+4)')
```



d) $a = 2$

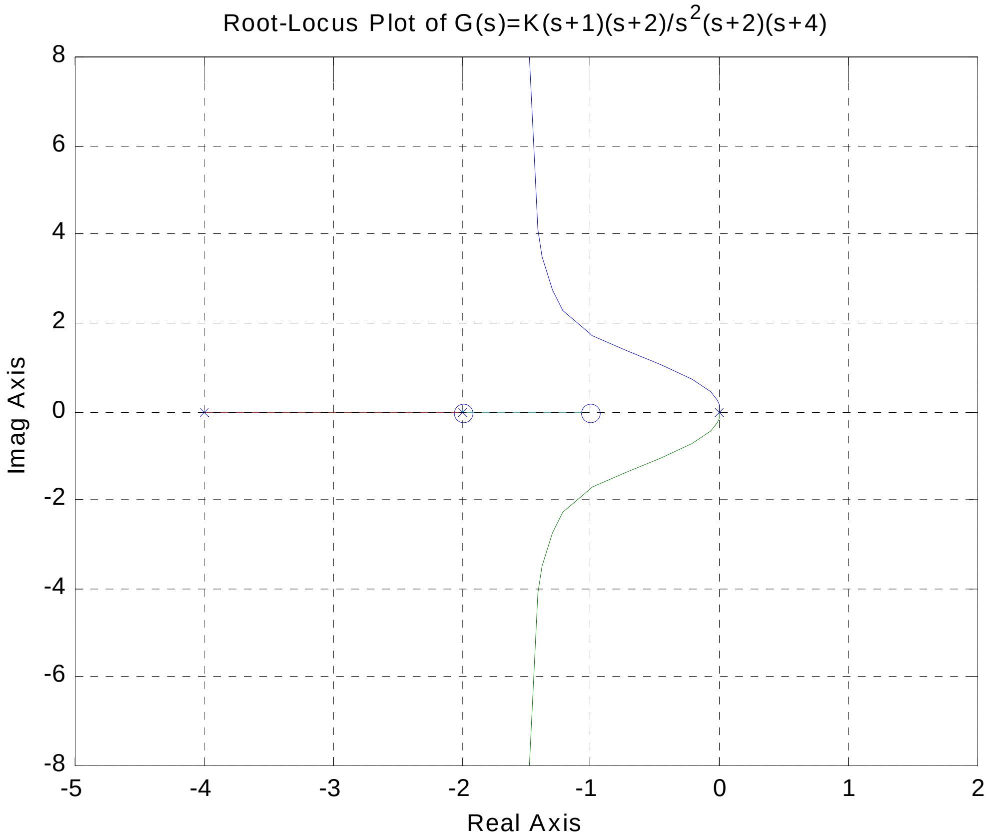
num = [1 3 2];

den = [1 6 8 0 0];

rlocus(num, den)

grid

title('Root - Locus Plot of G(s) = K(s + 1)(s + 2)/s^2(s + 2)(s + 4)')



e) $a = 2.1$

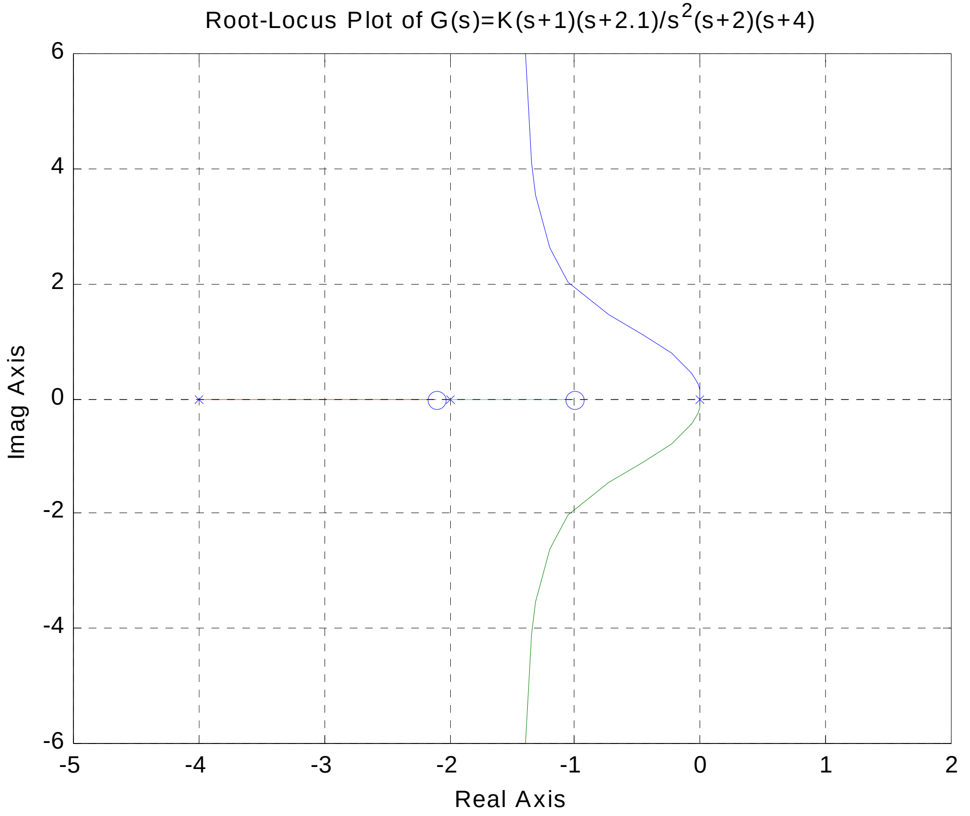
num = [1 3.1 2.1];

den = [1 6 8 0 0];

rlocus(num, den)

grid

title('Root - Locus Plot of G(s) = K(s+1)(s+2.1)/s^2(s+2)(s+4)')



f) a = 2.7

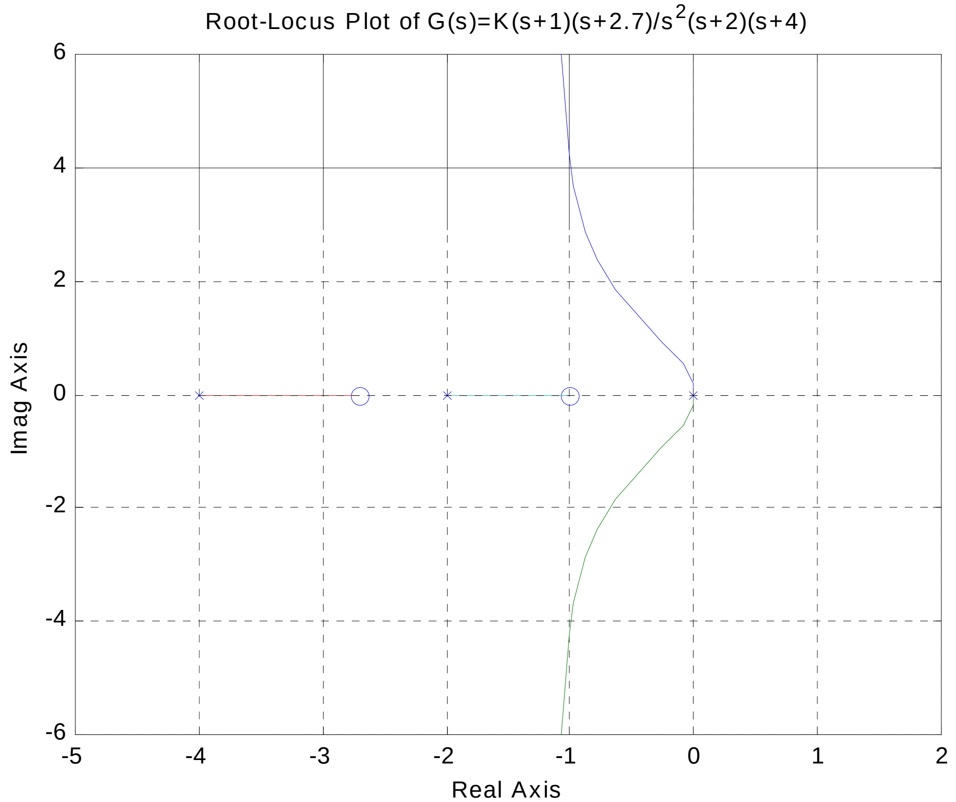
num = [1 3.7 2.7];

den = [1 6 8 0 0];

rlocus(num, den)

grid

title('Root - Locus Plot of G(s) = K(s + 1)(s + 2.7)/s^2(s + 2)(s + 4)')



g) a = 3

```
num = [1 4 3];
den = [1 6 8 0 0];
rlocus(num, den)
grid
title('Root - Locus Plot of G(s) = K(s+1)(s+3)/s^2(s+2)(s+4)')
```